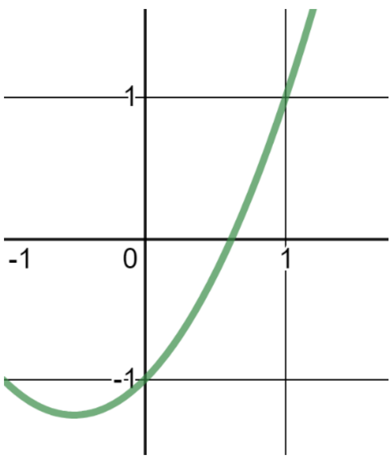
Linearization and Newton's Method

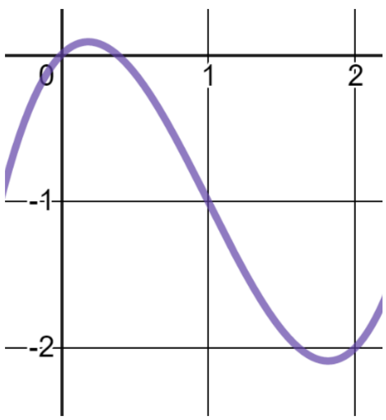
Goal:
• Understands linearization is just the tangent line at a point. • Understands that linearization "formula" is just point-slope form of tangent line. • Can use repeated linearization to approximate zeros using your calculator and ANS
Terminology: • Linearization • Newton's Method

On the board find the equation of the tangent line to the three curves below at $x = 1$ and sketch the curve and its tangent line.

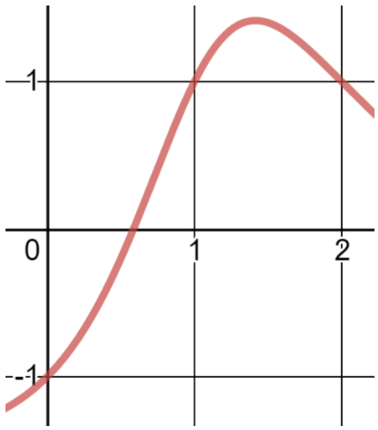
1. $f(x) = x^2 + x - 1, @\, x = 1$



2. $g(x) = x^3 - 3x^2 + x, @\, x = 1$



3. $h(x) = \frac{2x}{x^2 - 2x + 2} - 1,$ $@\, x = 1$



What is relevant or what stands out when you compare the tangent line to the original curve?

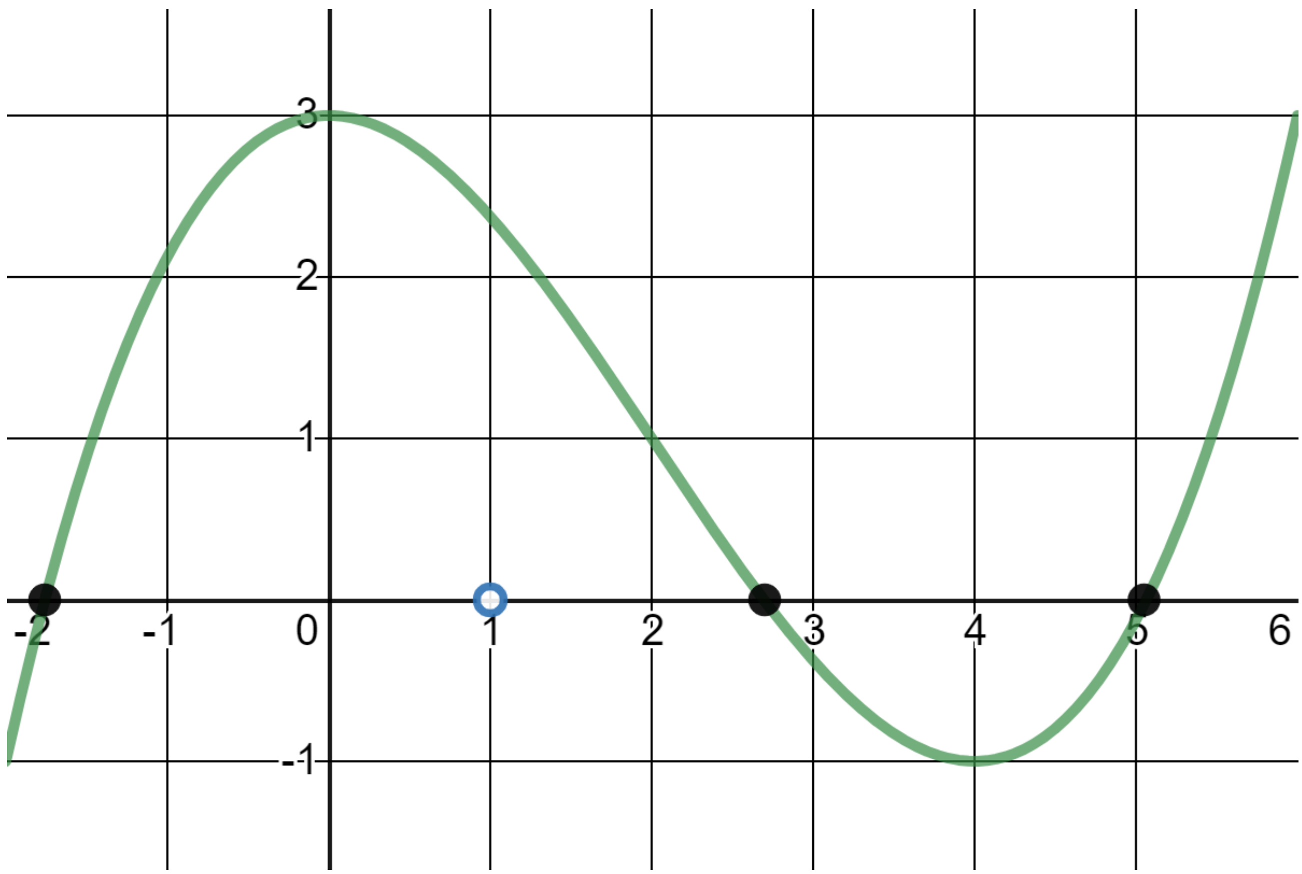
What we have done is create a **linearization** of the functions at the point $x = 1$

In general if we want to linearize a function at the point $x = a$ we will use point/slope form and have

Newton's Method looks to find the zeros of a function by repeatedly using linearization as follows

- Guess the zero to f, call it z_0
- Linearize around $x = z_0$, find $L_0(x) = f'(z_0)(x - z_0) + f(z_0)$
- Find the zero to the linearization, call it z_1
- Repeat and linearize around $x = z_1$, find $L_1(x)$
- Continue until z_n approaches a limit point

What does this look like?



So after 3 iterations, we are pretty close to the actual zero.

What if we started with $z_0 = -1$? What is we started with $z_0 = 4$?

In general we are solving for the zero of $L_k(x) = f'(z_k)(x - z_k) + f(z_k)$ and then using that to make a new linearization around $x = z_{k+1}$

Example: Find the zeros of $f(x) = x^2 + x - 1$

Practice: Find the zeros of $g(x) = x^3 - 3x^2 + x$

Practice Problems: 4.5: # 1-3, 8, (4 and 5 are good practice to if you need more)

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In Class Evidence

1. Start with $z_0 = 0$ and use Newton's method to find the solution to

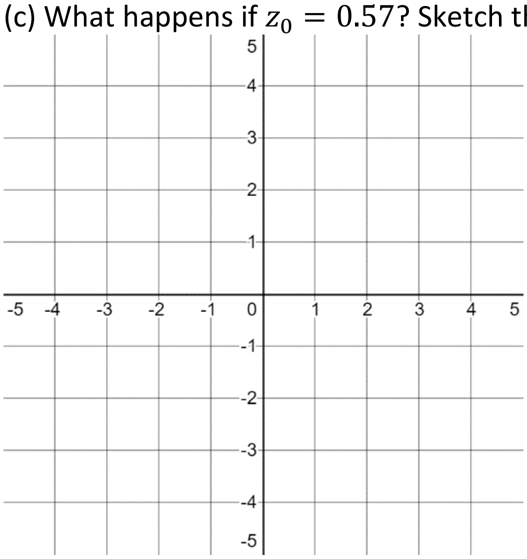
$$x^3 + 2x + 1 = 0$$

3. (a) Use Newton's method with $z_0 = 2$ to find the solution to

$$x^3 - x - 2 = 0$$

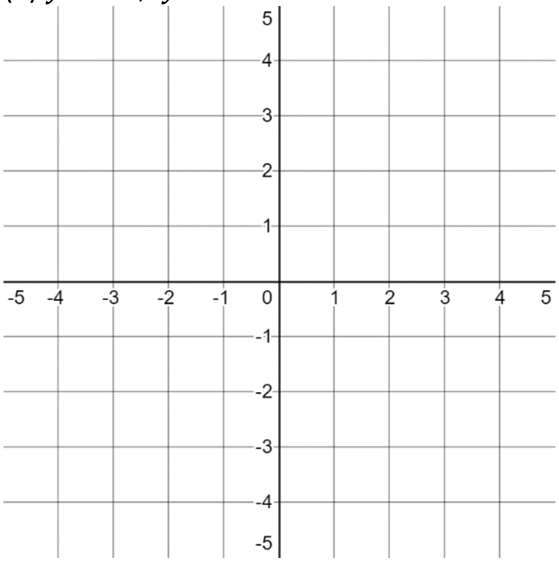
(b) What happens if $z_0 = 1$?

(c) What happens if $z_0 = 0.57$? Sketch the graph to help explain why this is such a poor choice.

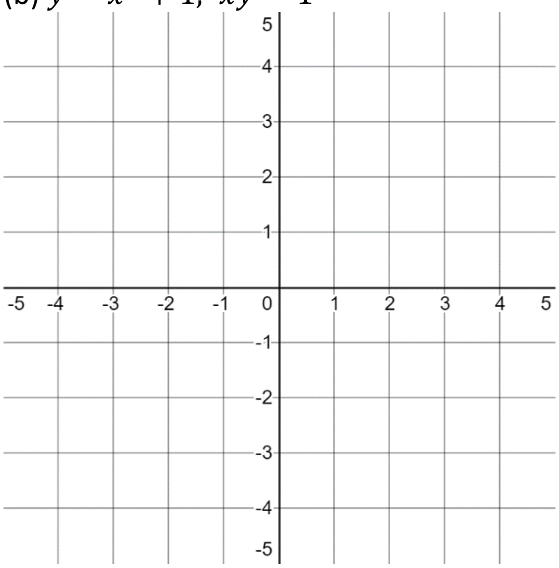


8. Sketch the following pairs of curves and find the coordinates of their point of intersection.

(a) $y = x^3,\ y = x + 1$



(b) $y = x^2 + 1,\ xy = 1$



(c) $y = x^5,\ y = 3x - 1$