Function Compositions

KNOW	DO	UNDERSTAND
Be able identify a function as a composition of other functions.	Use Desmos and Geogebra to graph compositions. Use correct notation when describing compositions. Evaluate compositions algebraically.	*Function Characteristics*: Be able to justify how the domain and range change after a composition.

Vocab & Notation

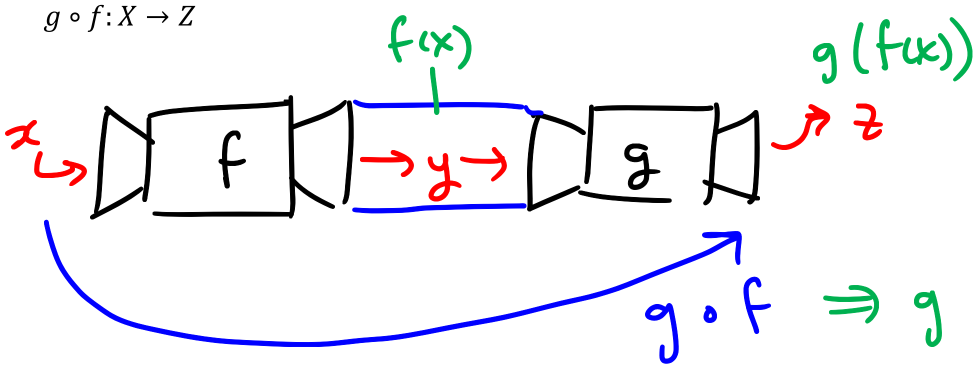
- Composition: $f \circ g$
- Subset: $A \subset B$
- Intersection of sets: $A \cap B$
- Union of sets: $A \cup B$

When we have two functions, it's natural to want to stitch these two functions together.

Definition: Consider the two functions

$$f: X \to Y \quad \text{and} \quad g: Y \to Z$$

Then the resulting function is called a **composition** and is denoted as
$g \circ f: X \to Z$



$g \circ f \Rightarrow$ g of f

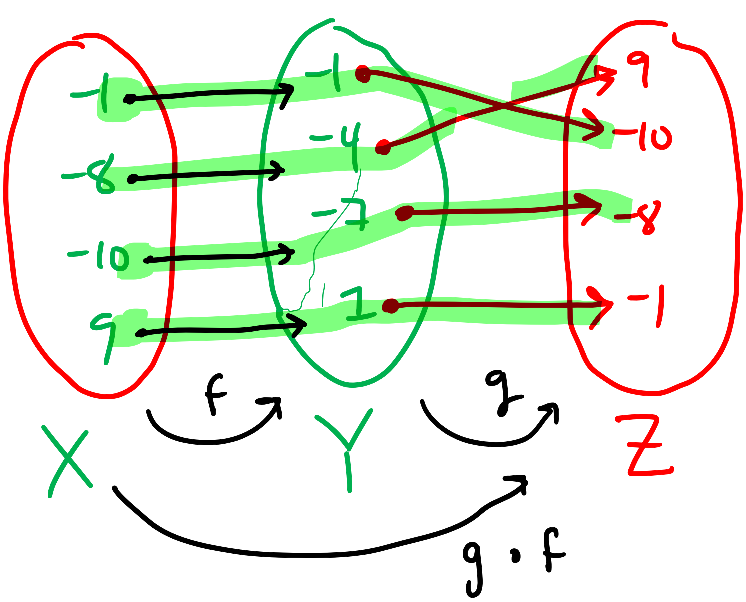
Example: Let $f = \{(-1,-1), (-8,-4), (-10,-7), (9,1)\}$ and $g = \{(-4,9), (-1,-10), (-7,-8), (1,-1)\}$

Illustrate $g \circ f$ using a map between sets and use it to determine $g(f(-1))$ and $(g \circ f)(-8)$

Set (-8, -4) vs point / coordinate (-8, -4)

range

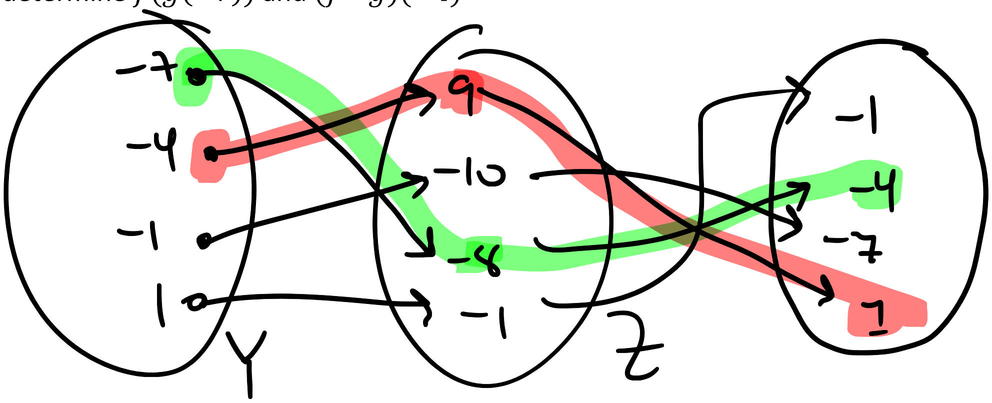
domain



$g(f(-1)) = -10$

$(g \circ f)(-8) = 9$

Practice: We still have $f: \{(-1, -1), (-8, -4), (-10, -7), (9, 1)\}$ and $g: \{(-4, 9), (-1, -10), (-7, -8), (1, -1)\}$ then determine $f(g(-7))$ and $(f \circ g)(-4)$



$f(g(-7)) = -4$

$(f \circ g)(-4) = 1$

Once we know how compositions work, we want to be able to recognize function compositions which comes down to recognizing recurring patterns and symbols. Typically, we will see either see the same thing occur more than once or we can see the layers of functions.

$$\sqrt{(\dots)}, \quad (\dots)^n, \quad |(\dots)|, \quad \frac{1}{(\dots)}$$

Example: If $h(x) = f\big(g(x)\big)$ then determine possible f and g.

$$h(x) = \frac{1}{x^2 - 2} + \sqrt{x^2 - 2}$$

let $g(x) = x^2 - 2$ $\rightarrow$ $h(x) = \frac{1}{g(x)} + \sqrt{g(x)} \sim \frac{1}{\boxed{x}} + \sqrt{\boxed{x}}$

$f(x) = \frac{1}{x} + \sqrt{x}$ $\Rightarrow$ $f(g(x)) = f(x^2 - 2) = \frac{1}{x^2-2} + \sqrt{x^2-2}$

Example: If $k(x) = f\big(g(h(x))\big)$ then determine possible f and g, and h. Note that F is a function

$$k(x) = -F\left(\left|\frac{(\sqrt{x} - 3)^3}{4}\right|\right) + 2$$

1.) $\sqrt{\ }$ it
2.) − by 3
3.) cube it | $h(x)$

4.) divide by 4
5.) abs it | $g(x)$

6.) F of it
7.) make it −ve
8.) + by 2 | $f(x)$

$h(x) = (\sqrt{x} - 3)^3$

$g(x) = \left|\frac{x}{4}\right|$

$f(x) = 2 - F(x)$

*Easily recognizing when you have a composition of functions is vital in calculus!

We also want to be able to compute compositions algebraically and simplify them accurately. This is like what we did last class where we looked at the input being something more complex than a single number.

Practice: Given that $f(x) = x^2 + 4$, $g(x) = 3 - 2x$, $h(x) = -x$, $k(x) = \sqrt{x-4}$

Determine:

$F_1 = f(f(x)) = (x^2+4)^2 + 4 = x^4 + 8x^2 + 20$

$F_2 = f(k(x)) = (\sqrt{x-4})^2 + 4 = x - \cancel{4} + \cancel{4} = x$

$x^2 = 4$

$x = \pm 2$

$F_3 = k(f(x)) = \sqrt{x^2 + \cancel{4} - \cancel{4}} = \sqrt{x^2} = |x|$

$F_3(-2) = \sqrt{4+4-4} = \sqrt{4} = 2$

$F_4 = f(h(x)) = (-x)^2 + 4 = x^2 + 4$

$F_5 = f\left(g\left(h(k(x))\right)\right) = \left(3 - 2\left(-\left(\sqrt{x-4}\right)\right)\right)^2 + 4$

$= (3 + 2\sqrt{x-4})^2 + 4$

$= 13 + 12\sqrt{x-4} + 4(x-4)$

Practice Problems: 10.3 page 507 – 509 # 1-10, 14, 17, 18, 22, C2
Building Understanding Worksheet 1