Steady States and Motion

Goal: • Can identify the steady states of a differential equation. • Can use the steady states to predict behaviour of a particle in motion.
Terminology: • Steady state • Stable, unstable

Discussion question: Three particles are moving along the y-axis. There vertical position are a, b, and c and their respective velocities are:

$$\frac{da}{dt} = 1 - t \qquad \frac{db}{dt} = 1 - b \qquad \frac{dc}{dt} = (1 - t) \cdot (1 - c)$$

Every particle is at $y = 2$ when $t = 0$, that is $a(0) = b(0) = c(0) = 2$. How does motion differ for each particle?

A differential of the form

$$\frac{dy}{dt} = f(t)$$

Likely has NO steady states even though $\frac{dy}{dt}$ may be 0 at some time t. A steady state is when $\frac{dy}{dt} \to 0$ as $t \to \infty$

Example: Consider the functions

$$\frac{dy}{dt} = t \qquad \frac{dy}{dt} = \frac{1}{t} \qquad \frac{dy}{dt} = \frac{1}{t^2}$$

Example: Determine the steady states and their stability of

$$\frac{dy}{dt} = y^2(y-1)(y+4)$$

Practice: Determine the steady states and stability of

$$\frac{dP}{dt} = (P^2 - 1)(2 - P)$$

Practice: Determine the steady states and stability of

$$\frac{da}{db} = (1 - b)(2a + 4)$$

Practice Problems: Steady state practice problems