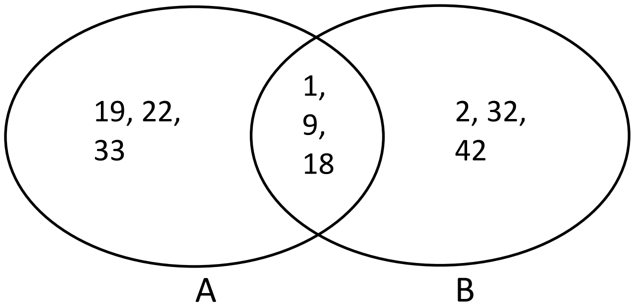
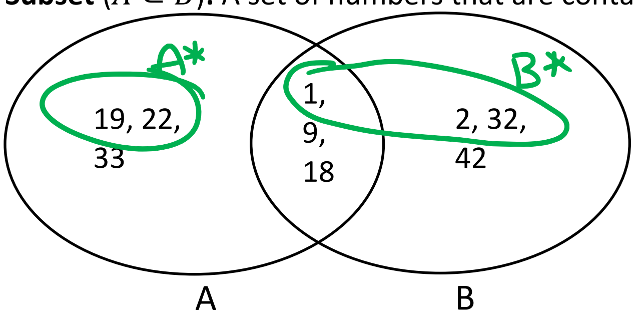
Function Composition: Domain and Range

Here are some vocabularies to know before we dive deep into domain and range of some composite functions:

Suppose there are two sets of numbers, set A and B:



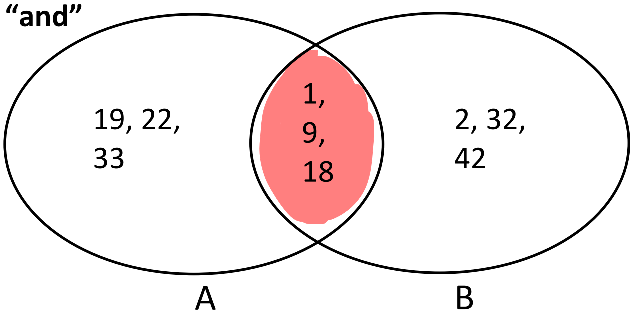
Subset ($A \subset B$)**:** A set of numbers that are contained in a larger set.



$\{19, 22\} \subset A$

$\{1, 2, 32\} \subset B$

Intersection of sets ($A \cap B$)**:** The intersection of two sets contains **only** the elements that are in **both** sets. – **"and"**

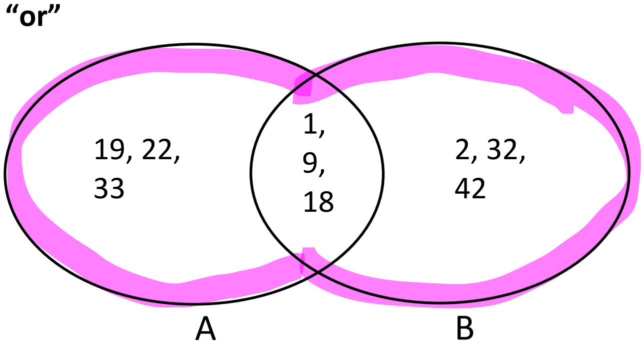


$A \cap B = \{1, 9, 18\}$

intersection

$2 \notin A \cap B$

Union of sets ($A \cup B$): The union of two sets contains all the elements contained in **either** set (or **both** sets). – **"or"**



$A \cup B = \{19, 22, 33, 1, 9, 18, 2, 32, 42\}$

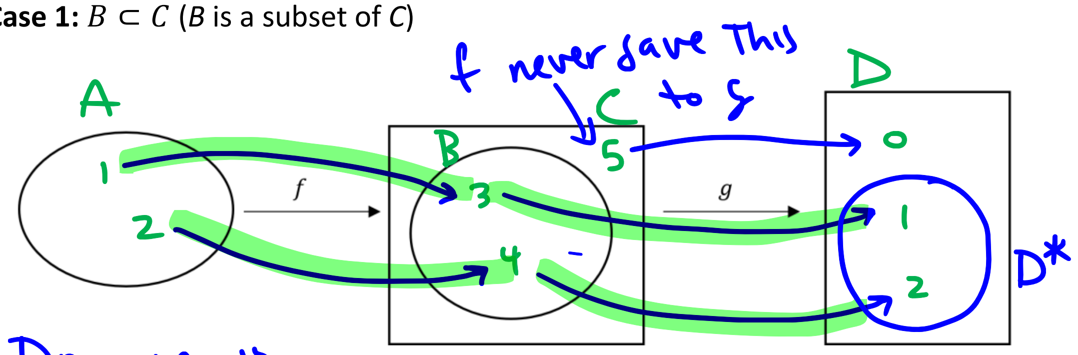
The big part that we need to **understand** this unit is how domain and range of individual function are changed when they combine (because they may not match perfectly in the middle).

Consider the functions:

$$f: A \to \boxed{B \text{ and } g: C} \to D$$

Where $B \neq C$

Case 1: $B \subset C$ (B is a subset of C)

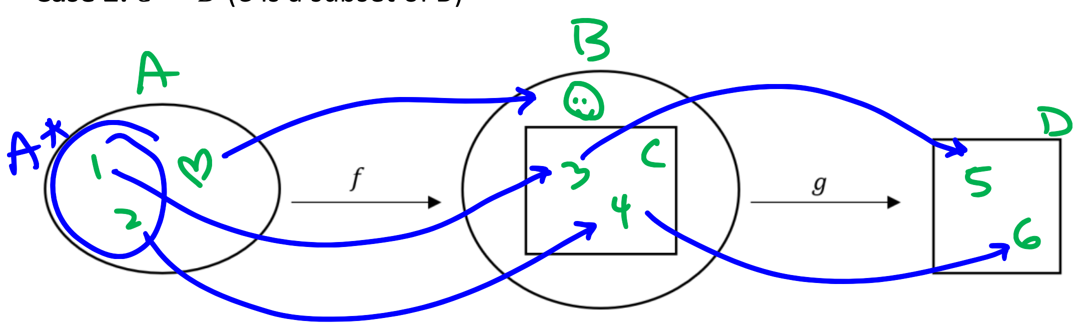


new Range

Domain is still A

D* = g(B) → These are all the things that f gave

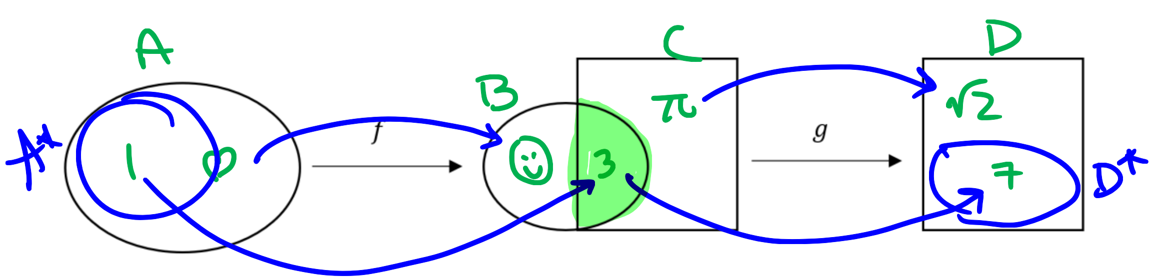
Case 2: $C \subset B$ (C is a subset of B)



Range is D

Must restrict Domain A*, f(A*) = C → whatever we give f must be okay for g

Case 3: $B \cap C$ Neither B nor C is a subset of each other.



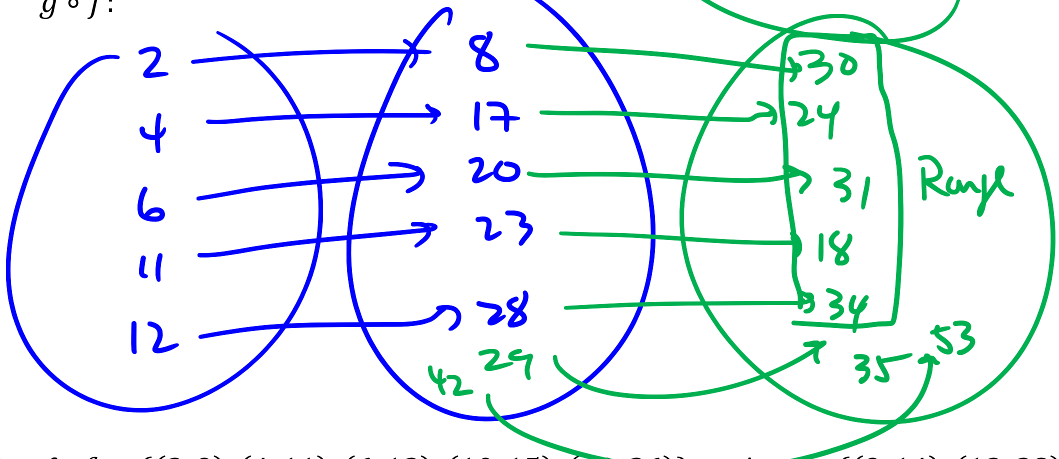
restricted range

restrict domain f(A*) = B∩C

g(B∩C) = D*

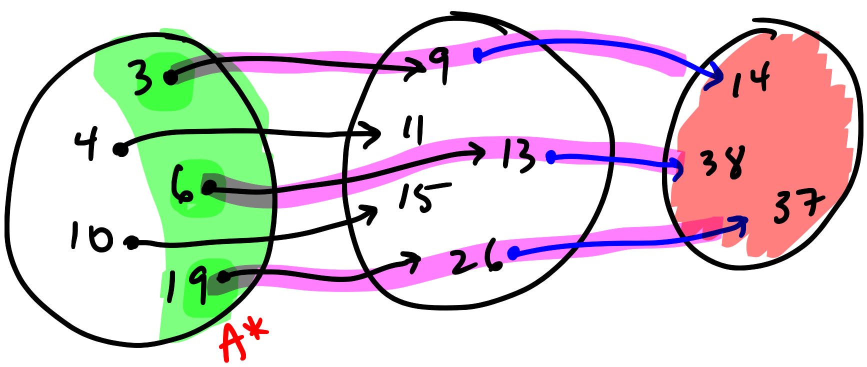
Questions:

1. If $f = \{(2,8),(4,17),(6,20),(11,23),(12,28)\}$ and $g = \{(8,30),(20,31),(17,24),(23,18),(28,34),(29,35),(42,53)\}$, determine domain and range of $g \circ f$:



Range = {30, 24, 31, 18, 34}

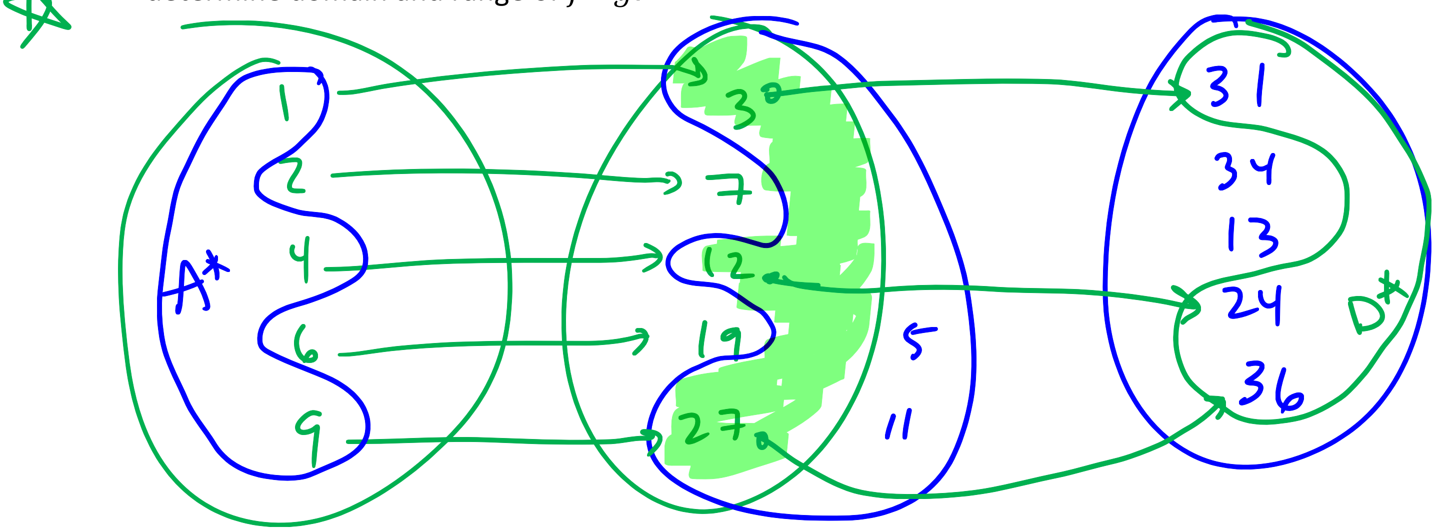
2. If $f = \{(3,9),(4,11),(6,13),(10,15),(19,26)\}$ and $g = \{(9,14),(13,38),(26,37)\}$, determine domain and range of $g \circ f$:



Domain is {3, 6, 19}

Range is not changed just {14, 38, 37}

3. If $g = \{(1,3),(2,7),(4,12),(6,19),(9,27)\}$ and $f = \{(3,31),(5,34),(11,13),(12,24),(27,36)\}$, determine domain and range of $f \circ g$:



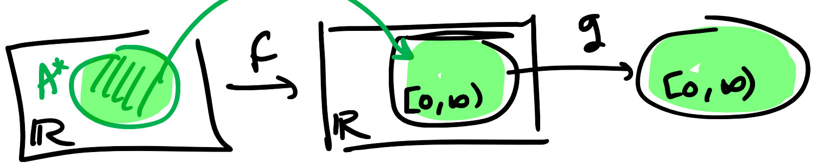
Domain = {1, 4, 9}

Range = {24, 31, 36}

Extra Practice:

1. If $f(x) = 8x$ and $g(x) = \sqrt{x}$, determine domain and range of $g \circ f$:

$f: \mathbb{R} \to \mathbb{R}$ $\quad g: [0,\infty) \to [0,\infty)$



must restrict domain so $f(A^*) = [0,\infty)$

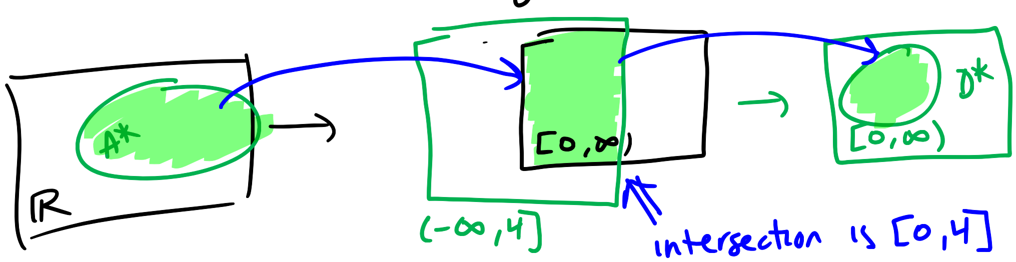
if $x \in A^*$ then $f(x) = 8x \in [0,\infty)$ $\Rightarrow$ $8x \geq 0$ $\boxed{\Rightarrow x \geq 0}$

new domain

$$g \circ f: [0,\infty) \longrightarrow [0,\infty)$$

2. If $f(x) = (x-2)^2$ and $g(x) = \sqrt{4-x}$, determine domain and range of $g \circ f$:

$f: \mathbb{R} \to [0,\infty)$ $\quad g: (-\infty,4] \to [0,\infty)$



$f(A^*) = [0,4]$ $\Rightarrow$ so if $x \in A^*$ then $f(x) \in [0,4]$

$\Rightarrow 0 \leq (x-2)^2 \leq 4$ $\Rightarrow$ $|x-2| \leq 2$ $\Rightarrow$ $-2 \leq x-2 \leq 2$

$\Rightarrow 0 \leq x \leq 4$ $\to$ new doman

also $g([0,4]) = D^*$ so if $x \in [0,4]$ then

$0 \leq x \leq 4 \Rightarrow 4 \geq 4-x \geq 0 \Rightarrow 2 \geq \sqrt{4-x} \geq 0$

✓ new range

$$\Rightarrow g \circ f: [0,4] \to [0,2]$$