Differential Equation Review

Name

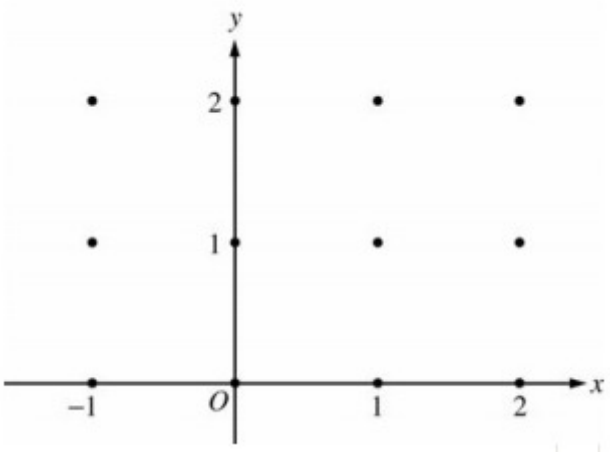
Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$

1. Find the values of the constants *m* and *b*, for which *y=mx+b* is a solution to the differential equation.

Please respond on separate paper, following directions from your teacher.

Consider the differential equation $dy/dx = -xy^2/2$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1)=2$.

2. On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



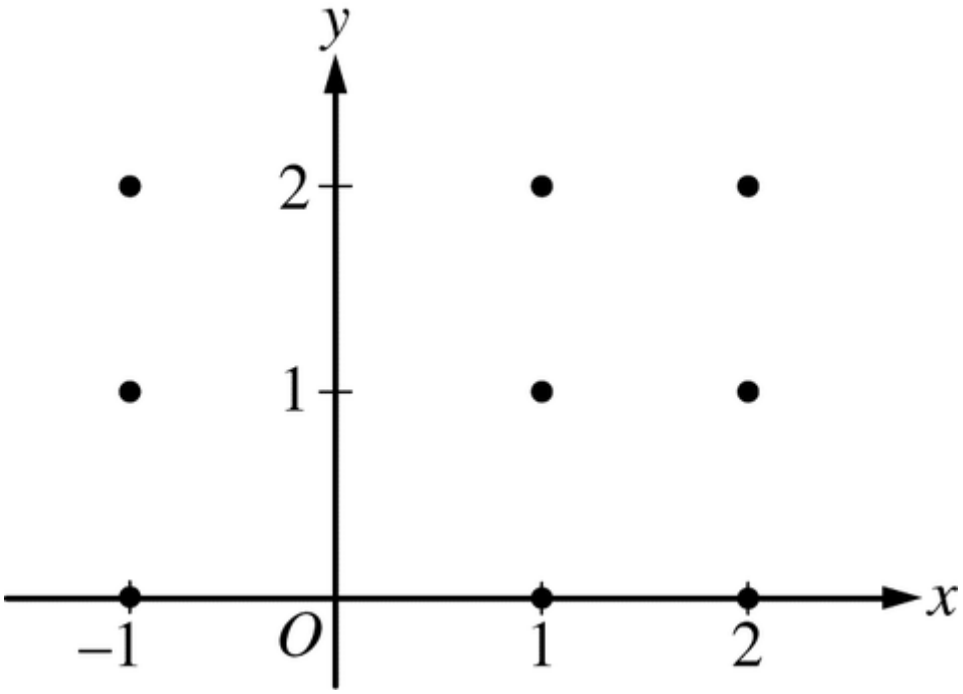
Please respond on separate paper, following directions from your teacher.

The following are related to this scenario:

Consider the differential equation $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y - 1)$,where x≠0 . Let *y=f(x)* be the particular solution to the differential equation with initial condition $f(1)=2$.

Differential Equation Review

3. On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



Please respond on separate paper, following directions from your teacher.

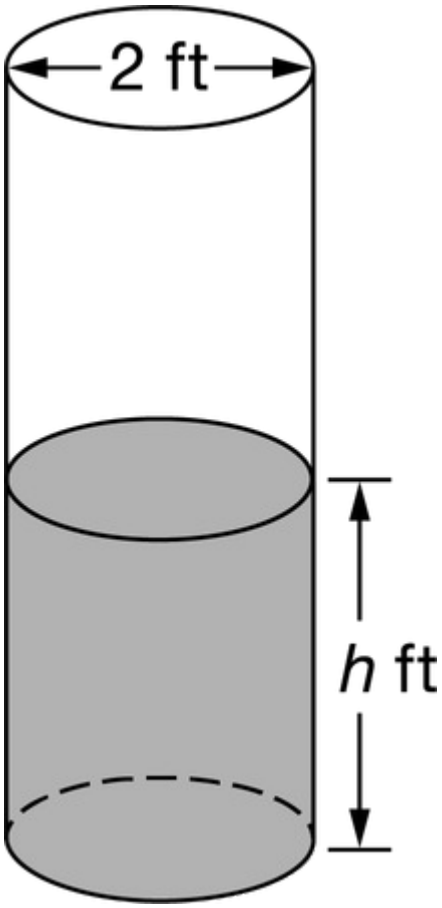
4. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Differential Equation Review



A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.

Please respond on separate paper, following directions from your teacher.

b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.

Please respond on separate paper, following directions from your teacher.

Differential Equation Review

c) At time $t = 0$ seconds, the height of the water is 5 feet. Use separation of variables to find an expression for h in terms of t.

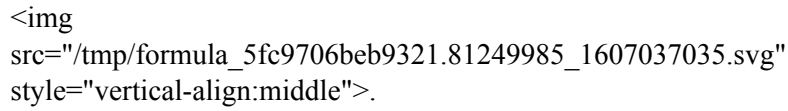
Please respond on separate paper, following directions from your teacher.

A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

5. Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

Please respond on separate paper, following directions from your teacher.

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then



Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

6. Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

Please respond on separate paper, following directions from your teacher.

Let f and g be functions that are differentiable for all real numbers x and that have the following properties.

i. $f'(x) = f(x) - g(x)$

ii. $g'(x) = g(x) - f(x)$
iii. $f(0) = 5$
iv. $g(0) = 1$

7. Find $f(x)$ and $g(x)$. Show your work.

Please respond on separate paper, following directions from your teacher.

Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

8. Find the particular solution $y=f(x)$ to the given differential equation with the initial condition $f(0)=0$.

Please respond on separate paper, following directions from your teacher.

Consider the differential equation $dy/dx = (y-1)^2\cos(\pi x)$.

9. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

Please respond on separate paper, following directions from your teacher.

Let f be the function that is defined for all real numbers x and that has the following properties.

(i) $f''(x)=24x-18$

(ii) $f'(1)=-6$

(iii) $f(2)=0$

10. Write an expression for $f(x)$.

Differential Equation Review

Please respond on separate paper, following directions from your teacher.

A particle moves along the x-axis in such a way that its acceleration at time t for $t \geq 0$ is given by $a(t)=4\cos(2t)$. At time $t = 0$, the velocity of the particle is $v(0)=1$ and its position is $x(0)=0$.

11. Write an equation for the position $x(t)$ of the particle.

Please respond on separate paper, following directions from your teacher.

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rte $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

12. Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

Please respond on separate paper, following directions from your teacher.

For $0 \leq t \leq 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time t=0.

13. To the nearest whole number, what is the maximum number of mosquitoes for $0 \leq t \leq 31$? Show the analysis that leads to your conclusion.

Please respond on separate paper, following directions from your teacher.

Differential Equation Review

A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t)=\ln(t^2-3t+3)$. The particle is at position $x=8$ at time $t=0$.

14. Find the position of the particle at time $t=2$.

Please respond on separate paper, following directions from your teacher.