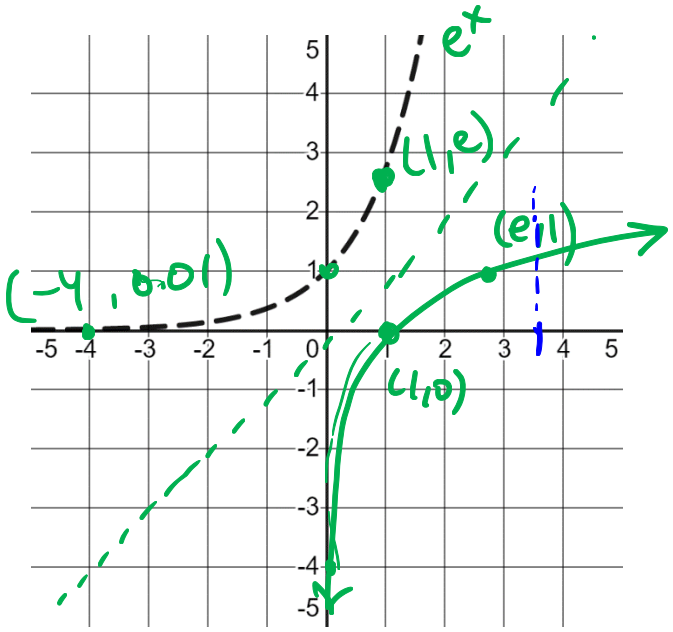
Exponential Inverses

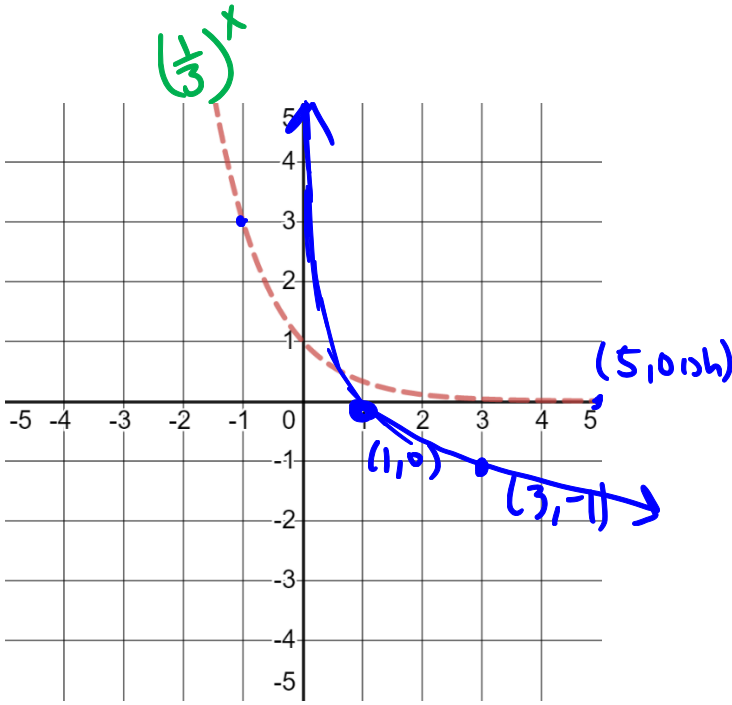
KNOW	DO	UNDERSTAND
The inverse of an exponential is a log of the same base and knows the domain and range of a log function.	Can find the exact equation in base e to an exponential. Can graph the basic $\ln x$.	*Function Characteristics*:

Vocab & Notation

- Logarithm, $\log x$
- Natural log, $\ln x$

Graph the inverse of $y = e^x$ and $y = (1/3)^x = 3^{-x}$





$(x,y) \mapsto (y,x)$

★ inverse is one-to-one (so its a function)

$f(x) = e^x$ $\quad f: \mathbb{R} \to (0,\infty) \iff f^{-1}: (0,\infty) \to \mathbb{R}$

★★ has a vertical asymptote at $x=0$

$g(x) = \sin x$ $\quad g^{-1}(x) = \sin^{-1} x = \arcsin x$

Since the exponential function $f(x) = b^x$ needs that the base $b > 0$ and $b \neq 1$, we have the same restriction on the function $f^{-1}(x) = \log_b x$.

log 100

There are three common bases that you will use depending on your field.

- Engineering: Base 10 $f(x)=10^x \rightarrow f^{-1}(x)=\log_{10} x = \log x$
- Science and Mathematics: Base e $g(x)=e^x \rightarrow g^{-1}(x)=\log_e x = \ln x$ (natural log)
- Computer Science: Base 2 $h(x)=2^x \rightarrow h^{-1}(x)=\log_2 x = \text{lb}\, x$

$= \log x$

Example: Solve for k

$\log(500 = 10^k)$

$\log 500 = \log 10^k$

$2.699\ldots = k$

$\ln(2 = e^k)$

$\ln 2 = k$

$0.693\ldots = k$

$x^2 = 2$

$x = \sqrt{2}$

Practice: Solve for x

$\log(1200 = 10^x)$

$\log 1200 = x$

$\ln(20 = e^x)$

$\ln 20 = x$

$2(9 = \log_2 x)$

$2^9 = 2^{\log_2 x}$

$512 = x$

$5 = \frac{1}{e^k} = \frac{1}{e^k}$

$\ln\left(\frac{1}{5} = e^k\right)$

$\ln\left(\frac{1}{5}\right) = k$

$10(3 = \log k)$

$10^3 = k$

$e(8 = \ln x)$

$e^8 = x$

$17 = \ln(e^k)$

$17 = k$

$32 = 10^{\log k}$

$32 = k$

$e(22 = \ln(\ln k))$

$e(e^{22} = \ln k)$

$e^{e^{22}} = k$

When we evaluate an exponential $2^6 = x$, we are asking:

2 to the power of 6 is what?

When we evaluate a logarithm, we are asking the inverse. For $\log_2 32 = x$ we are asking:

2 to what power is 32?

$2^x = 32 \rightarrow x = 5$

$\log_2 32 = x$

$2^x = 32$

Practice: Without a calculator evaluate the following:

$\log_3 729$

$\log_5 625 = x$

$\log_{19} 361 = x$

3 to the power of ??? is 729

$3^x = 729$

$3^6 = 729$

$x = 6$

$625 = 5^x$

$x = 4$

$361 = 19^x$

$x = 2$

Practice Problems: 8.1 page 380-381 # 1, 4-9, 12-15, 20-24