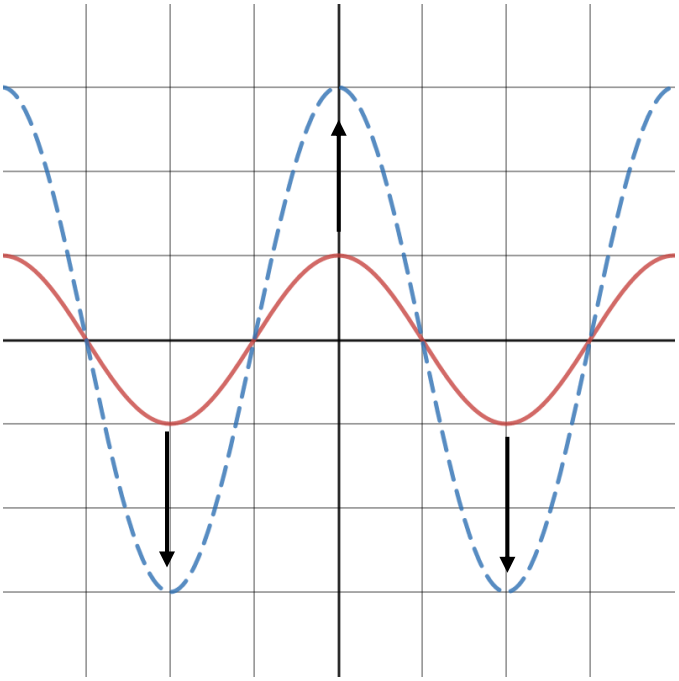
Function Expansions and Reflections

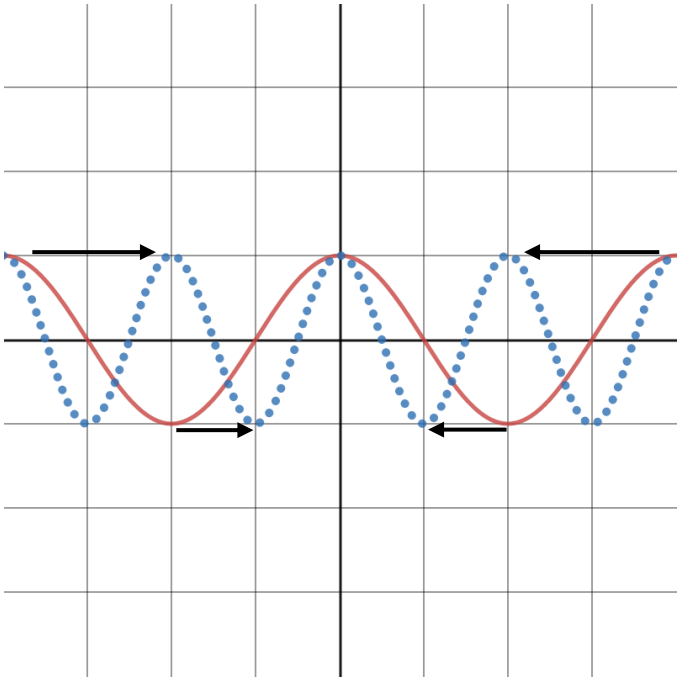
KNOW	DO	UNDERSTAND
Be able identify when a function was compressed, expanded, or reflected (vertically or horizontally) based on the mapping or function notation	Use Desmos and Geogebra to graph expansions and reflections. Use correct mapping and function notation to describe an expansion. Graph an expansion accurately by hand. Determine the expansion based on how points have moved.	*Transformations*: Can explain why horizontal expansions/compressions are opposite in function form. Can explain why and how domain and range change with an expansion/compression. Can explain how the intercepts and asymptotes move or stay in place after an expansion/compression.

Vocab & Notation

- Expansion
- Compression
- Reflection
- Parity (Odd or Even)
- Invariant Point

Aside from translating a function which preserves the general shape of the function (it just got moved around the graph) we can transform the graph in a more significant manner by stretching and compressing it relative to either axis.





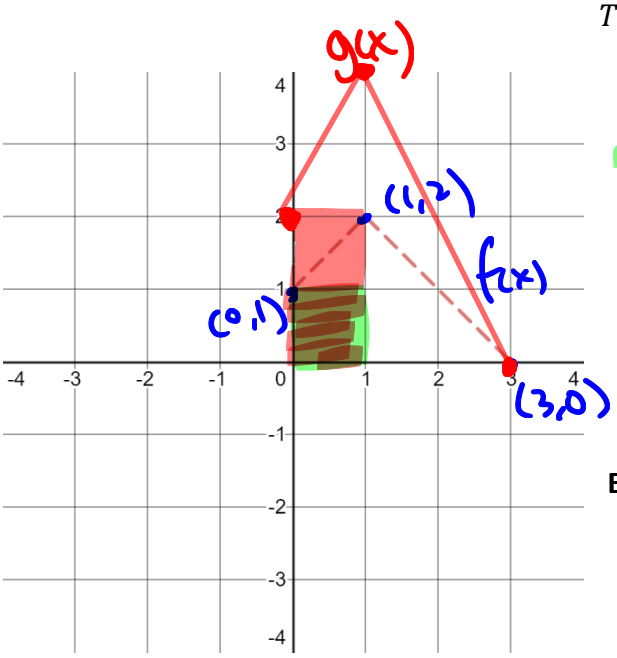
Definition: When a transformation stretches 2D space horizontally and vertically this is called an **expansion** or **compression** and the mapping notation looks like:

$$T: \mathbb{R}^2 \to \mathbb{R}^2$$

$$T: (x, y) \to (b \cdot x, a \cdot y)$$

For a **vertical stretch**, we expand or compress space up and down and we apply the transformation:

$$T: \mathbb{R}^2 \to \mathbb{R}^2$$
$$T: (x, y) \mapsto (x, ay)$$



$f: x \mapsto y$ $\quad f(x) = y$

$g: x \mapsto ay$ $\quad g(x) = a \cdot y$

$\Rightarrow g(x) = a \cdot f(x)$

★ $a > 1$ expansion
if $0 < a < 1$ compression

Example: $T: (x, y) \mapsto (x, 2y)$ → expand space vertically by 2

$g(x) = 2 \cdot f(x)$

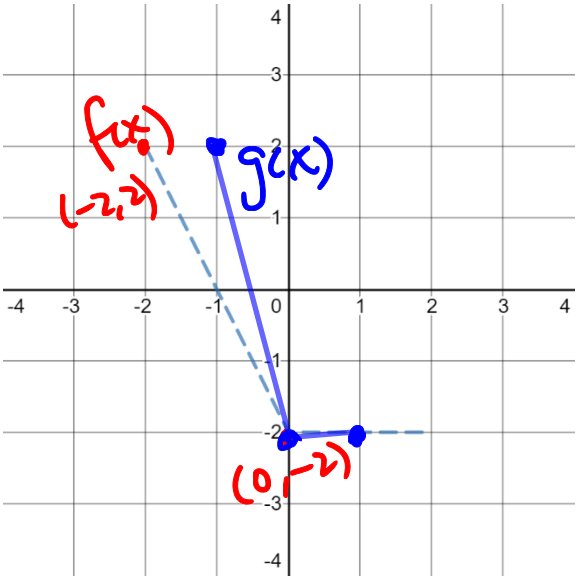
$T: (0,1) \mapsto (0,2)$

$T: (1,2) \mapsto (1,4)$

$T: (3,0) \mapsto (3,0)$

For a **horizontal stretch**, we expand or compress space up and down and we apply the transformation:

$$T: \mathbb{R}^2 \to \mathbb{R}^2$$
$$T: (x, y) \mapsto (bx, y)$$



$f: x \mapsto y$ $\quad f(x) = y$

$g: bx \to y$ $\quad g(bx) = y$

$X = bx$ $\quad \Rightarrow g(X) = f\left(\frac{1}{b} \cdot X\right)$

times by b became $\div$ by b

Example: $T: (x, y) \mapsto \left(\frac{1}{2}x, y\right)$ → horizontally compressed by 2

$g(x) = f(2x)$

$\Rightarrow$ stretched by $\frac{1}{2}$

$T: (-2,2) \mapsto (-1,2)$

$T: (0,-2) \mapsto (0,-2)$

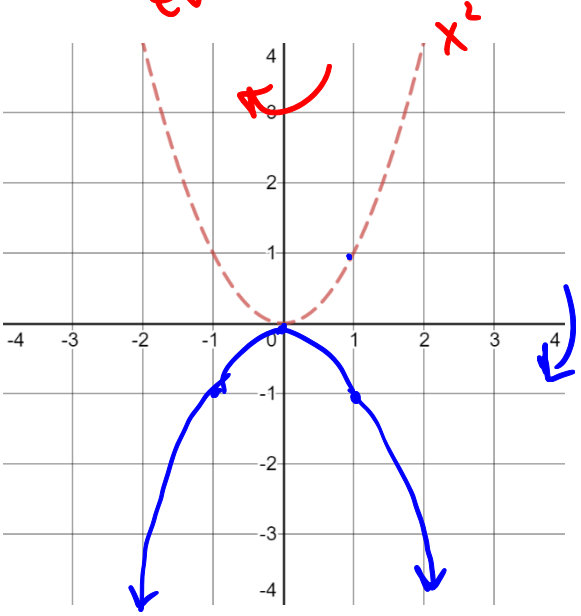
Definition: Under a transformation, T, if $T: (x_0, y_0) \mapsto (x_0, y_0)$ then the point (x_0, y_0) is **invariant**.

points on the axis $\perp$ to the stretch stay fixed!

Definition: A **reflection** occurs when we transform a point from one side of the axis to the other.

$$T: \mathbb{R}^2 \to \mathbb{R}^2$$
$$T: (x, y) \mapsto (-x, -y)$$

even



$T: \mathbb{R}^2 \to \mathbb{R}^2$

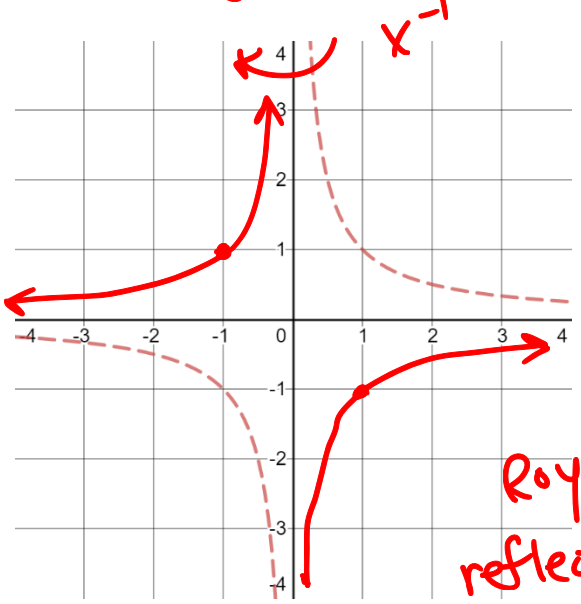
$T: (x,y) \mapsto (x,-y)$

$g(x) = -f(x)$

★ reflection over x-axis (R∘X)

$f(-x) = f(x)$ even

odd



$T: \mathbb{R}^2 \to \mathbb{R}^2$

$T: (x,y) \mapsto (-x,y)$

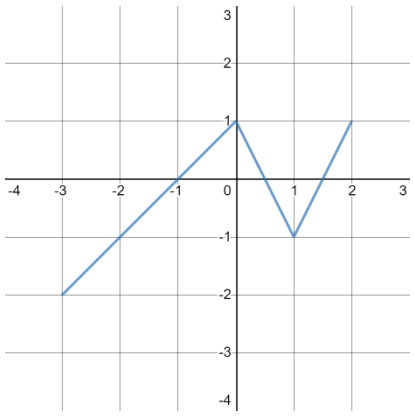
$g(x) = f\left(\frac{x}{-1}\right)$

$= f(-x)$

R∘Y reflection over y-axis

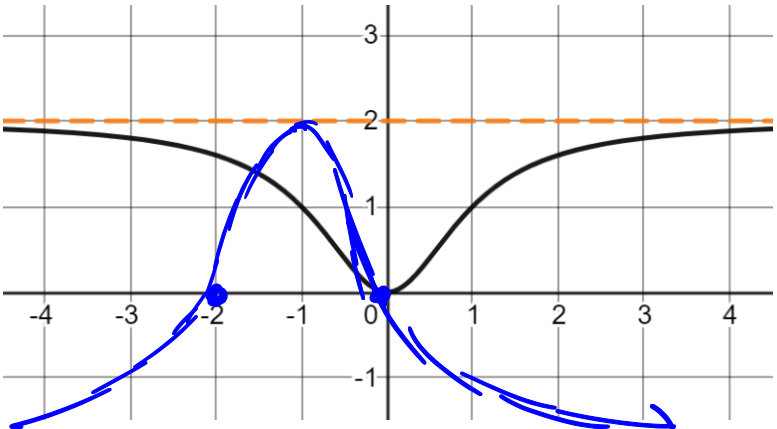
$f(-x) = -f(x)$ odd

Practice: Given the graph of f, ~~complete~~ the following table.



Words	horiz. compressed by 2	vert. expanded by 4, reflected over x-axis
Mapping Notation	$T: \mathbb{R}^2 \to \mathbb{R}^2$ $T: (x,y) \mapsto \left(\frac{1}{2}x, y\right)$	$T: \mathbb{R}^2 \to \mathbb{R}^2$ $T: (x,y) \mapsto (x,-4y)$
Function Notation	$h(x) = f(2x)$	$g(x) = -4f(x)$
Domain	$[-1.5, 1]$	$[-3, 2]$
Range	$[-2, 1]$	$[-4, 8]$
Zeros	$-0.5, 0.25, 0.75$	$-1, 0.5, 1.5$
y-intercept	1	-4

Practice: Given the graph of f complete the following table



Words	horiz. expanded by 3; shift down 1	vert. expanded by 3, ROY, shift up 2	vert expanded by 2, ROX, then up 2 and left 1
Mapping Notation	$T: \mathbb{R}^2 \to \mathbb{R}^2$ $T: (x, y) \mapsto (3x, y - 1)$	$T: \mathbb{R}^2 \to \mathbb{R}^2$ $T: (x, y) \mapsto (-x, 3y + 2)$	$T: \mathbb{R}^2 \to \mathbb{R}^2$ $T: (x, y) \mapsto (x - 1, -2y + 2)$
Function Notation	$k(x) = f(\frac{1}{3}x) - 1$	$g(x) = 3f(-x) + 2$	$m(x) = -2f(x + 1) + 2$
Horizontal Asymptote	$y = 1$	$y = 7$	$y = -2$
Range	$[-1, 1)$	$[1, 7)$	$y \in (-2, 2]$
Zeros	$x = -3, 3$	none	$x = -2, 0$
y-intercept	$y = -1$	$y = 1$	$y = 0$

* Note f is even $f(-x) = f(x)$

Practice Problems: 1.2 page 28 – 31 # 3-8, 9, 10, 14, 15, C1, C2
1.3 page 38 – 42 # 2-11, 13, 15, 17, C3