First Derivative Test and Newton's Method Practice

1. Find all maximum and minimums for the following functions:
   a. $f(x) = x^3 - 3x^2 + 5$

   b. $g(x) = -2x^3 + 24x$ on $[-1, 3]$

   c. $h(x) = \frac{1}{2}x^4 + 8x - 5$

2. Use Newton's method to find the zeros to the slope so you can find the local extremas. (Note: This function was made so the slope has zeros at very recognizable fractions in the interval $[-7, 4]$)

$$k(x) = \frac{1}{1000}(14.4x^5 + 169.5x^4 + 391.667x^3 - 2050x^2 - 10500x)$$

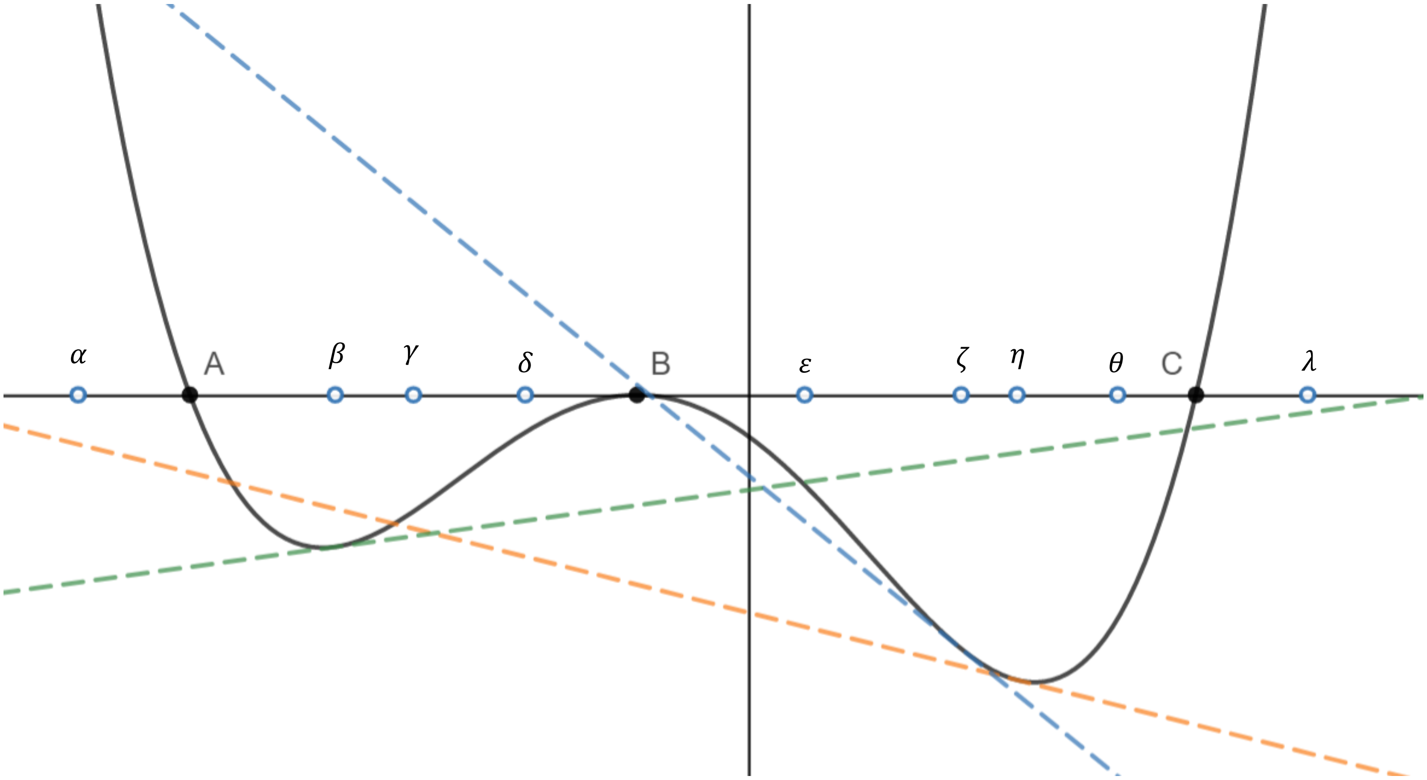
3. Use Newton's method to find the zeros of the slope so you can find ALL extremas of the function on the interval $[-2, 1]$ (This function was made so the coefficients look pretty and that was it)

$$L(x) = x^5 + 2x^4 - 3x^3 - 4x^2 + 5x$$

4. Why does Newton's method fail to find zeros of the following function?

$$M(x) = x^4 + x + 0.5$$

5. What zeros will the different choices of z_0 find using Newton's method? (the circled points labeled in Greek letters). Some tangent lines are given.



What would happen if $z_0 = \delta$ but the entire graph was shifted down very slightly so that there was no zero at B?

Solutions:

1.
   a. Local Max: $f(0) = 5$; Local Min: $g(2) = 1$
   b. Absolute Max: $g(2) = 32$; Absolute Min: $g(-1) = -22$; Local Min: $g(3) = 18$ (endpoint)
   c. Absolute Min: $h\left(-\sqrt[3]{4}\right) = -14.524$; No maximums
2. Local Max: $k(-5.25) = 13.281$; Local Min: $k(2.5) = -24.915$; Note that $x = -3.33$ is not a critical point since $k'(x)$ does not change signs around it
3. Absolute Max: $L(0.507) = 1.281$; Absolute Min: $L(-1.071) = -5.035$; Local Min: $L(0.934) = 0.969$; Local Min: $L(-2) = -2$ (endpoint); Local Max: $L(-1.971) = -1.986$; Local Max: $L(1) = 1$ (endpoint)
4. Hint: Think about the extremas of the curve. Another hint is in the next problem.
5. A is found by: α and η
   B is found by: $\gamma, \delta, \varepsilon$, and ζ
   C is found by: β, θ, and λ
   If the graph is shifted down then using $z_0 = \delta$ will result in what happens with the previous question.