Differential Equation and Slope Field Quiz Solutions

1. $y = A\sqrt{4x+1} - x + C$. Check and take derivative to find A
   $y' = \frac{A}{2}(4x+1)^{-\frac{1}{2}}(4) - 1 = \frac{2A}{\sqrt{4x+1}} - 1$ so $A = 1$. Letting $x = 12$ and $y = 1$ gives $C = 6$

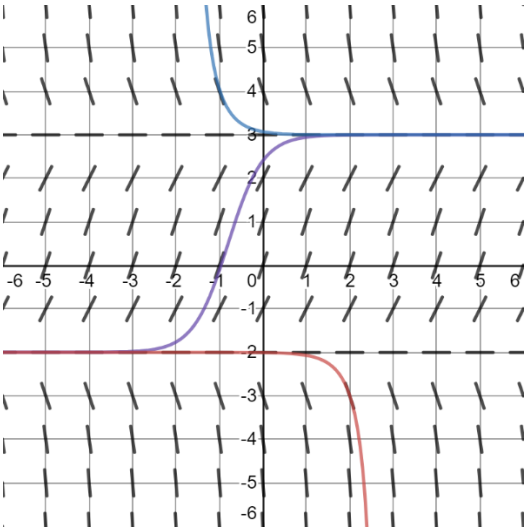
$$y(x) = \sqrt{4x+1} - x + 6$$

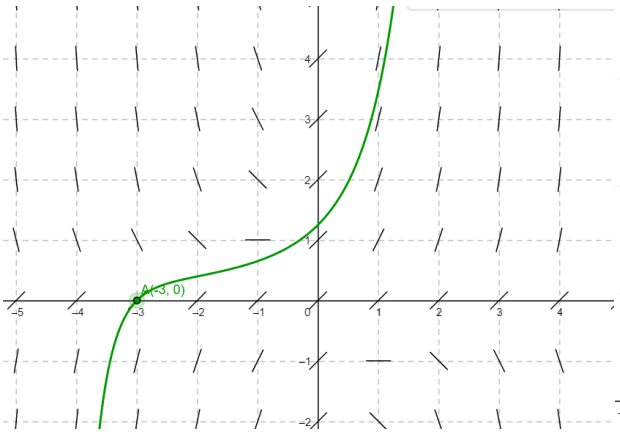
2. $f(x) = A \cdot \ln|1-2x| + \frac{B}{x^2} + C$, check the derivative
   $f'(x) = \frac{A}{1-2x}(-2) - \frac{2B}{x^3} = \frac{1}{1-2x} + \frac{1}{x^3}$ so $A = -\frac{1}{2}$ and $B = -\frac{1}{2}$. Solve for $C = -\frac{1}{2}$ by letting $x = 1$ and $f = -1$

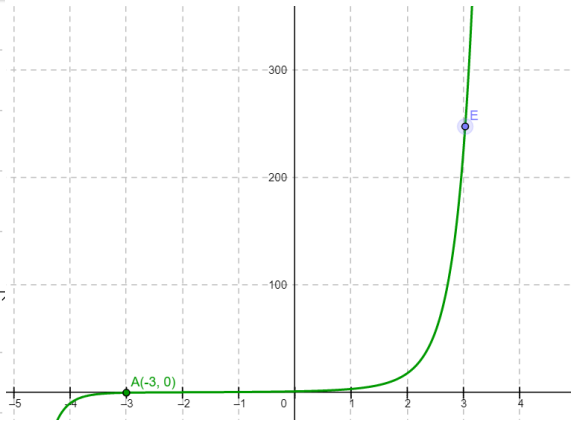
$$f(x) = -\frac{1}{2}\left(\ln|1-2x| + \frac{1}{x^2} + 1\right)$$

3. $x = At^{\frac{7}{3}} + Be^{-\frac{t}{2}} + C$, check the derivative
   $x' = \frac{7}{3}At^{\frac{4}{3}} - \frac{B}{2}e^{-\frac{t}{2}} = 7t^{\frac{4}{3}} - e^{-\frac{t}{2}}$ so $A = 3$ and $B = 2$. If $x = 0$ and $y = 1$ you will find $C = -1$

$$x(t) = 3t^{\frac{7}{3}} + 2e^{-\frac{t}{2}} - 1$$

4. 

5. 



When you zoom out it looks like $y(3) \approx 250$