Extra Optimization Problems

1. A rectangle is inscribed between the curve $y = -|x| + 1$ and the x-axis. What is the largest area that rectangle can have, and what are its dimensions?

2. What are the dimensions of the lightest open-top right circular cylindrical can, that will hold a volume of 1000 cm^3?

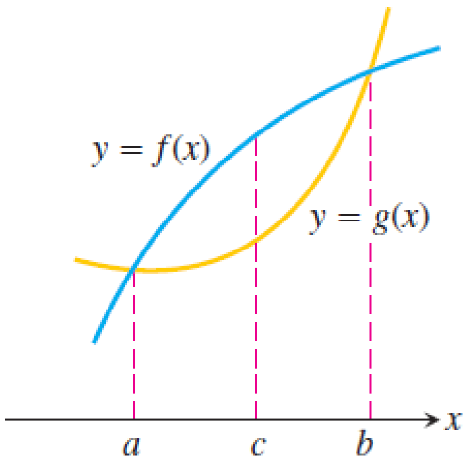
3. A window is in the form of a rectangle surmounted by a semicircle. The rectangle is of clear glass, whereas the semicircle is of tinted glass that transmits only half as much light per unit area as clear glass does. The total perimeter is fixed. Find the proportions of the window that will admit the most light. Neglect the thickness of the frame.

4. A silo (base not included) is to be constructed in the form of a cylinder surmounted by a hemisphere. The cost of construction per square unit of surface area is twice as great for the hemisphere as it is for the cylindrical sidewall. Determine the dimensions to be used if the volume is fixed and the cost of construction is to be kept to a minimum. Neglect the thickness of the silo and waste in construction.

5. Jane is 2 mi offshore in a boat and wishes to reach a coastal village 6 mi down a straight shoreline from the point nearest the boat. She can row 2 mph and can walk 5 mph. Where should she land her boat to reach the village in the least amount of time?

6. You operate a tour service that offers the following rates: $200 per person if 50 people (the minimum number to book the tour) go on the tour. For each additional person, up to a maximum of 80 people total, the rate per person is reduced by $2. It costs $6000 (a fixed cost) plus $32 per person to conduct the tour. How many people does it take to maximize your profit?

7. Let $f(x)$ and $g(x)$ be the differentiable functions graphed below. Point c is the point where the vertical distance between the curves is the greatest. Is there anything special about the tangents to the two curves at c? Give reasons for your answer.



Solutions:

1. Area 0.5 square units; dimensions 1 by 0.5
2. Minimal surface area $r, h = 10/\sqrt[3]{\pi}$
3. Proportions $2r = \frac{8h}{4+\pi}$
4. Minimal cost $h = \left(\frac{3V}{\pi}\right)^{\frac{1}{3}}$
5. Land $\frac{4}{\sqrt{21}} = 0.87$ miles down the shoreline
6. 67 people
7. Consider the difference function and minimize it.