Function Review

KNOW	DO	UNDERSTAND
Be able to recognize a function vs relation. Be able to identify the domain from the range.	Use Desmos and Geogebra to graph functions. Use correct language and notation when describing functions and sets.	No Big Ideas, but understand that a function is just a list of instructions that changes an input into a new thing.

Vocab & Notation

- Set: $\mathbb{R}, \mathbb{Q}, \mathbb{Z}, \mathbb{N}$
- Element: $x \in A$
- Mapping for sets: the function f from X to $Y \equiv f: X \to Y$
- Mapping for elements: the function f maps x to $y \equiv f: x \mapsto y$

Definition: A **set** is a collection of objects called **elements** that have a common property. Typically sets are collections of numbers, but they can be collections of anything really (even other sets!).

Example: The set of students in the front row and the set of months.

such that

$F = \{\text{Melvin}, \text{Paul}, \text{Carsen}\}$ or $F = \{x \mid x \text{ is a student in the front}\}$

$M = \{\text{Jan}, \text{Feb}, \text{Mar}, \dots, \text{Dec}\}$

Zahlen

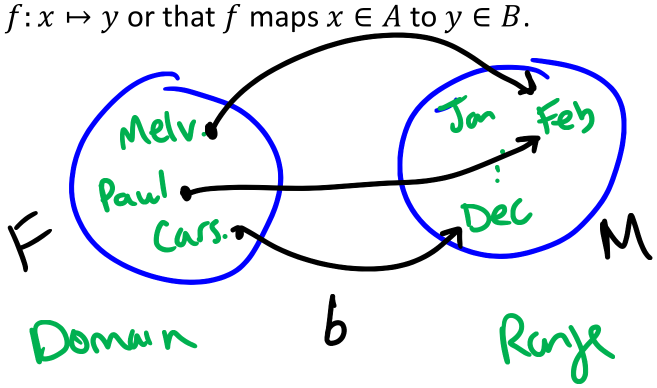
Some **very** commonly used sets are $\mathbb{N}$ (the set of natural numbers), $\mathbb{Z}$ (the set of integers), $\mathbb{Q}$ (the set of rational numbers), and $\mathbb{R}$ (the set of real numbers)

$1 \in \mathbb{N} \Rightarrow 1$ is a natural # $\mathbb{N} = \{1, 2, 3, \dots\}$

$\sqrt{2} \notin \mathbb{Z} \Rightarrow \sqrt{2}$ is NOT an integer $\mathbb{Z} = \{0, -1, 1, -2, -2, \dots\}$

$\mathbb{Q} = \{x \mid x = \frac{a}{b}, a, b \in \mathbb{Z}\}$ $\sqrt{2} \in \mathbb{R}$, $\pi \in \mathbb{R}$ $\sqrt{-1} \notin \mathbb{R}$

Definition: A **mapping** f, denoted as $f: A \longrightarrow B$, is a relation between the set A and the set B. We say that $f: x \mapsto y$ or that f maps $x \in A$ to $y \in B$.

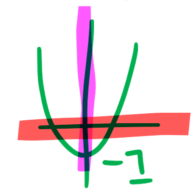


$b: F \to M$ sets

$b: \text{student} \mapsto \text{birth month}$ elements

cap for elements

Example: Write the relation $y = x^2 - 1$ in mapping notation using the function g.



$g: \mathbb{R} \to Y$

can be -1

can't be ∞

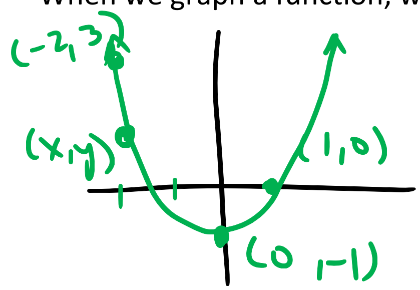
$Y = \{y \mid y \geq -1\}$ or $Y = [-1, \infty)$

$g: x \mapsto x^2 - 1$

Domain Range

x	y
$0 \mapsto$	-1
$3 \mapsto$	8
$-2 \mapsto$	3

When we graph a function, we are illustrating this relation with the coordinate (x, y).



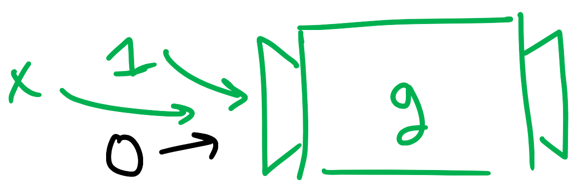
$g: x \mapsto y$

$g: x \mapsto x^2 - 1$

I want you to learn to be comfortable thinking about evaluating functions at abstract points. It can help to think of the function as an *action* that operates on an input x in a predictable way and transforms it into a new output $f(x)$.

g changed 1

$1 \in$ domain



$0 = g(1)$

$-1 = g(0)$

come from $\mathbb{R}$

$g(x) = x^2 - 1 \equiv g: x \mapsto x^2 - 1$

Example: Consider the function from before $g(x) = x^2 - 1$, determine $g(\pi)$, $g(\sqrt{2})$, $g(\beta)$ and $g(x^2 - 1)$

$g(\pi) = \pi^2 - 1$ $g: \pi \mapsto \pi^2 - 1$

$g(\sqrt{2}) = 1$ $g: \sqrt{2} \mapsto \sqrt{2}^2 - 1 = 1$

$g(\beta) = \beta^2 - 1$ $g: \beta \to \beta^2 - 1$

$g(x^2 - 1) = x^4 - 2x^2$ $g: x^2 - 1 \mapsto (x^2 - 1)^2 - 1$

Practice: Write the relation in mapping notation as a function F. State that the domain is all positive numbers and the range is all positive numbers less than 1.

$$y = \frac{x}{1+x}$$

$F: (0, \infty) \to (0, 1)$

$F: x \mapsto \frac{x}{1+x}$

OR $F: \{x \mid x > 0\} \to \{y \mid 0 < y < 1\}$

Determine $F(2), F\left(\frac{4}{3}\right), F(-3), F(\pi), F(\alpha), F\left(\frac{x}{1+x}\right)$

$F(2) = \frac{2}{3}$ $\quad 2 \mapsto \frac{2}{1+2} = \frac{2}{3}$

$F(4/3) = \frac{4}{7}$ $\quad 4/3 \mapsto \frac{4/3}{1+4/3} = \frac{4}{7}$

$F(-3) =$ undefined $\quad -3 \notin (0, \infty)$

$F(\pi) = \frac{\pi}{1+\pi}$ $\quad \pi \mapsto \frac{\pi}{1+\pi}$

$F(\alpha) = \frac{\alpha}{1+\alpha}$ $\quad \alpha \mapsto \frac{\alpha}{1+\alpha}$

$F\left(\frac{x}{1+x}\right) = \frac{x}{1+2x}$ $\quad \frac{x}{1+x} \mapsto \frac{\frac{x}{1+x}}{1+\frac{x}{1+x}} = \frac{\frac{x}{\cancel{1+x}}}{\frac{1+2x}{\cancel{1+x}}}$