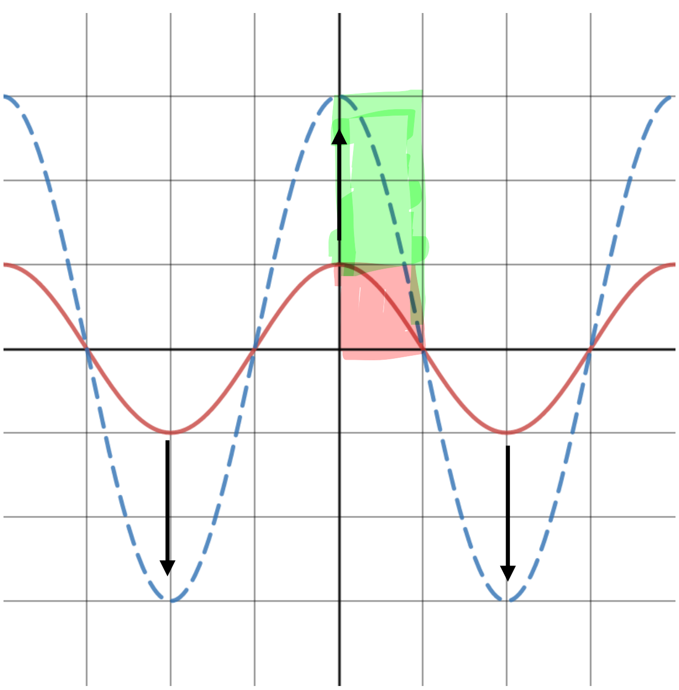
Function Expansions and Reflections

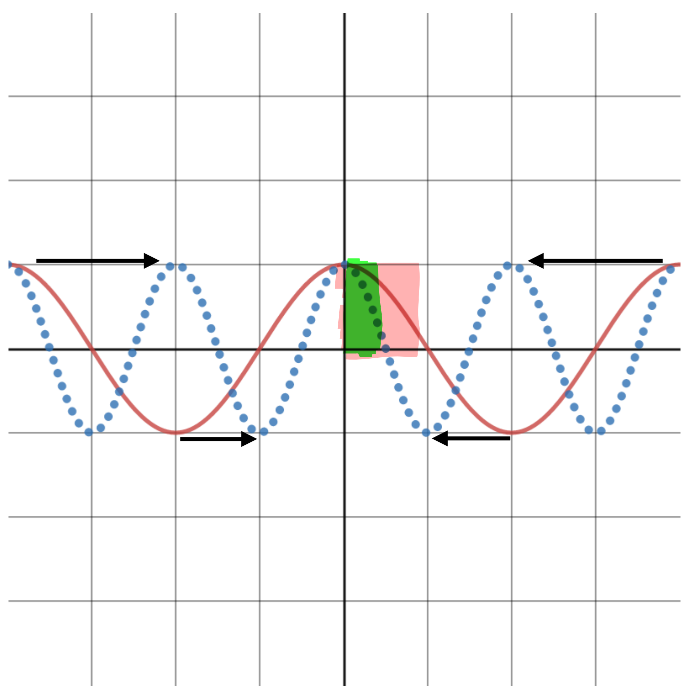
KNOW	DO	UNDERSTAND
Be able identify when a function was compressed, expanded, or reflected (vertically or horizontally) based on the mapping or function notation	Use Desmos and Geogebra to graph expansions and reflections. Use correct mapping and function notation to describe an expansion. Graph an expansion accurately by hand. Determine the expansion based on how points have moved.	*Transformations*: Can explain why horizontal expansions/compressions are opposite in function form. Can explain why and how domain and range change with an expansion/compression. Can explain how the intercepts and asymptotes move or stay in place after an expansion/compression.

Vocab & Notation

- Expansion
- Compression
- Reflection
- Parity (Odd or Even)
- Invariant Point

Aside from translating a function which preserves the general shape of the function (it just got moved around the graph) we can transform the graph in a more significant manner by stretching and compressing it relative to either axis.





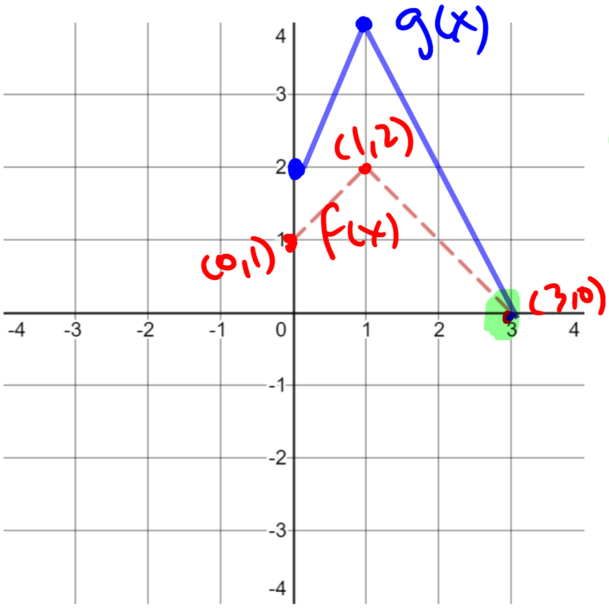
plane

Definition: When a transformation stretches 2D space horizontally and vertically this is called an **expansion** or **compression** and the mapping notation looks like:

$$T: \mathbb{R}^2 \to \mathbb{R}^2$$
$$T: (x, y) \to (b \cdot x, a \cdot y)$$

For a **vertical stretch**, we expand or compress space up and down and we apply the transformation:

$$T: \mathbb{R}^2 \to \mathbb{R}^2$$
$$T: (x, y) \mapsto (x, ay)$$



$f: x \mapsto y$

$f(x) = y$

$g: x \mapsto ay$

$g(x) = ay$

$\Rightarrow g(x) = a \cdot f(x)$

* if $a > 1$ we get vert. expansion
* if $0 < a < 1$ we get compression

Example: $T: (x, y) \mapsto (x, 2y)$ vertically expand by 2

$T: (0,1) \mapsto (0,2)$

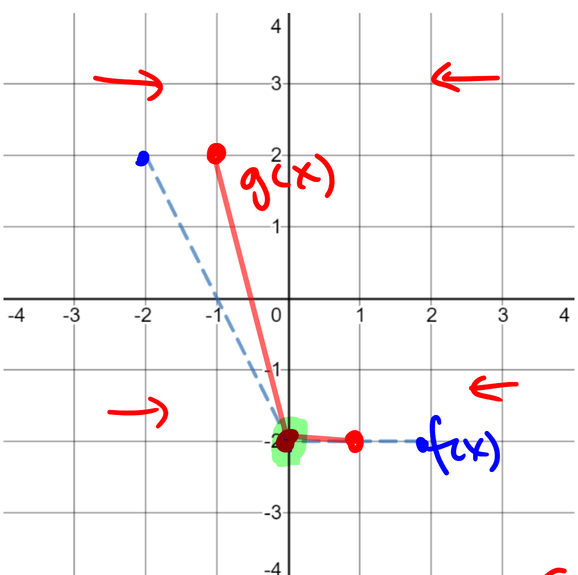
$T: (1,2) \mapsto (1,4)$

$T: (3,0) \mapsto (3,0)$

$\Rightarrow g(x) = 2f(x)$

For a **horizontal stretch**, we expand or compress space up and down and we apply the transformation:

$$T: \mathbb{R}^2 \to \mathbb{R}^2$$
$$T: (x, y) \mapsto (bx, y)$$



$f: x \mapsto y$

$f(x) = y$

$g: bx \mapsto y$

$g(bx) = y = f(x)$

let $X = bx$

$\Rightarrow g(X) = f(\frac{1}{b}X)$

Example: $T: (x, y) \mapsto \left(\frac{1}{2}x, y\right)$ → compressed horizontally by 2

$g(x) = f(2x)$

$\Rightarrow$ stretched horiz. by $\frac{1}{2}$

$2 > 1$ but this is a compression.

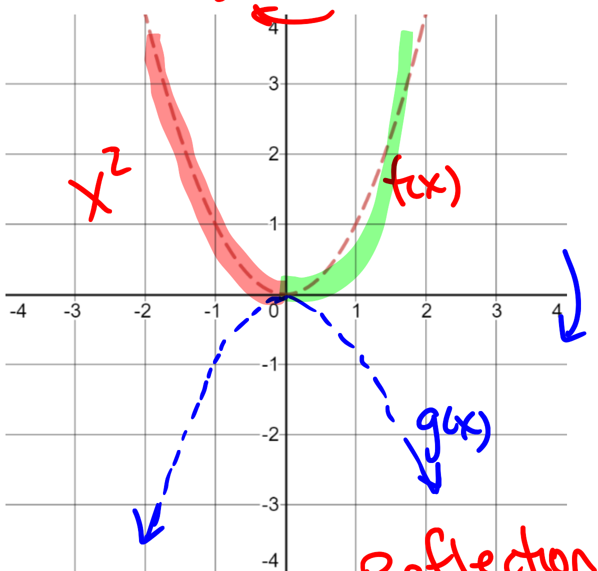
Definition: Under a transformation, T, if $T: (x_0, y_0) \mapsto (x_0, y_0)$ then the point (x_0, y_0) is **invariant**.

Points on the axis $\perp$ to the stretch stay fixed.

Definition: A **reflection** occurs when we transform a point from one side of the axis to the other.

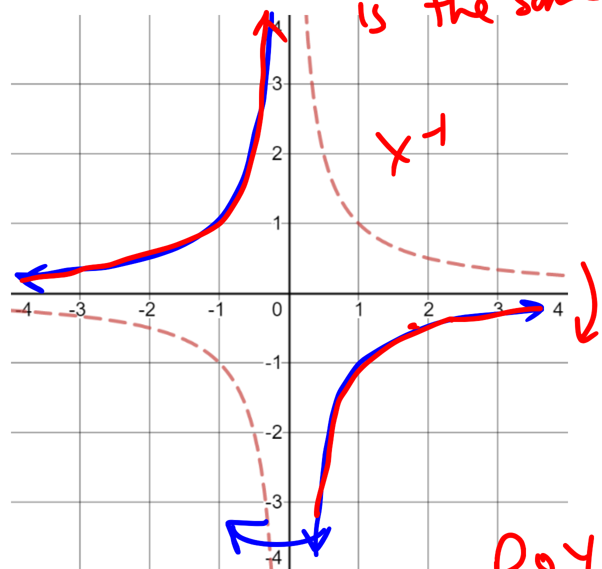
$$T: \mathbb{R}^2 \to \mathbb{R}^2$$
$$T: (x, y) \mapsto (-x, -y)$$

either reflection is the same



Reflection over x-axis (Rox)

$T: \mathbb{R}^2 \to \mathbb{R}^2$
$(x, y) \mapsto (x, -y)$
$g(x) = -f(x)$



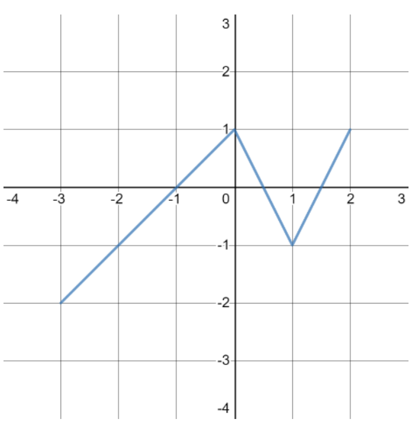
Roy

$T: \mathbb{R}^2 \to \mathbb{R}^2$
$(x, y) \mapsto (-x, y)$
$g(x) = f\left(\frac{x}{-1}\right) = f(-x)$

Even $f(-x) = f(x)$

ODD $f(-x) = -f(x)$

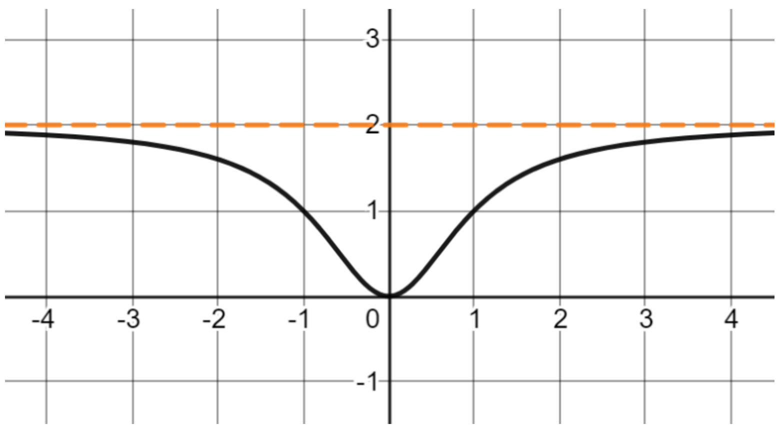
Practice: Given the graph of f, complete the following table.



Words		
Mapping Notation	$T: \mathbb{R}^2 \to \mathbb{R}^2$ $T: (x, y) \mapsto \left(\frac{1}{2}x, y\right)$	
Function Notation		$g(x) = -4f(x)$
Domain		
Range		
Zeros		
y-intercept		

see morning notes

Practice: Given the graph of f complete the following table



Words			
Mapping Notation	$T: \mathbb{R}^2 \to \mathbb{R}^2$ $T: (x, y) \mapsto (3x, y - 1)$		
Function Notation		$g(x) = 3f(-x) + 2$	
Horizontal Asymptote			$y = -2$
Range			$y \in (-2, 2]$
Zeros			$x = -2, 0$
y-intercept			$y = 0$

see morning notes

Practice Problems: 1.2 page 28 – 31 # 3-8, 9, 10, 14, 15, C1, C2
1.3 page 38 – 42 # 2-11, 13, 15, 17, C3