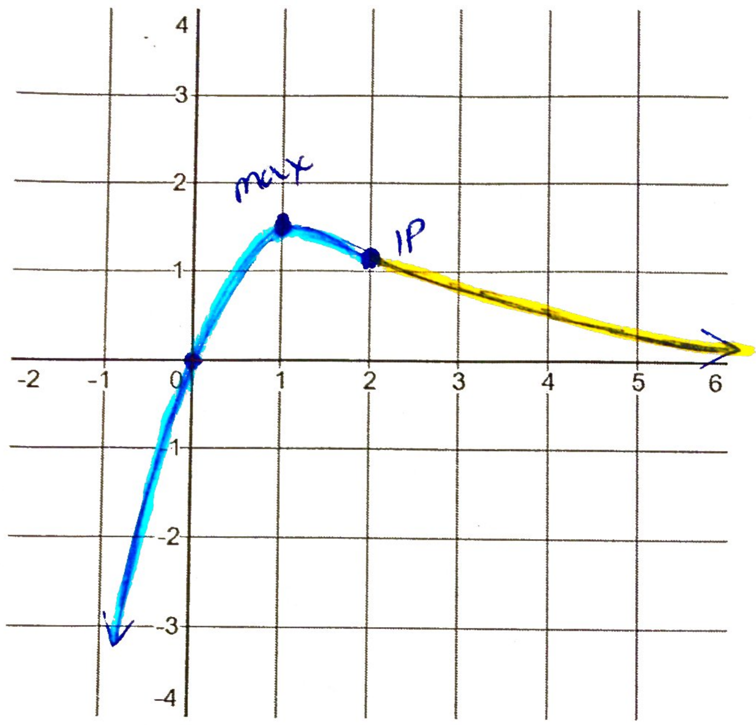
Derivative of ln and e Extra Practice and Evidence

1. Graph the following functions accurately

a. $f(x) = 4xe^{-x}$



$f'(x) = 4\left[e^{-x} - xe^{-x}\right]$

$\Rightarrow e^{-x}(1-x)=0 \Rightarrow x=1$

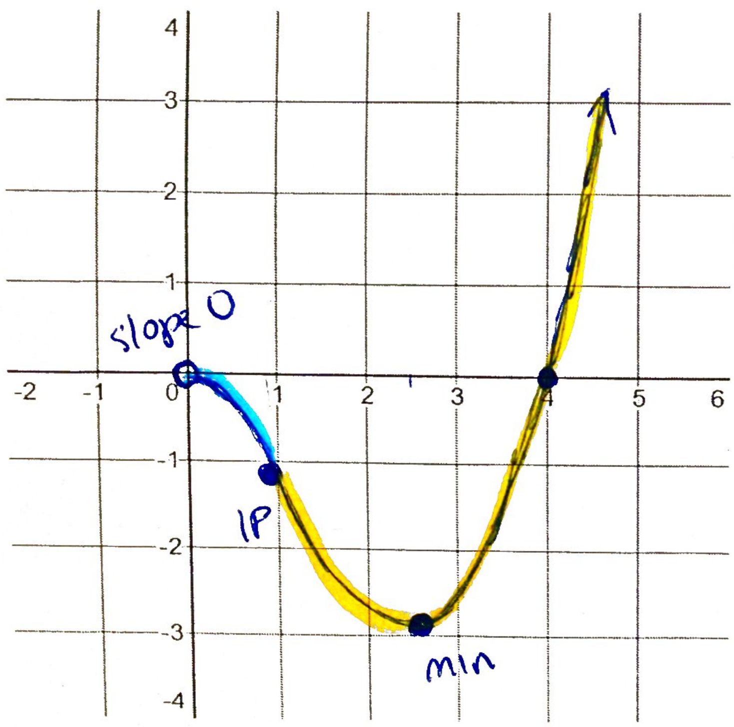
		1	
$f'(x)$	$+$	0	$-$
		max	

$f''(x) = 4\left[-e^{-x} - e^{-x} + xe^{-x}\right]$

$\Rightarrow -e^{-x}(2-x)=0 \Rightarrow x=2$

x		2	
$f''(x)$	$-$	0	$+$

b. $g(x) = x^2 \cdot \ln\frac{x}{4}$



$g'(x) = 2x\ln\frac{x}{4} + \frac{\cancel{4}}{\cancel{x}}\cdot\frac{1}{\cancel{4}}\cdot x^2$

$= x\left(2\ln\frac{x}{4} + 1\right) = 0$

$x = 0 \qquad x = 4e^{-1/2} = 2.4$

		0		2.4	
$g'(x)$	$+$	0	$-$	0	$+$
		max		min	

$g''(x) = 2\ln\frac{x}{4} + 1 + 2x\,\frac{\cancel{4}}{\cancel{x}}\cdot\frac{1}{\cancel{4}}$

$= 2\ln\frac{x}{4} + 3 = 0$

$\Rightarrow x = 4e^{-3/2} = 0.9$

2. Find the solutions to the following equations (all solutions are in the interval $x \in (-5, 5)$)

a. $x + 2 = e^x$

$f(x) = e^x - x - 2 \qquad f'(x) = e^x - 1$

$$x = \frac{-(\cancel{A} e^A - A - 2)}{e^A - 1} + A$$

$$\boxed{\begin{aligned} x &= -1.84140566 \\ x &= 1.146193221 \end{aligned}}$$

b. $5 - e^{\sqrt{x}} = \ln x - 2$

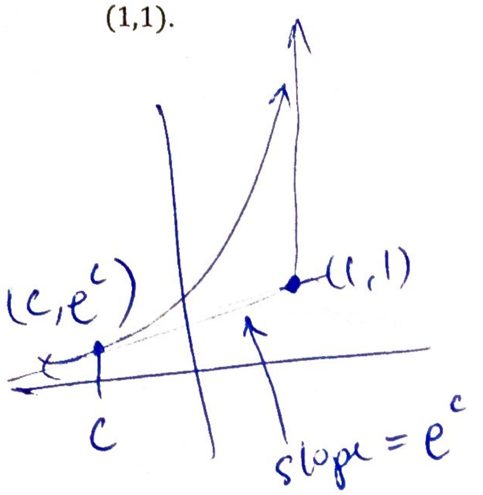
$f(x) = \ln x + e^{\sqrt{x}} - \cancel{+} 7$

$$f'(x) = \frac{1}{x} + \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

$$x = \frac{-(\ln A + e^{\sqrt{A}} - \cancel{+} 7)}{\frac{1}{A} + \frac{e^{\sqrt{A}}}{2\sqrt{A}}} + A$$

$x = \underline{\underline{3.126420217}}$

3. Determine the equation of all lines that are tangent to the curve $y = e^x$ and pass through the point (1,1).



$$y = e^c(x-1)+1$$
$$\equiv$$
$$y = e^x \quad @ \quad x = c$$

$$e^c = e^c(c-1)+1$$

$$\Rightarrow \quad 1 = c - 1 + \frac{1}{e^c}$$

$$f(c) = e^{-c} + c - 2 = 0$$

$$f'(c) = -e^{-c} + 1$$

$$c = \frac{-2 + A + e^{-A}}{e^{-A} - 1} + A$$

$$\boxed{\begin{array}{l} c = 1.8414 \\ c = -1.14619 \end{array}}$$

$$\Rightarrow \boxed{\begin{array}{l} y = 0.318(x-1)+1 \\ y = 6.305(x-1)+1 \end{array}}$$

4. Use log laws to find $\frac{dy}{dx}$ of the following function (next class stuff)

$$\ln y = \ln\left(\frac{(x-2)^3\sqrt{x^2+1}}{x^4+3x}\right)$$

$$\ln y = 3\ln(x-2) + \frac{1}{2}\ln(x^2+1) - \ln(x^4+3x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{x-2} + \frac{x}{x^2+1} - \frac{4x^3+3}{x^4+3x}$$

$$\frac{dy}{dx} = \left[\frac{3}{x-2} + \frac{x}{x^2+1} - \frac{4x^3+3}{x^4+3x}\right] \cdot \boxed{\frac{(x-2)^3\sqrt{x^2+1}}{x^4+3x}}$$

y

5. Since we found $\frac{d}{dx}\ln x$ using implicit differentiation after we knew what the derivative of it's inverse was find the derivative of the inverse function f^{-1} generally.
That is, if $\frac{d}{dx}f(x) = f'(x)$ is known and $y = f(x)$, what is $\frac{d}{dx}f^{-1}(x)$?

$$y = f(x) \leftrightarrow f^{-1}(y) = x$$

~~want~~ $\cancel{\frac{d}{dx}f^{-1}(x) = \frac{d}{dx}f(y) = \frac{d}{dx}(x)}$, since

$$\cancel{= (f'(y))\frac{dy}{dx}}$$

let $y = f^{-1}(x)$ want $\frac{dy}{dx}$

$\rightarrow f(y) = x$

$$\frac{d}{dx}\left(f(y) = x\right) \Rightarrow f'(y)\frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{f'(y)} = \frac{1}{f'(f^{-1}(x))}$$