Slope Fields

Goal:
• Can sketch a rough slope field by hand • Can use technology to draw slope fields and predict solutions to differential equations.
Terminology:
• Slope field

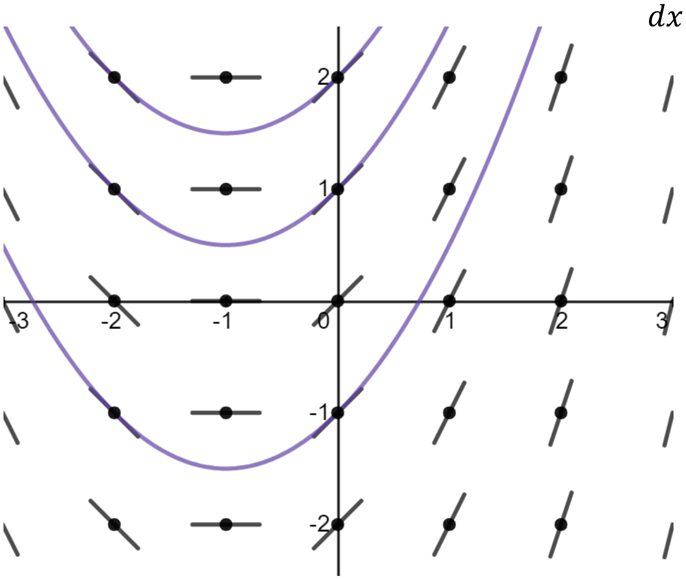
Discussion question: What would the solution curve $y(x)$ look like if $y(0) = 0$

$$\frac{dy}{dx} = y - x$$

If we are given a differential equation, we can still guess what the solution curve will look like and even make predictions about future values despite not being able to solve the differential equation at times. This is the power of **slope fields**.

Example: Determine the solution to the following differential equation and graph the slope field.

$$\frac{dy}{dx} = x + 1$$



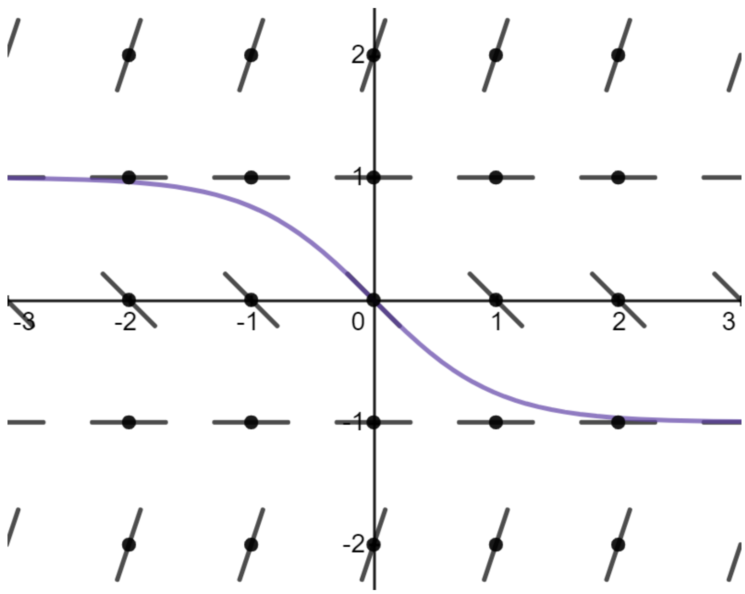
We can solve this easily now:

$$y(x) = \frac{x^2}{2} + x + C$$

The slope field maps parabolas opening up and we see that $\frac{dy}{dx} = 0$ when $x = -1$

Example: Graph the slope field for the following differential equation and predict $y(2)$ if $y(0) = 0$

$$\frac{dy}{dt} = y^2 - 1$$

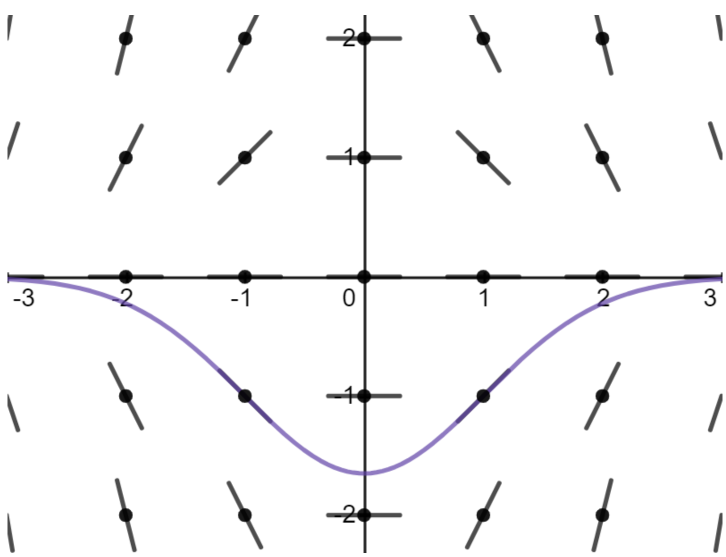


It looks like we have a horizontal asymptote of $y = -1$. Note that $y = \pm 1$ makes $\frac{dy}{dt} = 0$ and that the slope does not change as t changes (only moving up and down)

When $y(0) = 0$ it looks like $y(2) \approx -0.9$

Example: Graph the slope field for the following differential equation and predict $y(2)$ if $y(-1) = -1$

$$\frac{dy}{dx} = -xy$$

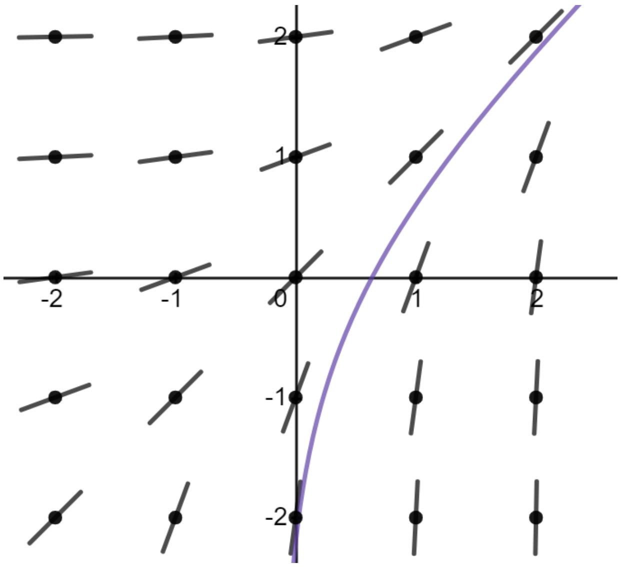


It looks like we have a horizontal asymptote of $y = 0$. Note that $y = 0$ and $x = 0$ makes $\frac{dy}{dx} = 0$ and then for most values of x and y the slope has a large magnitude and is positive in quadrants 2 and 4.

When $y(-1) = -1$ it looks like $y(2) \approx -0.3$

Practice: Graph the slope field for the following differential equation and predict $y(2)$ if $y(0) = -2$

$$\frac{dy}{dx} = e^{x-y}$$



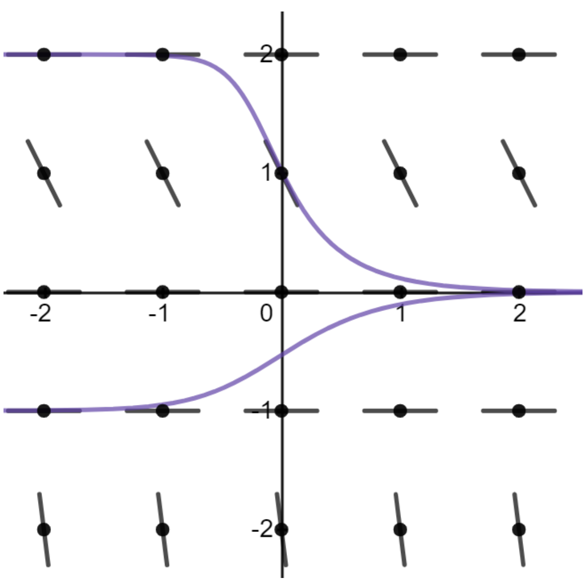
It looks like we have a slant asymptote of $y = x$.

On the line $y = x$ the slope is 1 and as $y \to \infty$ we have $\frac{dy}{dx} \to 0$, and as $y \to -\infty$ we have $\frac{dy}{dx} \to \infty$. The opposite is true for x, so as $x \to \infty$ then $\frac{dy}{dx} \to \infty$

When $y(0) = -2$ it looks like $y(2) \approx 1.8$

Practice: Graph the slope field for the following differential equation and predict the valie of $y(2)$ if $y(0) = 1$ OR $y(0) = -0.5$

$$\frac{dy}{dt} = y(y-2)(y+1)$$



It looks like we have a horizontal asymptote on the right side, that is as $t \to \infty$ we have $y \to 0$. Note that $\frac{dy}{dt} = 0$ when $y = 0, 2,$ and -1. The slope also does not depend on t and only changes as y changes.

When $y(0) = 1$ it approaches 0 from above and $y(2) \approx 0.01$ and when $y(0) = -0.5$ it approaches from below and $y(2) \approx -0.01$

Practice Problems: Geogebra slope field problems