25. Even, since the function involves only even powers of x.

26. Odd, since the function is a sum of odd powers of x.

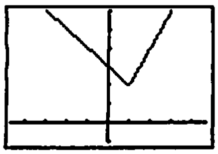
27. Odd, since the function is a quotient of an odd function (x^3) and an even function (x^2-1).

28. Neither, since, (for example), $y(-2) = 4^{1/3}$ and $y(2) = 0$.

29. Neither, since, (for example), $y(-1)$ is defined and $y(1)$ is undefined.

30. Even, since the function involves only even powers of x.

31. (a)

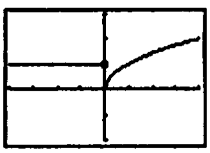


[–4.7, 4.7] by [–1, 6]

(b) $(-\infty, \infty)$ or all real numbers

(c) $[2, \infty)$

32. (a)

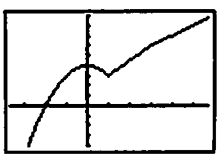


[–4, 4] by [–2, 3]

(b) $(-\infty, \infty)$ or all real numbers

(c) $[0, \infty)$

33. (a)

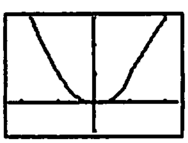


[–3.7, 5.7] by [–4, 9]

(b) $(-\infty, \infty)$ or all real numbers

(c) $(-\infty, \infty)$ or all real numbers

34. (a)



[–2.35, 2.35] by [–1, 3]

(b) $(-\infty, \infty)$ or all real numbers

(c) $[0, \infty)$

35. Because if the vertical line test holds, then for each x-coordinate, there is at most one y-coordinate giving a point on the curve. This y-coordinate would correspond to the value assigned to the x-coordinate. Since there is only one y-coordinate, the assignment would be unique.

36. If the curve is not $y = 0$, there must be a point (x, y) on the curve where $y \neq 0$. That would mean that (x, y) and $(x, -y)$ are two different points on the curve and it is not the graph of a function, since it fails the vertical line test.

37. No

38. Yes

39. Yes

40. No

41. Line through (0, 0) and (1, 1): $y = x$

Line through (1, 1) and (2, 0): $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ -x+2, & 1 < x \le 2 \end{cases}$$

42. $$f(x) = \begin{cases} 2, & 0 \le x < 1 \\ 0, & 1 \le x < 2 \\ 2, & 2 \le x < 3 \\ 0, & 3 \le x \le 4 \end{cases}$$

43. Line through (0, 2) and (2, 0): $y = -x + 2$

Line through (2, 1) and (5, 0): $m = \frac{0-1}{5-2} = \frac{-1}{3} = -\frac{1}{3}$,

so $y = -\frac{1}{3}(x-2) + 1 = -\frac{1}{3}x + \frac{5}{3}$

$$f(x) = \begin{cases} -x+2, & 0 < x \le 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \le 5 \end{cases}$$

44. Line through (–1, 0) and (0, –3):

$m = \frac{-3-0}{0-(-1)} = \frac{-3}{1} = -3$, so $y = -3x - 3$

Line through (0, 3) and (2, –1):

$m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$, so $y = -2x + 3$

$$f(x) = \begin{cases} -3x-3, & -1 < x \le 0 \\ -2x+3, & 0 < x \le 2 \end{cases}$$

45. Line through (−1, 1) and (0, 0): $y = -x$
Line through (0, 1) and (1, 1): $y = 1$
Line through (1, 1) and (3, 0):

$$m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2},$$

$$\text{so } y = -\frac{1}{2}(x-1) + 1 = -\frac{1}{2}x + \frac{3}{2}$$

$$f(x) = \begin{cases} -x, & -1 \le x < 0 \\ 1, & 0 < x \le 1 \\ -\frac{1}{2}x + \frac{3}{2}, & 1 < x < 3 \end{cases}$$

46. Line through (−2, −1) and (0, 0): $y = \frac{1}{2}x$

Line through (0, 2) and (1, 0): $y = -2x + 2$
Line through (1, −1) and (3, −1): $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x, & -2 \le x \le 0 \\ -2x+2, & 0 < x \le 1 \\ -1, & 1 < x \le 3 \end{cases}$$

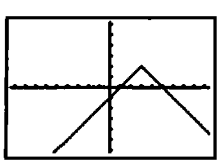
47. Line through $\left(\frac{T}{2}, 0\right)$ and $(T, 1)$:

$$m = \frac{1-0}{T-(T/2)} = \frac{2}{T}, \text{ so } y = \frac{2}{T}\left(x - \frac{T}{2}\right) + 0 = \frac{2}{T}x - 1$$

$$f(x) = \begin{cases} 0, & 0 \le x \le \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \le T \end{cases}$$

48. $$f(x) = \begin{cases} A, & 0 \le x < \frac{T}{2} \\ -A, & \frac{T}{2} \le x < T \\ A, & T \le x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \le x \le 2T \end{cases}$$

49. **(a)**



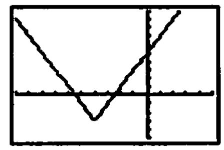
[−9.4, 9.4] by [−6.2, 6.2]

Note that $f(x) = -|x-3| + 2$, so its graph is the graph of the absolute value function reflected across the x-axis and then shifted 3 units right and 2 units upward.

(b) $(-\infty, \infty)$

(c) $(-\infty, 2]$

50. **(a)** The graph of $f(x)$ is the graph of the absolute value function stretched vertically by a factor of 2 and then shifted 4 units to the left and 3 units downward.



[−10, 5] by [−5, 10]

(b) $(-\infty, \infty)$ or all real numbers

(c) $[-3, \infty)$

51. **(a)** $f(g(x)) = (x^2 - 3) + 5 = x^2 + 2$

(b) $g(f(x)) = (x+5)^2 - 3$
$= (x^2 + 10x + 25) - 3$
$= x^2 + 10x + 22$

(c) $f(g(0)) = 0^2 + 2 = 2$

(d) $g(f(0)) = 0^2 + 10 \cdot 0 + 22 = 22$

(e) $g(g(-2)) = [(-2^2) - 3]^2 - 3 = 1^2 - 3 = -2$

(f) $f(f(x)) = (x+5) + 5 = x + 10$

52. **(a)** $f(g(x)) = (x-1) + 1 = x$

(b) $g(f(x)) = (x+1) - 1 = x$

(c) $f(g(x)) = 0$

(d) $g(f(0)) = 0$

(e) $g(g(-2)) = (-2-1) - 1 = -3 - 1 = -4$

(f) $f(f(x)) = (x+1) + 1 = x + 2$

53. **(a)** Since $(f \circ g)(x) = \sqrt{g(x) - 5} = \sqrt{x^2 - 5}$, $g(x) = x^2$.

(b) Since $(f \circ g)(x) = 1 + \frac{1}{g(x)} = x$, we know that

$$\frac{1}{g(x)} = x - 1, \text{ so } g(x) = \frac{1}{x-1}.$$

(c) Since $(f \circ g)(x) = f\left(\frac{1}{x}\right) = x$, $f(x) = \frac{1}{x}$.

(d) Since $(f \circ g)(x) = f(\sqrt{x}) = |x|$, $f(x) = x^2$.

The completed table is shown. Note that the absolute value sign in part (d) is optional.

$g(x)$	$f(x)$	$(f \circ g)(x)$
x^2	$\sqrt{x-5}$	$\sqrt{x^2-5}$
$\frac{1}{x-1}$	$1+\frac{1}{x}$	$x, x \ne -1$
$\frac{1}{x}$	$\frac{1}{x}$	$x, x \ne 0$
$\sqrt{x}$	x^2	$\|x\|, x \ge 0$