Logistic Growth

Goal: • Can build differential equations that reflect logistic growth
Terminology: • Logistic

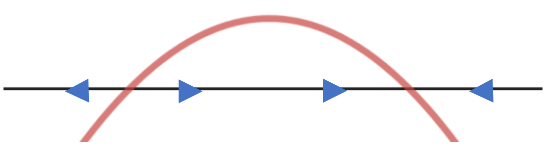
Discussion question: Consider the situation we are currently in. COVID-19 is currently spreading exponentially (i.e. proportional to the amount of people who have the virus), but it can't go on forever. Eventually, either a vaccine will be developed or it burns itself out and everyone on Earth gets it but we cannot say that as $t \to \infty$ that the number of COVID-19 cases, $n \to \infty$. Write a differential equation that could be used to describe the how the number of cases changes over time.

We know that we have the situation were as soon as one person gets the virus then it will start spreading, but if no one ever got it then it wouldn't spread and the number of people that have it would stay at $n = 0$. Therefore, we have an unstable steady state at $n = 0$.

As well, the population growth is going to slow down and eventually not change, so there should be a steady state at $n = M > 0$ (stable)

$$\frac{dn}{dt} = kn(n - M)$$

And we want M to be stable and n to be unstable so using an arrow diagram we want $k < 0$. Alternatively, we know that while $n \in (0, M)$ we want the population to grow so $n' > 0$ and we can match our signs, $\frac{dn}{dt} = k(+)(-)$



We will NOT be solving this differential equation (although it does have a solution). Instead we can use technology to predict the solution curve.

Example: A new meme is kind of like a virus. It will get shared and shared until everyone has either seen it or a person was never aware the meme existed. Write a differential equation for the number of people who have seen the following meme. Three years after "*I can has cheezburger*?" was created 16 million people had seen it. Today 20 million people know it (assume this is the limit). Use technology to predict k.

What are the two steady states? In this case $P = 0$ and $P = 20$ in million are the two extremes. If no one sees the meme it won't spread ($P = 0$ is unstable) and as soon as it spreads it will converge to the limit number ($P = 20$ is stable).

$$\frac{dP}{dt} = kP(P - 20)$$

We have the initial condition that $P(3) = 16$. Using geogebra we can play around with the value of k and see that it would be around $k = -0.07$ if we have 1 million people see it after 1 month.

When typing into geogebra, use geogebra.org/classic. Enter the differential equation with $\frac{dy}{dt} = f(x, y)$ and then use solveODE$\left(f, (3,16)\right)$ to graph the solution curve. You can play around with the value of k to change the shape.

Practice: A chemical reaction procceds as follows:

$$A + B \to C + g$$

Where chemical A and B react to form chemicals C and g where g is a gas. The important thing is that the mass of the system does not stay fixed as the chemical g will leave the system. Before anything happens the mass of $A + B$ is 500 g and once the reaction is complete (and A, B have disapeared) the mass of C is 300 g. The rate of the reaction is proportional to how much has reacted and how much there is left to react.

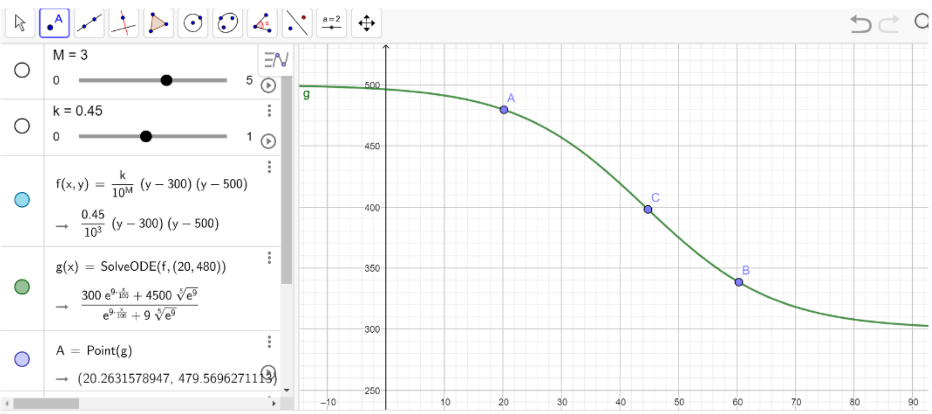
a. Make a differential equation that describes the change in the mass of the system

Let the mass be m, and we have two steady states $m = 500$ (unstable) and $m = 300$ (stable)

$$\frac{dm}{dt} = k(m - 300)(m - 500)$$

And since we want $\frac{dm}{dt} < 0$ (i.e. mass decreases from 500 to 300) we have $\frac{dm}{dt} = k(+)(-)$ so $k > 0$. We can see this using an arrow diagram too.

b. After 20 minutes the mass of the system is 480 g and after 60 minutes the mass of the system is 340 g. Use technology to predict k and the point where the reaction is half way done.

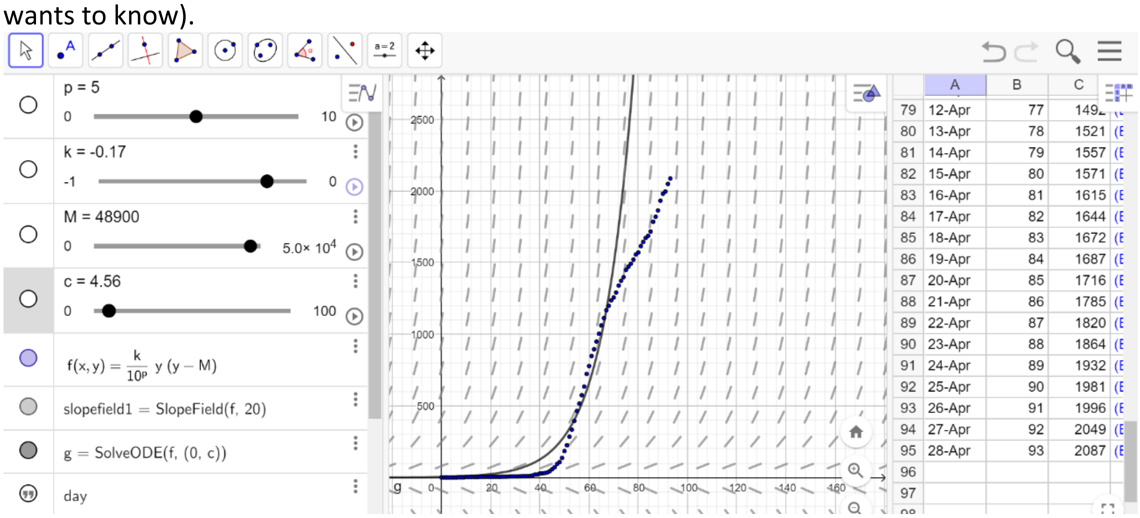


We have two initial conditions we need to match. First $m(20) = 480$ and second $m(60) = 340$.
Playing around I get

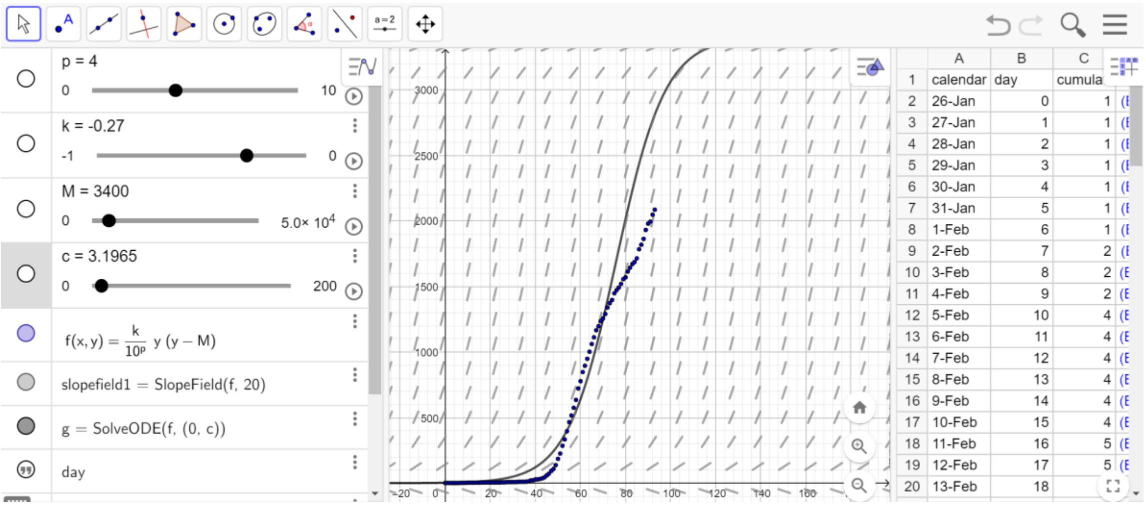
$$k = \frac{0.45}{1000} = 0.00045$$

Once I have this fixed, I just look for when $m = 400$ and that happens between 44 and 45 minutes.

Practice: Follow the link given on Teams that has data of the number of confirmed cases in BC of COVID-19 over the past 90 days. Write the differential equation for the change in the number of confirmed cases. There are three variable that are unknown. In your group come to an agreement on the best choice of C, k, M (and M is obviously the value everyone wants to know).



This is a great illustration for why it is so hard to predict this early into a pandemic. This is a graph with a final confirmed infected population of near 50K



This is an illustration with a final confirmed cases of 3400. We can make a lot of close graphs but because we haven't seen much of a decrease in cases there is not a lot we can do to predict without using other analytical methods

Practice Problems: 9.6 # 1-5 (make the differential equation and use technology to make your prediction)