Exponential Inverses

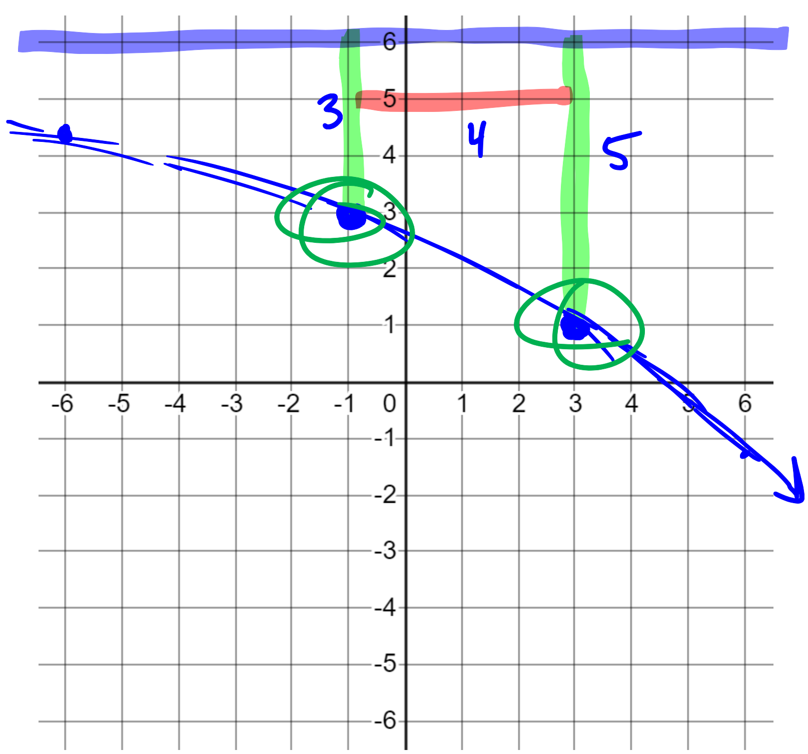
KNOW	DO	UNDERSTAND
The inverse of an exponential is a log of the same base and knows the domain and range of a log function.	Can find the exact equation in base e to an exponential. Can graph an exponential function. Can use logs to solve exponentials.	*Inverse Characteristics*: Can analyze a log using exponential functions and then inverting.
Vocab & Notation • Logarithm, $\log x$ • Natural log, $\ln x$		

To graph an exponential, we work backwards. Identify the transformations that took place and then add those to the graph.

- Vertical shift → the asymptote
- Horizontal shift → the x-coordinate of reference point
- Vertical stretch → distance between asymptote + ref. point
- Horizontal stretch → horiz. distance from ref. point to next point
- Base → ratio between distances to the asymptote

Example: Graph

$$f(x) = -3\left(\frac{5}{3}\right)^{\frac{x+1}{4}} + 6$$

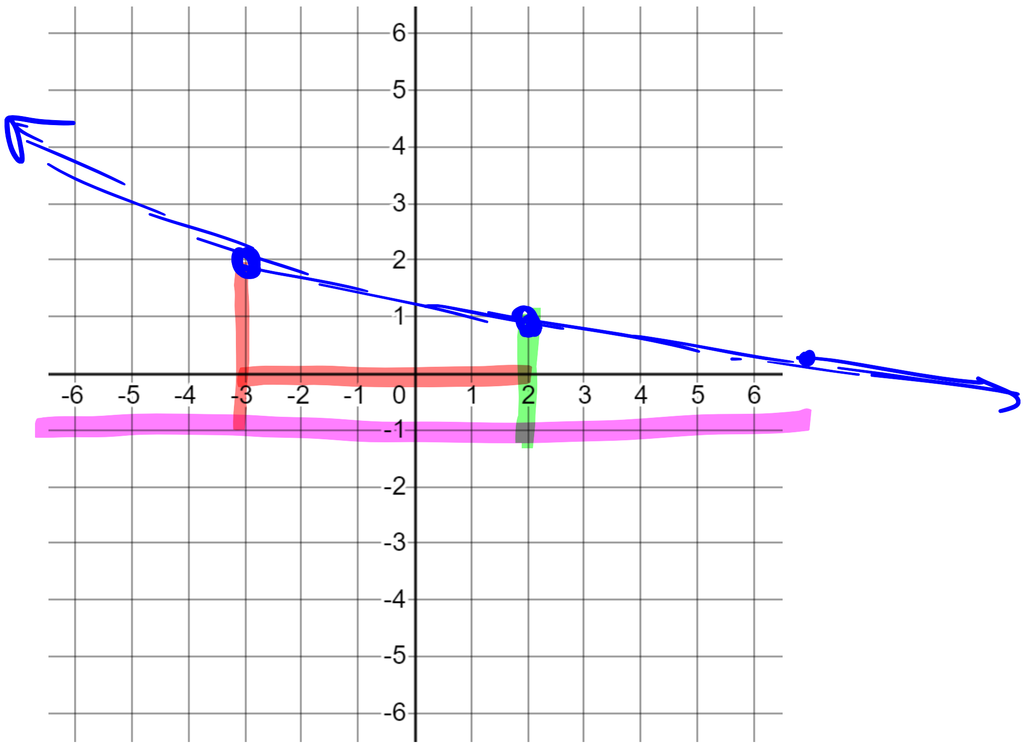


$f(-6) = -3\left(\frac{5}{3}\right)^{-5/4} + 6 = 4.4$

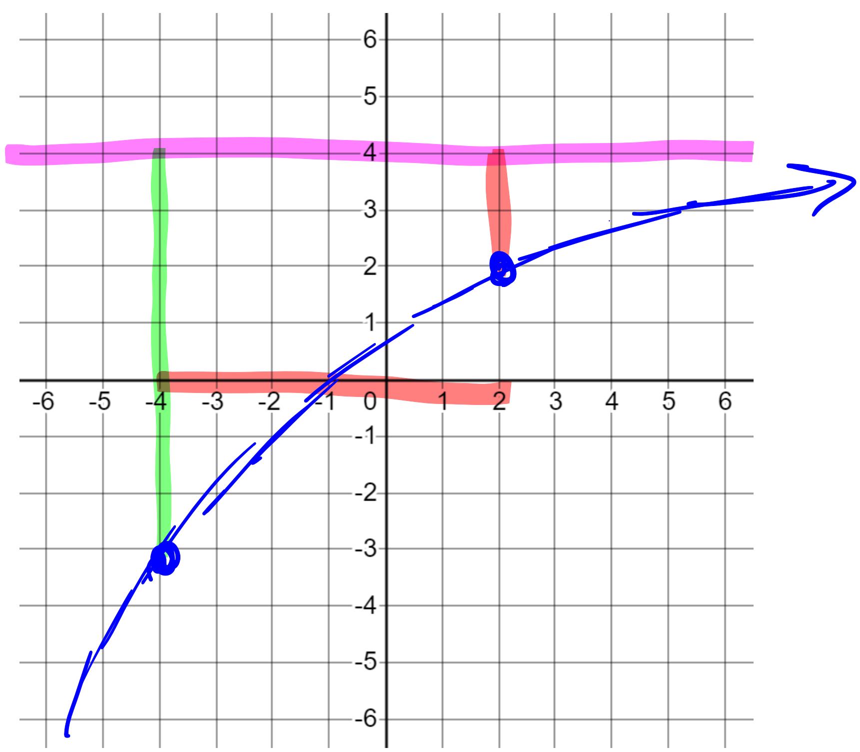
$f(6) = -1.3$

Practice: Graph the following functions.

$$g(x) = 2\left(\frac{2}{3}\right)^{\frac{x-2}{5}} - 1$$



$$h(x) = -7\left(\frac{2}{7}\right)^{\frac{x+4}{6}} + 4$$



Notice that the exponential function is 1-to-1 so it must have an inverse function.

$f(x) = b^x \longrightarrow f^{-1}(x) = \log_b x$

Since the exponential function $f(x) = b^x$ needs that the base $b > 0$ and $b \neq 1$, we have the same restriction on the function $f^{-1}(x) = \log_b x$.
There are three common bases that you will use depending on your field.

- Engineering: Base 10

  $f(x) = 10^x \rightarrow f^{-1}(x) = \log_{10} x = \log x$ common log

- Science and Mathematics: Base e

  natural log
  $f(x) = e^x \rightarrow f^{-1}(x) = \log_e x = \ln x = \log x$

- Computer Science: Base 2

  binary log
  $f(x) = 2^x \rightarrow f^{-1}(x) = \log_2 x = \text{lb}\, x = \log x$

Example: Solve for k

$\log(500 = 10^k)$

$\log 500 = \cancel{\log} 10^k$

$2.69.... = k$

$\ln(2 = e^k)$

$\ln 2 = \cancel{\ln} e^k$

$0.69.. = k$

$2 = e^{\ln 2}$

$e(5 = \ln k)$

$e^5 = \cancel{e^{\ln}} k$

$e^5 = k = 148.4...$

Practice: Solve for x

$1200 = 10^x$

$\log(1200) = x$

$\ln(20 = e^x)$

$\ln 20 = x$

$9 = \log_2 x$

$2^9 = x$

$5 = \frac{1}{4^k}$

$4^k = \frac{1}{5}$

$4 = e^x$

$x = \ln 4$

$\ln(\cancel{e}^{\ln 4 \cdot k} = \frac{1}{5})$

$\Rightarrow \ln 4 \cdot k = \ln(1/5)$

$3 = \log k$

$10^3 = k$

$8 = \ln x$

$e^8 = x$

$17 = \cancel{\ln(e}^{k})$

$17 = k$

$32 = \cancel{10^{\log}}{}^{k}$

$32 = k$

$22 = \cancel{\ln}(\cancel{\ln} k)$

$e^{e^{22}}$

$e^{(e^{22})} = k \sim 10^{1.5 \text{ billion}}$

This now gives us the tool to put an exponential function b^x exactly in the form of e^{kx} and we can solve exponential functions.

Example: Find the zeros of the functions we graphed

$$f(x) = -3\left(\frac{5}{3}\right)^{\frac{x+1}{4}} + 6$$

$e^k = 5/3 \Rightarrow k = \ln(5/3)$

$f(x) = -3\boxed{e^{k\frac{(x+1)}{4}}} + 6 = 0$

$\ln\left(\cancel{e}^{\frac{k}{4}(x+1)} = 2\right)$

$\frac{1}{4}\ln(5/3)(x+1) = \ln 2$

$x = \frac{4\ln 2}{\ln(5/3)} - 1$

$= 4.428$

$$g(x) = 2\left(\frac{2}{3}\right)^{\frac{x-2}{5}} - 1$$

$e^k = 2/3 \Rightarrow k = \ln(2/3)$

$2e^{\frac{k}{5}(x-2)} - 1 = 0$

$\ln\left(\cancel{e}^{\frac{k}{5}(x-2)} = \frac{1}{2}\right)$

$\frac{\ln(2/3)}{5}(x-2) = \ln(1/2)$

$x = \frac{5\ln(1/2)}{\ln(2/3)} + 2$

$= 10.548$

Practice Problems: 8.1 page 380-381 # 1, 4-9, 12-15, 20-24