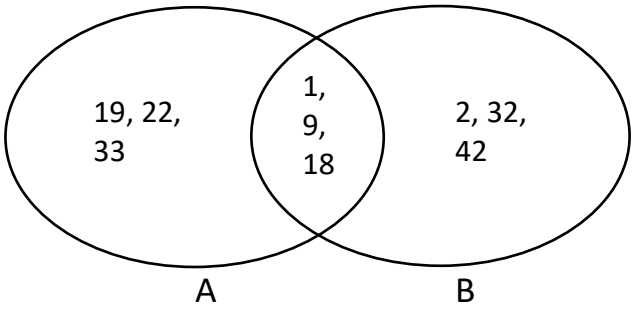
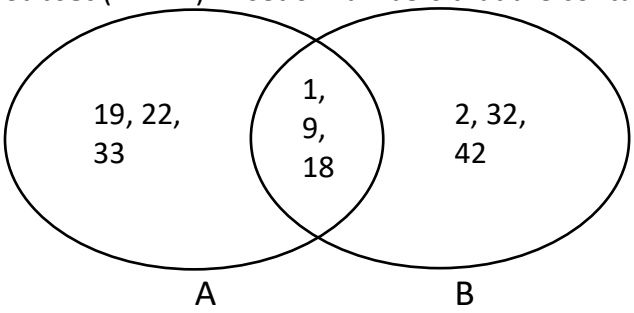
Function Composition: Domain and Range

Here are some vocabularies to know before we dive deep into domain and range of some composite functions:

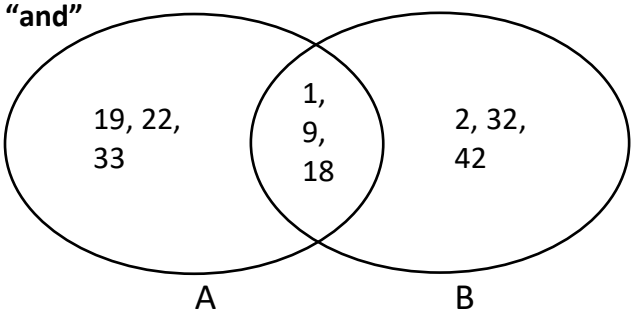
Suppose there are two sets of numbers, set A and B:



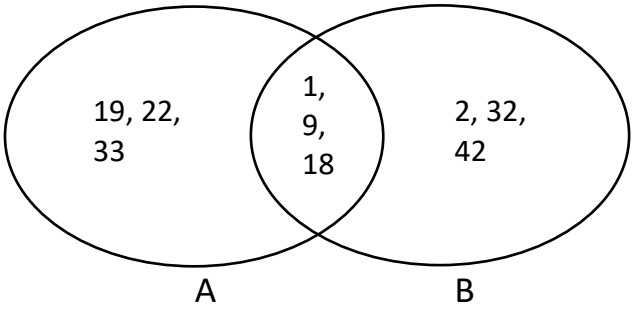
Subset ($A \subset B$)**:** A set of numbers that are contained in a larger set.



Intersection of sets ($A \cap B$)**:** The intersection of two sets contains **only** the elements that are in **both** sets. – **"and"**



Union of sets ($A \cup B$): The union of two sets contains all the elements contained in **either** set (or **both** sets). – **"or"**



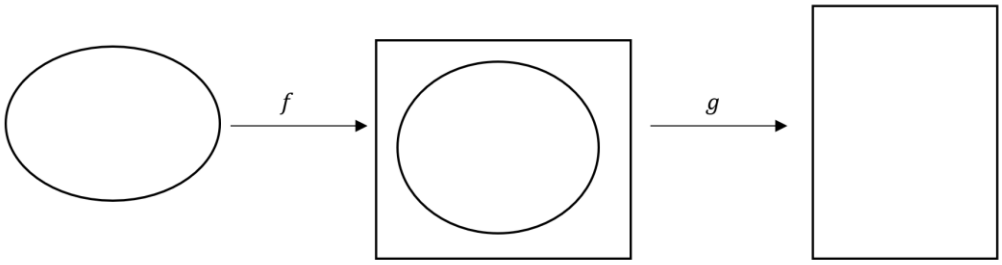
The big part that we need to **understand** this unit is how domain and range of individual function are changed when they combine (because they may not match perfectly in the middle).

Consider the functions:

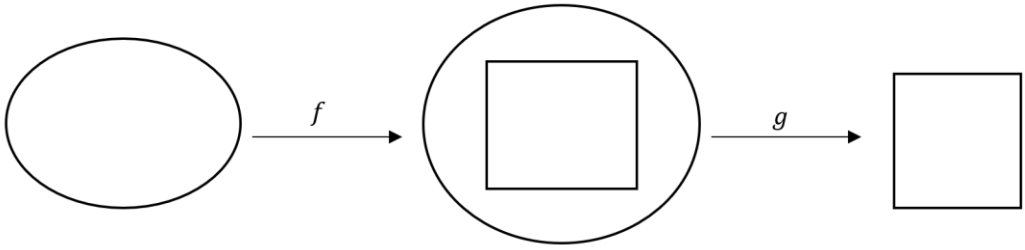
$$f: A \to B \text{ and } g: C \to D$$

Where $B \neq C$

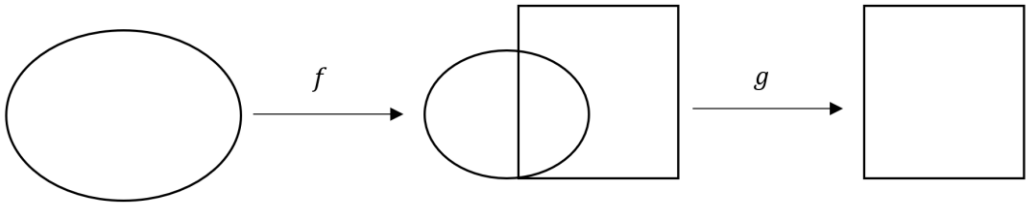
Case 1: $B \subset C$ (*B* is a subset of *C*)



Case 2: $C \subset B$ (*C* is a subset of *B*)



Case 3: $B \cap C$ Neither B nor C is a subset of each other.



Questions:

1. If $f = \{(2,8),(4,17),(6,20),(11,23),(12,28)\}$ and $g = \{(8,30),(20,31),(17,24),(23,18),(28,34),(29,35),(42,53)\}$, determine domain and range of $g \circ f$:

2. If $f = \{(3,9),(4,11),(6,13),(10,15),(19,26)\}$ and $g = \{(9,14),(13,38),(26,37)\}$, determine domain and range of $g \circ f$:

3. If $g = \{(1,3),(2,7),(4,12),(6,19),(9,27)\}$ and $f = \{(3,31),(5,34),(11,13),(12,24),(27,36)\}$, determine domain and range of $f \circ g$:

Extra Practice:

1. If $f(x) = 8x$ and $g(x) = \sqrt{x}$,determine domain and range of $g \circ f$:

2. If $f(x) = (x-2)^2$ and $g(x) = \sqrt{4-x}$, determine domain and range of $g \circ f$: