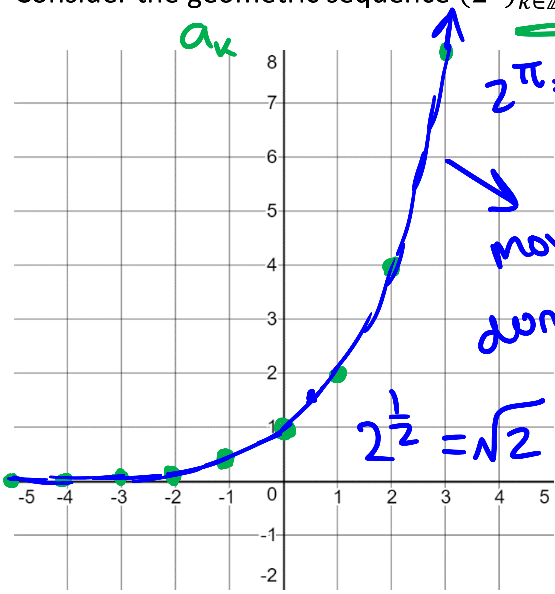
Exponential Intro

KNOW	**DO**	**UNDERSTAND**
The general shape of an exponential graph depending on if the base is larger than 1 or less than 1.	Can graph exponential function of the form $f(x) = b^{\frac{x}{T}}$ and can graph e^x	*Function Characteristics*: The base is the rate of growth/decay and the horizontal stretch is the length of one growth/decay period

Vocab & Notation
- Exponential growth/decay
- Euler's number

Consider the geometric sequence $(2^k)_{k\in\mathbb{Z}}$. Plot it on the graph below.



$2^\pi = 8.8\ldots$ $(2^k)_{k\in\mathbb{Z}} = \left(\ldots, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, \ldots\right)$

$\ldots a_{-2}, a_{-1}, a_0, a_1, a_2, \ldots$

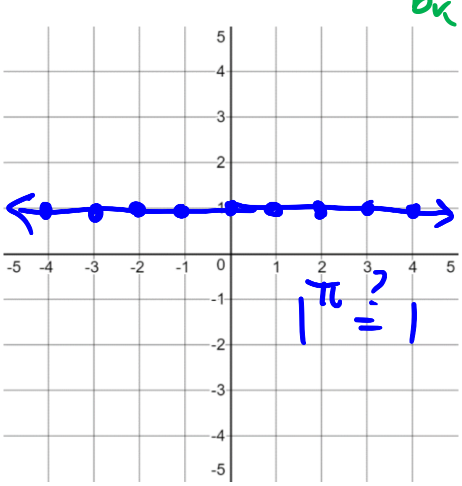
move domain into $\mathbb{R}$

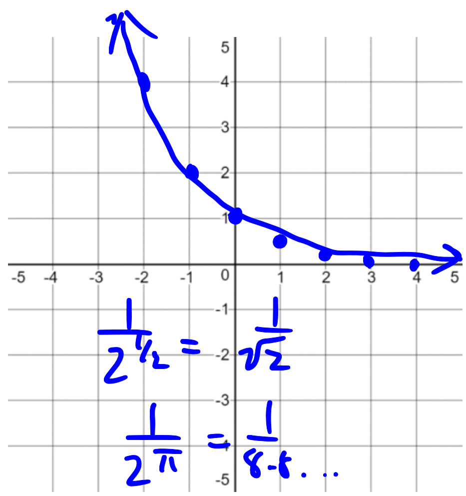
$a_k = 0$ $k = ?$ CAN'T

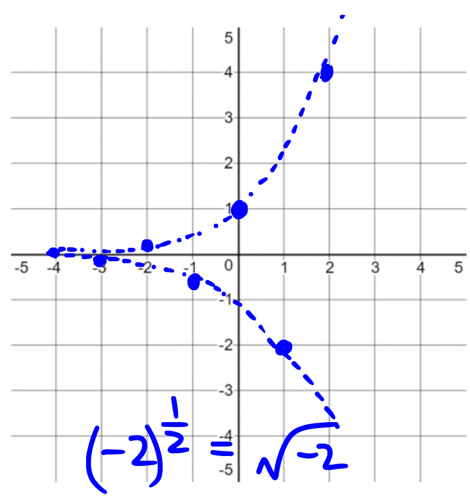
$a_k = \frac{1}{2^{-k}} > 0$ * will it reach 0 irl ??

Do the same for the sequences $(1^k)_{k\in\mathbb{Z}}$; $\left(\frac{1}{2^k}\right)_{k\in\mathbb{Z}}$; and $((-2)^k)_{k\in\mathbb{Z}}$ and graph them below.

b_k c_k d_k







BORING

$(-2)^{\frac{1}{3}} = \sqrt[3]{-2}$

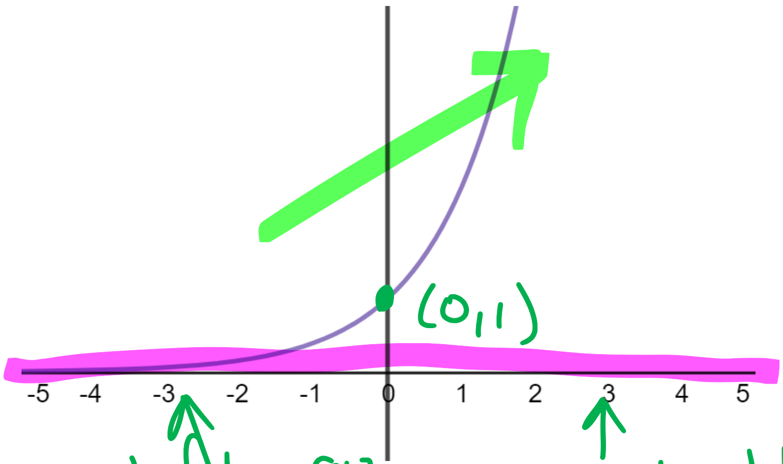
$(-2)^\pi =$ Out of domain

DONT LIKE -VE BASE

Base

$(b^k)_{k\in\mathbb{Z}}$

The general graph form $y = b^x$ for $b > 1$

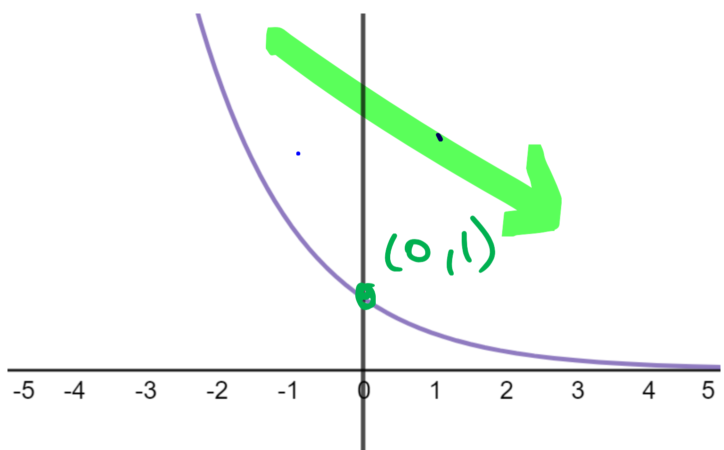


☆ always pass thru (0,1)

☆ increasing

☆ horizontal asymptote at $y=0$

If $b \in (0,1)$, then



☆ b^x Reflected over y axis

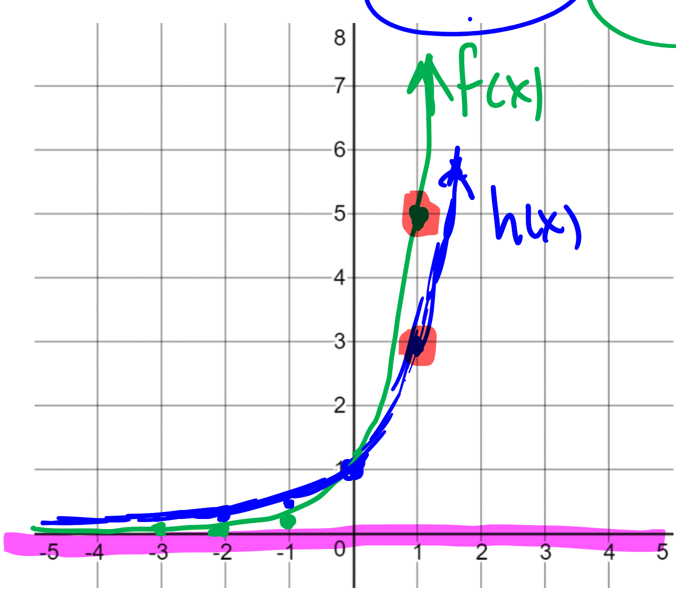
$\left(\frac{1}{b}\right)^x = b^{-x}$

☆ still thru (0,1)

☆ decreasing

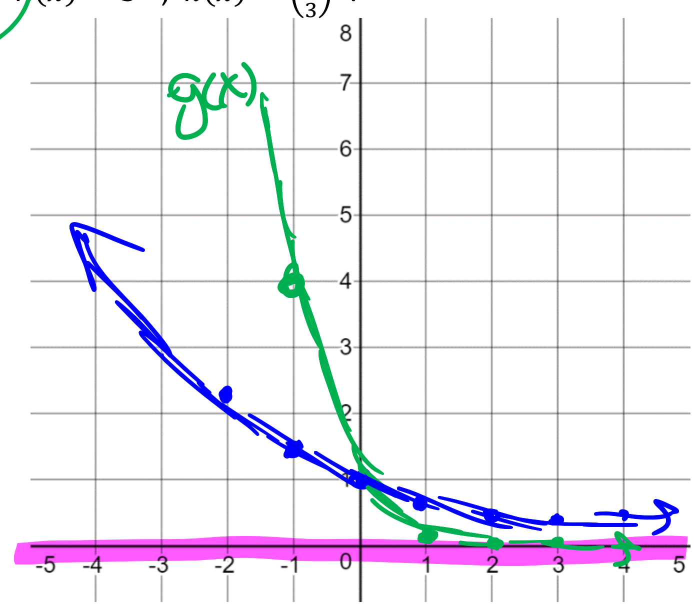
☆ horiz. asymptote @ $y=0$

Practice: Graph the functions $f(x) = 5^x$; $g(x) = \left(\frac{1}{4}\right)^x$; $h(x) = 3^x$; $k(x) = \left(\frac{2}{3}\right)^x$.



Increasing functions: **Growth**

base > 1



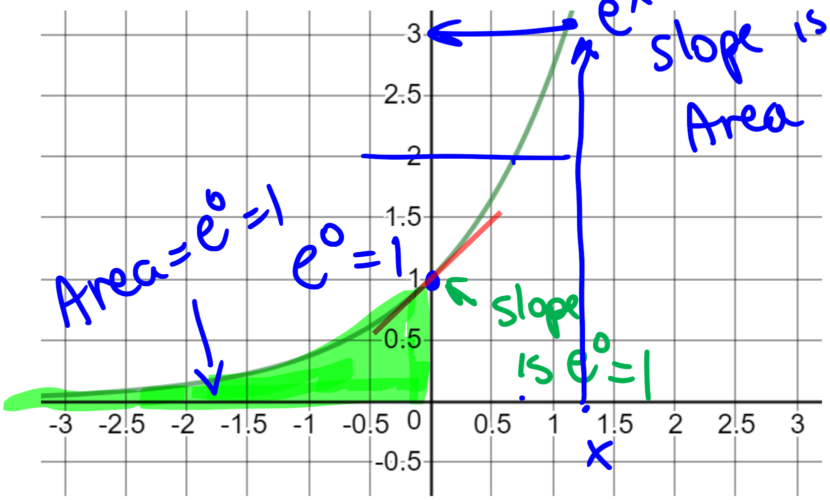
Decreasing functions: **Decay**

base $\in (0,1)$

Arguably the most important exponential function uses **Euler's Number:**

$$f(x) = (2.71828\ldots)^x = e^x$$

$$e = \sum_{k=0}^{\infty} \frac{1}{k!}$$



$2 = e^{c}$

$c \sim 0.7$

And the important thing is that we can change from one base to another.

$$f(x) = 2^x = (e^{0.7})^x = e^{0.7x}$$

We'll talk more about this next week on finding that power for e, but right now just be comfortable that we can do this at least through guess and check and when you start doing calculus you will only want to work exclusively with base e.

For now, let's look to see how any exponential function could be defined by an arbitrary base.

Consider the specific case where a population doubles every year.

Population	1	2	4	8	16	...	2^t
Time	0	1	2	3	4	...	t

And now, the general case where a population grows at a rate r and it takes T years to grow that much or that little.

$\times r$

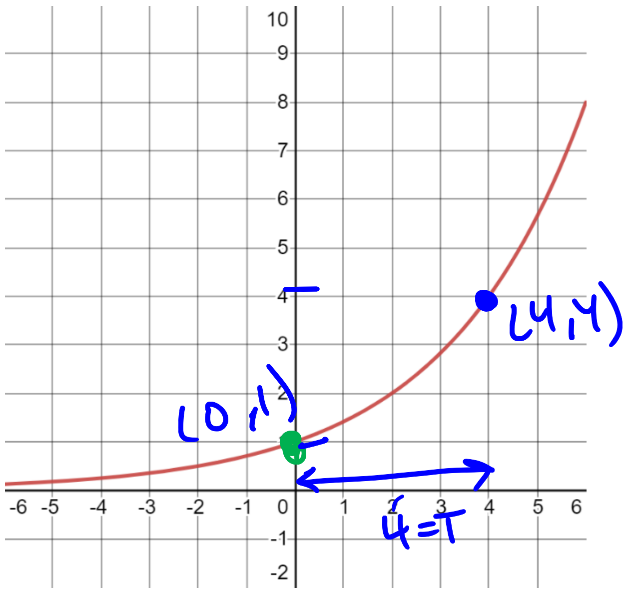
Population	1	r	r^2	r^3	r^4	...	$r^n = r^{\frac{t}{T}}$
Time	0	T	$2T$	$3T$	$4T$	...	$nT = t$

$n = \frac{t}{T}$

$y(x) = r^{\frac{x}{T}}$ ← how long to grow/decay by r

↑ rate of growth

Example: Determine two equations for the following graph, and one equation in base e.



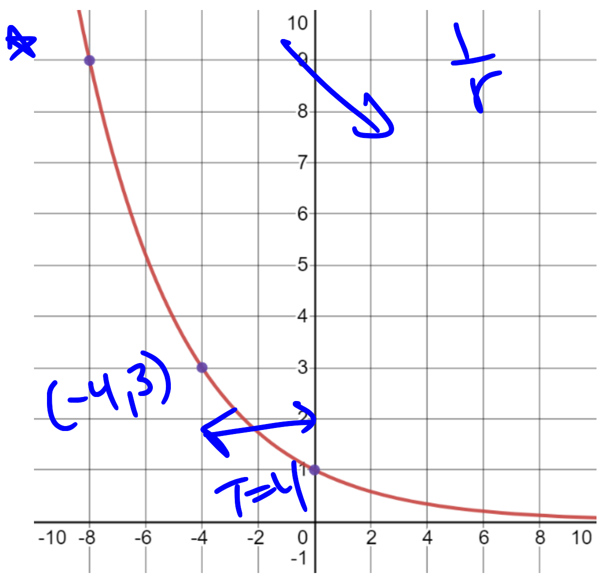
$a_0 = 1 \xrightarrow{\times 4 = r} a_1 = 4$

T = period of time to $\times 4$

$y = 4^{\frac{x}{4}} \approx \left(e^{1.4}\right)^{x/4}$

$= e^{0.35x}$

Practice: Determine two equations for the following graph, and one equation in base e.



$a_0 = 1 \quad a_1 = 3$

$\times 3 = r$

$y = \left(\frac{1}{3}\right)^{\frac{x}{4}} = 3^{-\frac{x}{4}} = \left(\frac{1}{9}\right)^{\frac{x}{8}}$

$= \left(e^{1.1}\right)^{-\frac{x}{4}} = e^{-0.275x}$

Practice: Determine an equation in base e for an exponential that passes through $(0,1)$ and the point $(5,7)$

• (5,7)

$\times 7$ $\quad$ $+5$

• (0,1)

$r = 7 \quad T = 5$

$y = 7^{x/5} = \left(e^{1.95}\right)^{x/5} = e^{0.39x}$

Practice Problems: 7.1 page 342-345 # 2-5, 7, 15 (modelling practice # 6, 8-10, 12) 7.2 page 354-357 # 9, 11, 12-14

Practice Building Equations

Build an exponential function in base something and base e given that it passes through the indicated point. For base e, just get the exponent correct to one decimal (although you could use a graphing calculator to solve it more precisely). **They all have a horizontal asymptote of $y = 0$**

1. Through $(0,1)$ and $(9, 5)$

2. Through $(0,1)$ and $(-8, 7)$

3. Through $(0,1)$ and $(8, 2)$

4. Through $(0,1)$ and $(-5, 0.5)$

5. Through $(0,1)$ and $(0.25, 6)$

6. Through $(0,1)$ and (a, b)

Solutions: 1) $y_1 = 5^{x/9} = e^{0.179x}$; 2) $y_2 = \left(\frac{1}{7}\right)^{x/8} = 7^{-x/8} = e^{-0.243x}$; 3) $y_3 = 2^{x/8} = e^{0.087x}$; 4) $y_4 = \left(\frac{1}{2}\right)^{-x/5} = 2^{x/5} = e^{0.139x}$; 5) $y_5 = 6^{4x} = e^{7.167x}$; 6) $y_6 = b^{x/a}$. Also e^{kx} with $k > 0$ if $a > 0$ and vice versa