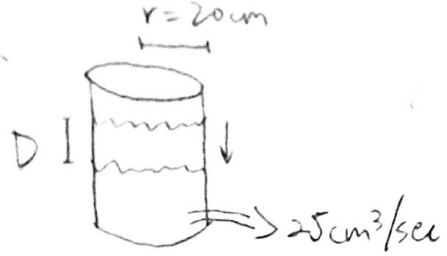
1.



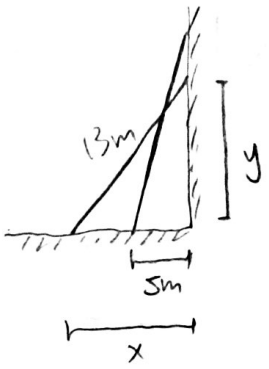
$V = \pi r^2 \cdot D$ $\quad \frac{dV}{dt} = 25 cm^3/sec$

$\frac{dV}{dt} = \pi r^2 \cdot \frac{dD}{dt}$

$25 cm^3/sec = \pi \cdot (20cm)^2 \cdot \frac{dD}{dt}$

$\frac{dD}{dt} = \frac{1}{16\pi}$

2.



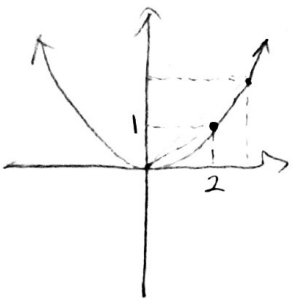
$x^2 + y^2 = 13^2$ $\quad \frac{dy}{dt} = 0.1 m/s$ $\quad x = 5m$ $\quad y = 12m$

$2x \cdot \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$2 \times 5 \frac{dx}{dt} + 2 \times 12 \times 0.1 m/s = 0$

$\frac{dx}{dt} = -0.24 m/s$

3.



$y = \frac{1}{4}x^2$ $\quad \frac{dx}{dt} = 5 cm/s$

$\frac{dy}{dt} = \frac{1}{2} x \cdot \frac{dx}{dt}$

$\frac{dy}{dt} = 5 cm/s$

4.

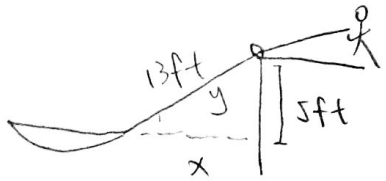
$V = \frac{1}{3}\pi r^2 h$ $\quad \frac{dV}{dt} = 15 cm^3/sec$ $\quad r = \frac{1}{2}h = 1.5 cm$

$\frac{dV}{dt} = \frac{1}{3}\pi r^2 \cdot \frac{dh}{dt}$

$15 = \frac{1}{3}\pi (1.5)^2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{20}{3\pi} = 2.12 cm/s$

5.



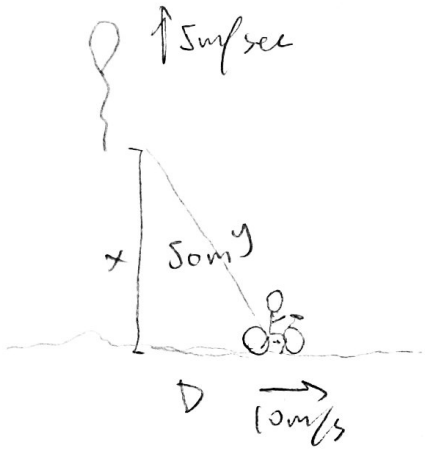
$x^2+5^2=y^2$ $\quad \frac{dy}{dt}=0.6\,ft/sec$ $\quad y=13$ $\quad x=12$

$2x\cdot\frac{dx}{dt}=2y\cdot\frac{dy}{dt}$

$24\frac{dx}{dt}=26\times0.6$

$\frac{dx}{dt}=0.65\,ft/s$

6.



$x^2+D^2=y^2$ $\quad \frac{dx}{dt}=5m/sec$ $\quad \frac{dD}{dt}=10m/sec$

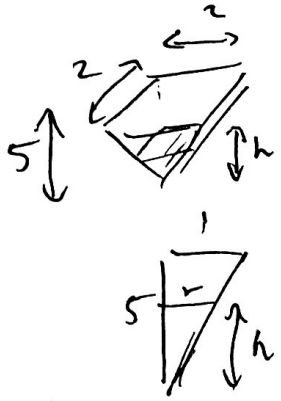
$2x\cdot\frac{dx}{dt}+2D\cdot\frac{dD}{dt}=2y\frac{dy}{dt}$

$2\times60\times5+2\times20\times10=2\times\sqrt{60^2+20^2}\cdot\frac{dy}{dt}$

$\frac{dy}{dt}=7.9\,m/s$

(x after 2 sec is $50m+5m/s\times2s=60m$
y is $10m/s\times2s=20m$)

7.)

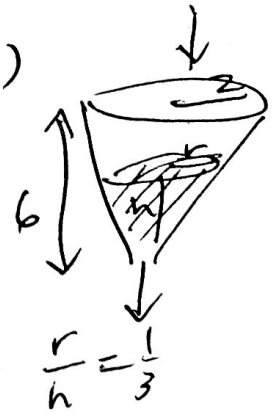


$V=\frac{1}{3}\pi r^2h$ $\quad \frac{dV}{dt}=3m^3/min$

$\frac{h}{r}=5$ $\quad r=\frac{h}{5}$ $\Rightarrow V=\frac{4h^3}{75}$

$\frac{dV}{dt}=\frac{4}{25}h^2\frac{dh}{dt}=3$ $\Rightarrow \left.\frac{dh}{dt}\right|_{h=4}=\frac{75}{16\cdot4}=\frac{75}{64}\,m/min$

8.)

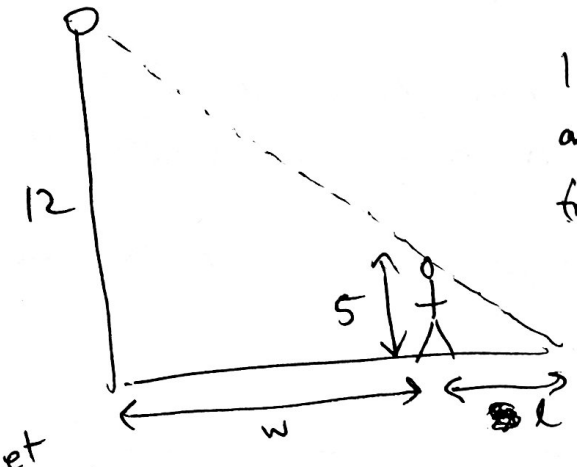


$\left.\frac{dV}{dt}\right_{out}=-0.1\,m^3/min$ $\quad \left.\frac{dV}{dt}\right_{in}=K$ $\quad \left.\frac{dh}{dt}\right|_{h=2}=0.2\,m/min$

$V=\frac{1}{3}\pi r^2h=\frac{\pi}{27}h^3$

$\frac{dV}{dt}=K-0.1=\frac{\pi}{9}h^2\frac{dh}{dt}$ $\Rightarrow K=\frac{4\pi}{9}\cdot\frac{1}{5}+0.1$

9



let l be length of shadow and T be distance of tip of shadow from the pole.

$$\frac{l}{5} = \frac{l+w}{12} = \frac{w}{7}$$

w feet
t seconds

$$\frac{dw}{dt} = 3.5$$

$$\frac{d}{dt}(7l = 5w)$$

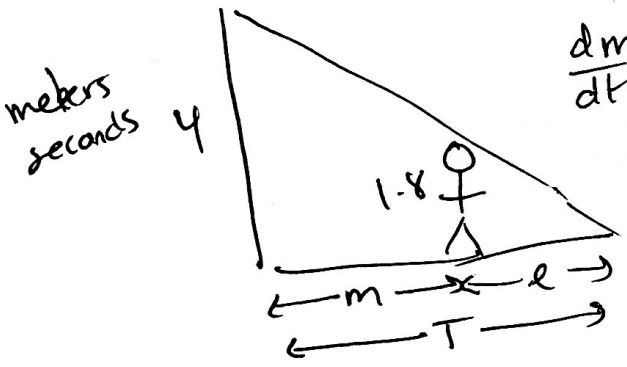
$$\frac{dl}{dt} = \frac{5}{7}\frac{dw}{dt} = 2.5 \text{ feet/s}$$

and $T = w + l$

$$\frac{dT}{dt} = \frac{dw}{dt} + \frac{dl}{dt}$$

$$= 3.5 + 2.5 = 6 \text{ feet/s}$$

10



$$\frac{dm}{dt} = -1$$

$$\frac{l}{1.8} = \frac{l+m}{4} = \frac{m}{2.2}$$

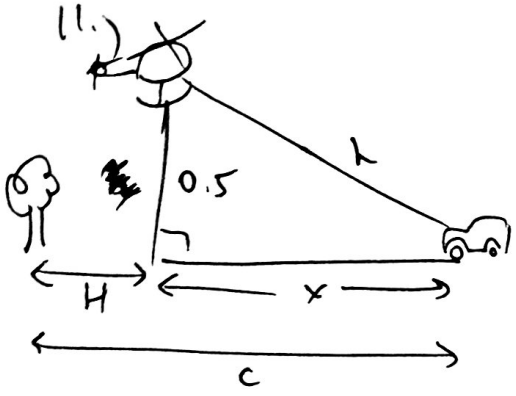
$$\frac{d}{dt}(2.2l = 1.8m)$$

$$2.2\frac{dl}{dt} = 1.8\frac{dm}{dt} \Rightarrow \frac{dl}{dt} = -\frac{9}{11} \text{ m/s}$$

$$\frac{dT}{dt} = \frac{dm}{dt} + \frac{ds}{dt}$$

$$= -1 - \frac{9}{11} = -\frac{20}{11} \text{ m/s}$$

11.)



let H be dist from helicop. to a tree

let c be dist from car to tree

$c = x + H$

$\frac{dH}{dt} = 150 \quad \left.\frac{dh}{dt}\right|_{h=1} = -190$

$\frac{d}{dt}\left(0.5^2 + x^2 = h^2\right) \Rightarrow 2x\frac{dx}{dt} = 2h\frac{dh}{dt}$

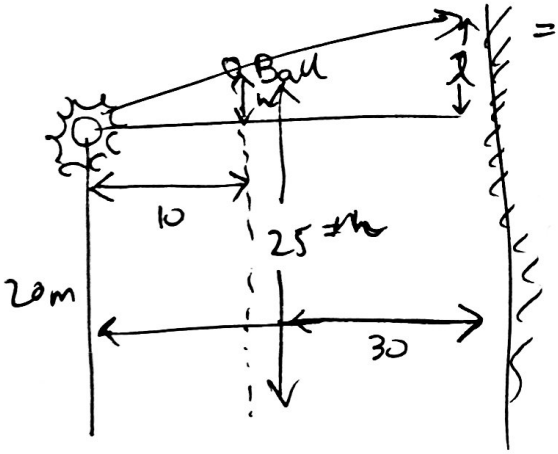
$x\left(\frac{dc}{dt} - \frac{dH}{dt}\right) = h\frac{dh}{dt}$

$\Rightarrow \frac{dc}{dt} = \frac{h}{x}\frac{dh}{dt} + \frac{dH}{dt}$

$\left.\frac{dc}{dt}\right|_{h=1} = \frac{1}{\sqrt{3/4}}(-190) + 150$

$= \frac{-380}{\sqrt{3}} + 150 \text{ mph}$

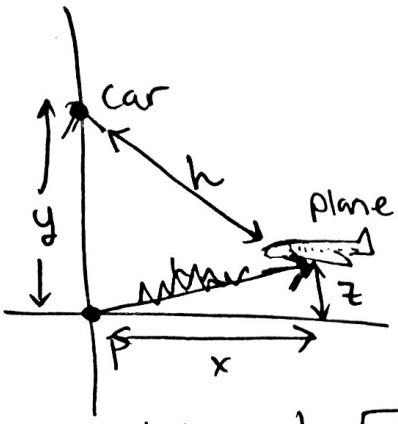
12.)



$\frac{l}{30} = \frac{h}{10} \Rightarrow \frac{dl}{dt} = 3\frac{dh}{dt}$

$= -18 \text{ m/s}$

13

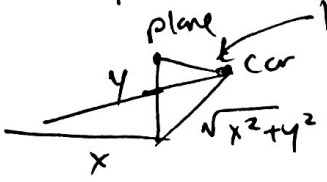


$\left.\frac{dy}{dt}\right|_{\substack{y=10\\x=15}} = 80$

$\left.\frac{dx}{dt}\right|_{\substack{y=10\\x=15}} = 200$

$\frac{dz}{dt} = 0$

$z = 2$



$h = \sqrt{x^2+y^2+z^2}$

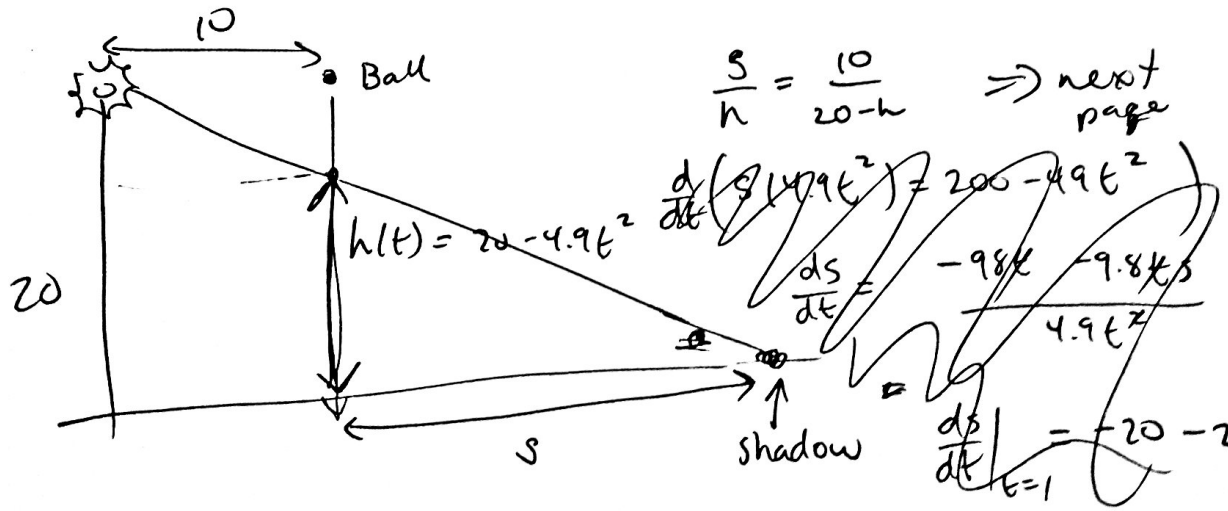
$$\frac{dh}{dt} = \frac{1}{h}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$$

$$= \frac{15(200) + 10(80)}{\sqrt{15^2+10^2+2^2}}$$

14.) now $\frac{dz}{dt} = 10$ $\frac{dx}{dt} = -200$

$$\frac{dh}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt}}{\sqrt{x^2+y^2+z^2}} = \frac{15(-200) + 10(80) + 2(10)}{\sqrt{15^2+10^2+2^2}}$$

15.)



$\frac{s}{h} = \frac{10}{20-h}$ ⇒ next page

$\frac{d}{dt}(s(20-4.9t^2)) = 200 - 49t^2$

$\frac{ds}{dt} = \frac{-98t - 9.8t^3}{4.9t^2}$

$\left.\frac{ds}{dt}\right|_{t=1} = -20 - 2s$

15.)

$$\frac{s}{h} = \frac{10}{20-h} \Rightarrow \frac{d}{dt}(20s - sh = 10h)$$

$$20\frac{ds}{dt} - s\frac{dh}{dt} - h\frac{ds}{dt} = 10\frac{dh}{dt}$$

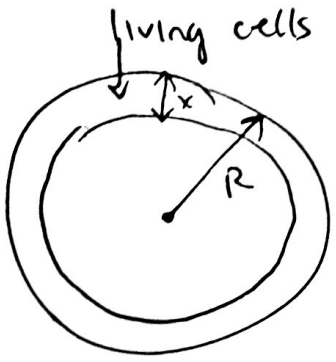
$$\frac{ds}{dt} = \frac{10\frac{dh}{dt} + s\frac{dh}{dt}}{20-h}$$

$h(1) = 15.1 \qquad s = \frac{15.1}{4.9}$

$$\frac{dh}{dt}\Big|_{t=1} = -9.8$$

$$= 81.6$$

16.)



$$F = \frac{\pi R^2 - \pi(R-x)^2}{\pi(R)^2}$$

$$F = \frac{2Rx - x^2}{R^2} \qquad \frac{dF}{dt}\Big|_{R=5x} = ?$$

$A = \pi r^2$

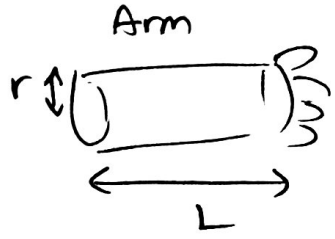
$$\frac{dF}{dt} = \frac{2x(R^2)\frac{dR}{dt} - 2R\frac{dR}{dt}(2Rx - x^2)}{R^4}$$

$$\frac{dF}{dt}\Big|_{R=5x} = \frac{-2R^{\not x}x\frac{dR}{dt} + 2\not R x^2\frac{dR}{dt}}{R^{\not 3}}$$

$$\frac{dF}{dt}\Big|_{R=5x} = \frac{(-10x^2 + 2x^2)\frac{dR}{dt}}{125x^3}$$

$$= \frac{-8}{125x}\frac{dR}{dt}$$

17.)



$\frac{L}{r} = C$

$\frac{dr}{dt} = a \quad \frac{dm}{dt} = ? \quad \rho = 1 g/cm^3$

$\Rightarrow m = V$

$V = \pi r^2 L$

$\Rightarrow m = \pi r^2 \rho L$

$L = Cr$

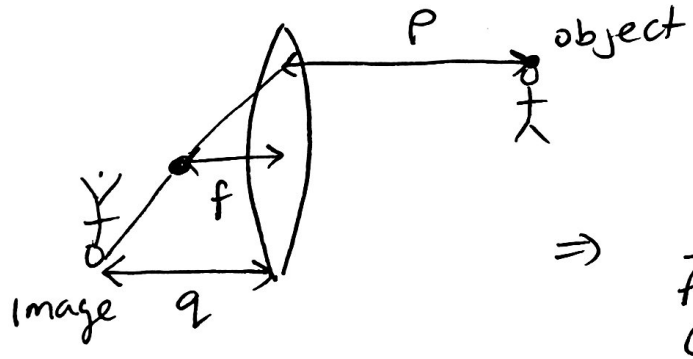
$\frac{dL}{dt} = C\frac{dr}{dt} = aC = \frac{aL}{r}$

$\frac{dm}{dt} = \pi\rho\left(2r\frac{dr}{dt}L + r^2\frac{dL}{dt}\right)$

$= \pi\left(2raL + r^2\frac{aL}{r}\right)$

$= 3\pi arL$

18.)



$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

$\Rightarrow \frac{1}{f^2}\frac{df}{dt} = \frac{1}{p^2}\frac{dp}{dt} + \frac{1}{q^2}\frac{dq}{dt}$

0

$\frac{dq}{dt} = -\frac{q^2}{p^2}\frac{dp}{dt}$

$p = 30 \quad \frac{dp}{dt} = 4$

$\frac{1}{10} = \frac{1}{30} + \frac{1}{q} \Rightarrow q = 15$

$\frac{dq}{dt} = -\frac{1}{4} \times 4 = -1 \text{ cm/s}$

↑ toward lens