Function Translations

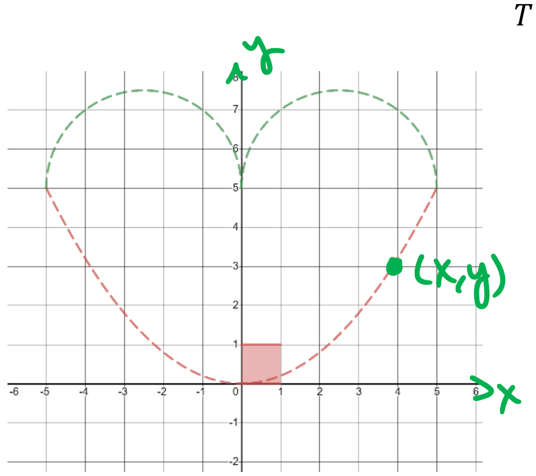
KNOW	DO	UNDERSTAND
Be able identify when a function was shifted left or right (and up or down) based on the mapping or function notation	Use Desmos and Geogebra to graph translations. Use correct mapping and function notation to describe a translation. Graph a translation accurately by hand. Determine the translation based on how points have moved.	*Transformations*: Can explain why translations left/right are opposite in function form. Can explain how vertical characteristics (range, y-intercepts, horizontal asymptotes) change by shifting up/down and how horizontal characteristics (domain, zeros and vertical asymptotes) change by shifting left/right

Vocab & Notation

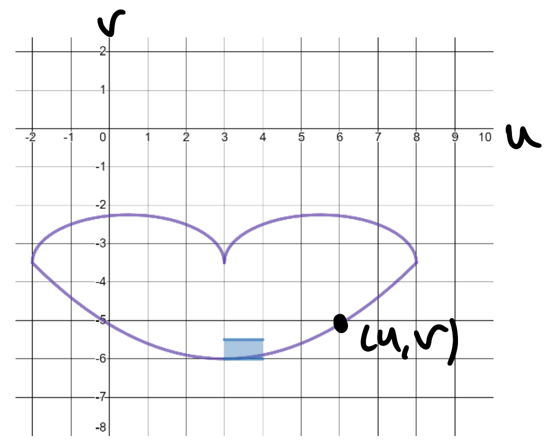
- The plane of real numbers: $\mathbb{R}^2$
- Translation
- Function Characteristics

We are going to be looking at how we can transform 2D space and functions that occupy space using mapping.

$$T: \mathbb{R}^2 \to \mathbb{R}^2$$
$$T: (x, y) \mapsto (u, v)$$



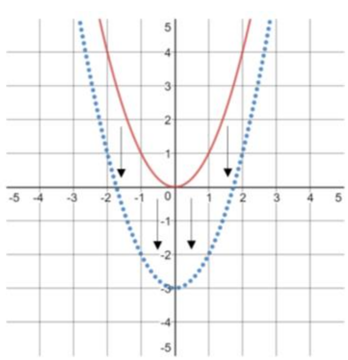
$$f: \mathbb{R} \to \mathbb{R}$$
$$f: x \mapsto y$$

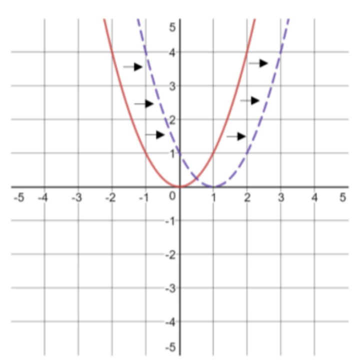


$$g: \mathbb{R} \to \mathbb{R}$$
$$g: u \mapsto v$$

Definition: When a transformation moves 2D space horizontally and vertically this is called a **translation** and the mapping notation looks like:

$$T: \mathbb{R}^2 \to \mathbb{R}^2$$
$$T: (x, y) \mapsto (x + c, y + d)$$





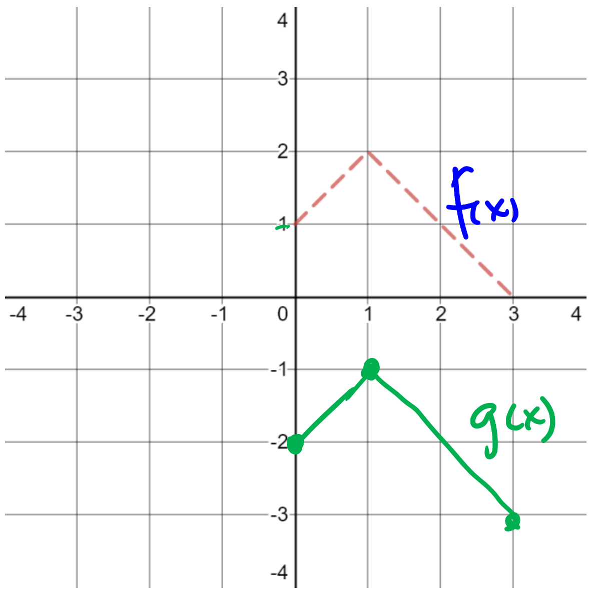
For a **vertical translation**, we shift space up and down and we apply the transformation:

$$T: \mathbb{R}^2 \to \mathbb{R}^2$$
$$T: (x, y) \mapsto (x, y + d)$$

$f: x \mapsto y$ $g: x \mapsto y + d$

$f(x) = y$ $g(x) = y + d$

$\Rightarrow$ $g(x) = f(x) + d$



Example: $T: (x, y) \mapsto (x, y - 3)$ → down 3

$g(x) = f(x) - 3$

For a **horizontal translation**, we shift the function left and right and apply the transformation:

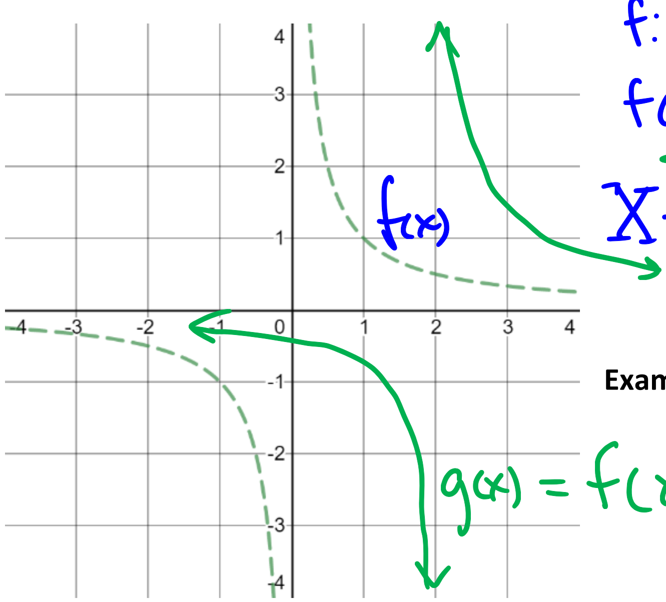
$$T: \mathbb{R}^2 \to \mathbb{R}^2$$
$$T: (x, y) \mapsto (x + c, y)$$

$f: x \mapsto y$ $g: x + c \mapsto y$

$f(x) = y$ $g(x + c) = y = f(x)$

$X = x + c$

$g(X) = f(X - c)$



Example: $T: (x, y) \mapsto (x + 2, y)$ → right by 2

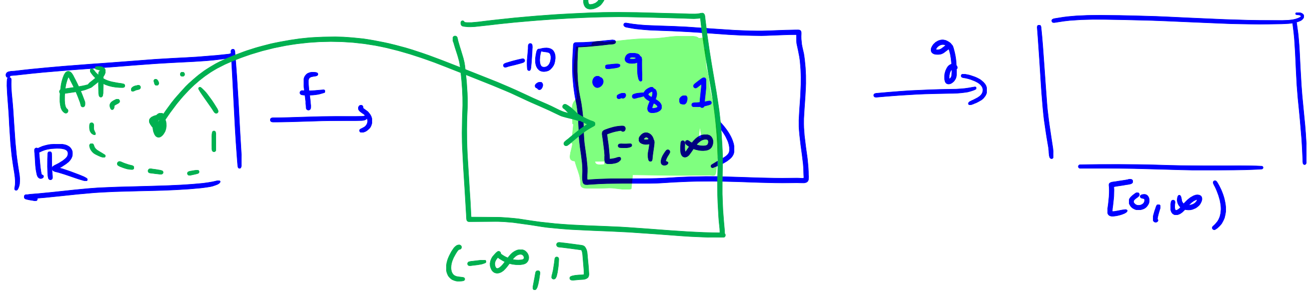
$g(x) = f(x - 2)$ ☆ looks deceptive

Practice Problems: 1.1 page 12 – 15 # 1-11, 17-19, C1-C4

Composition Domain/Range Practice:

Given $f(x) = 2|x| - 9$ and $g(x) = \sqrt{1-x}$ determine the domain and range of $g \circ f$

$f: \mathbb{R} \to [-9, \infty)$ $g: (-\infty, 1] \to [0, \infty)$



$(-\infty, 1] \cap [-9, \infty) = [-9, 1]$

★ Range take $x \in [-9, 1]$ where is $g(x)$?

$g(x) = \sqrt{1-x}$

$\Rightarrow -9 \leq x \leq 1$

$\Rightarrow 9 \geq -x \geq -1$

~~Determine the domain and range of $f \circ g$~~ $\Rightarrow 10 \geq 1 - x \geq 0$

$\Rightarrow \sqrt{10} \geq \sqrt{1-x} \geq 0$

$\Rightarrow$ Range is $[0, \sqrt{10}]$

Domain if $x \in A^*$ then $f(x) \in [-9, 1]$

$\Rightarrow -9 \leq 2|x| - 9 \leq 1$

$0 \leq 2|x| \leq 10$

$0 \leq |x| \leq 5 \Rightarrow -5 \leq x \leq 5$ Domain

Domain is $[-5, 5]$