Fundamental Theorem Practice Solutions

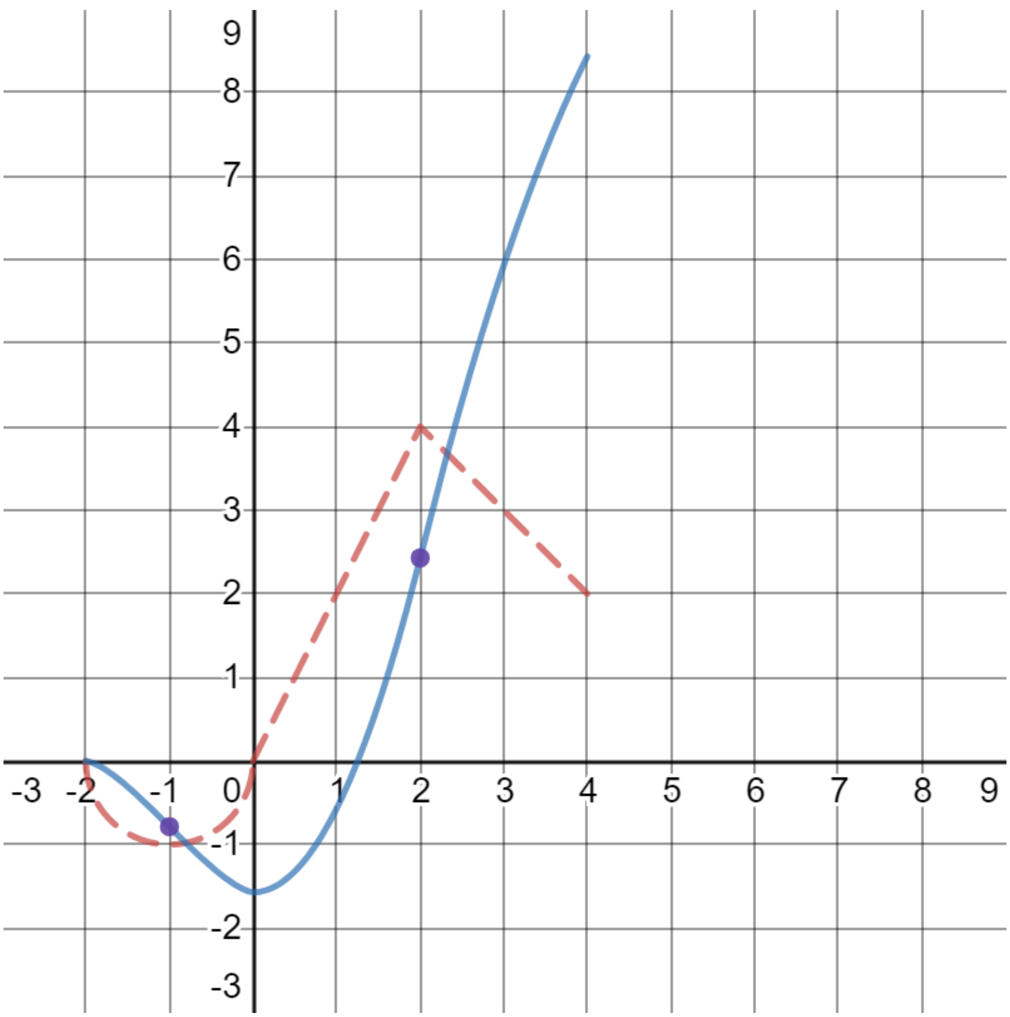
Evaluate the following functions defined using the given the graph f at the indicated points. Find the x value where it has an extreme value and an inflection point.

1.

$$F(x) = \int_{-4}^{x} f(t)dt$$

x	-4	0	4	8
$F(x)$	0	$-\frac{1}{2}\pi$	$-\frac{1}{2}\pi + 4$	$-\frac{1}{2}\pi + 10$

Maximums of F	Minimums of F	Inflection Points of F
No pure local max ($F'(x) = f(x)$ never goes positive to negative) Absolute max at $x = 4$	Local min at $x = 0$ ($f(x)$ goes negative to positive) and this is the absolute min too.	$F'' = f'$ and the slope changes sign at $x = -1$ and at $x = 2$

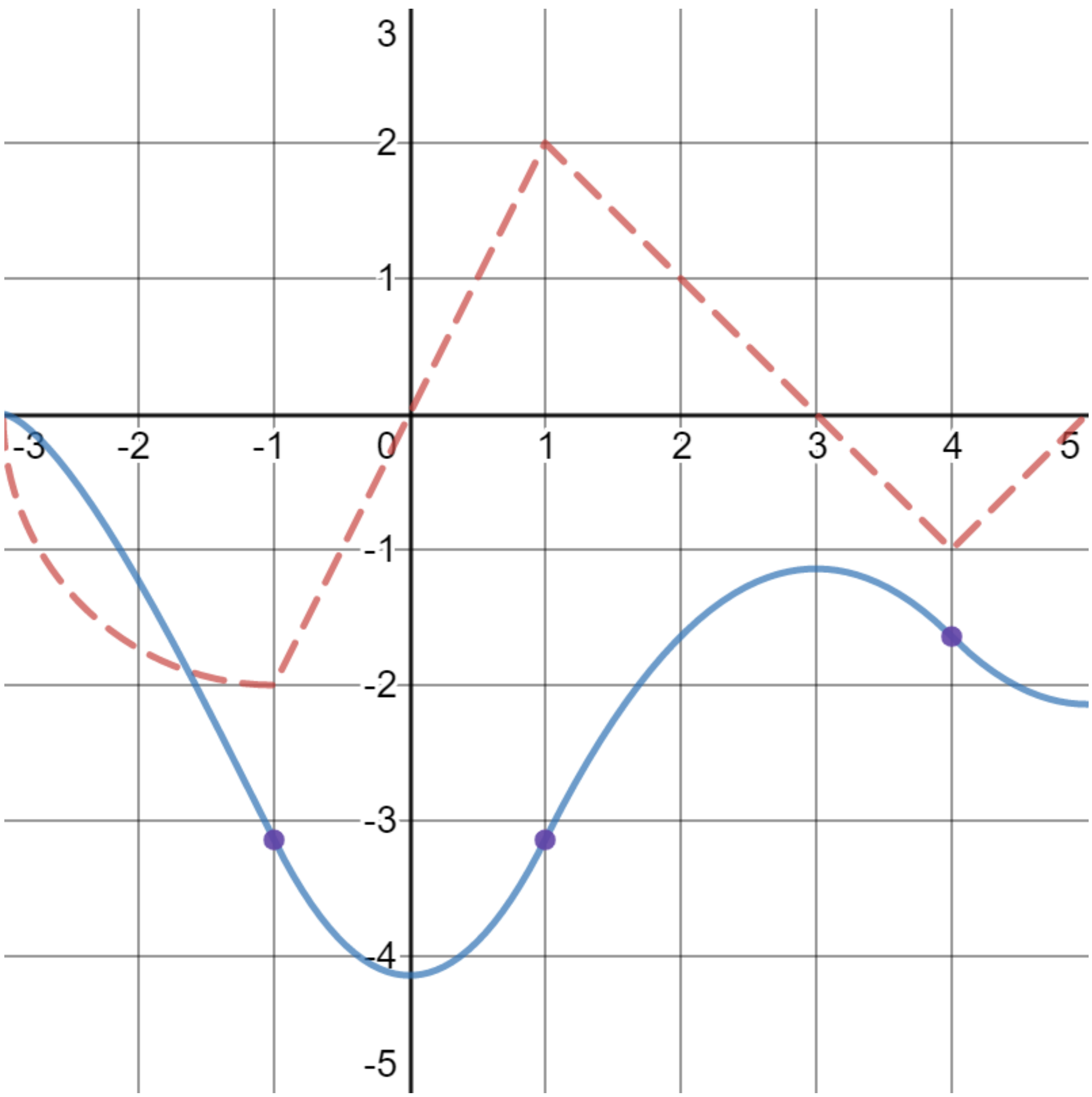


2.

$$g(x) = \int_{-3}^{x} f(t)dt$$

x	-3	-1	1	5
$g(x)$	0	$-\pi$	$-\pi$	$-\pi + 1$

Maximums of g	Minimums of g	Inflection Points of g
Local when $x = 3$ ($g'(x)$ goes positive to negative We can see that $g(3) > g(5)$ but $g(3) = -\pi + 2 < 0$ so $g(-3)$ will be the absolute max	Local when $x = 0$ ($g'(x)$ goes negative to positive) This is also the absolute since $g(0) = -\pi - 1 < g(-3), g(5)$	$g''(x) = f'(x)$ and the slope changes sign at $x = -1, 1, \text{and } 4$

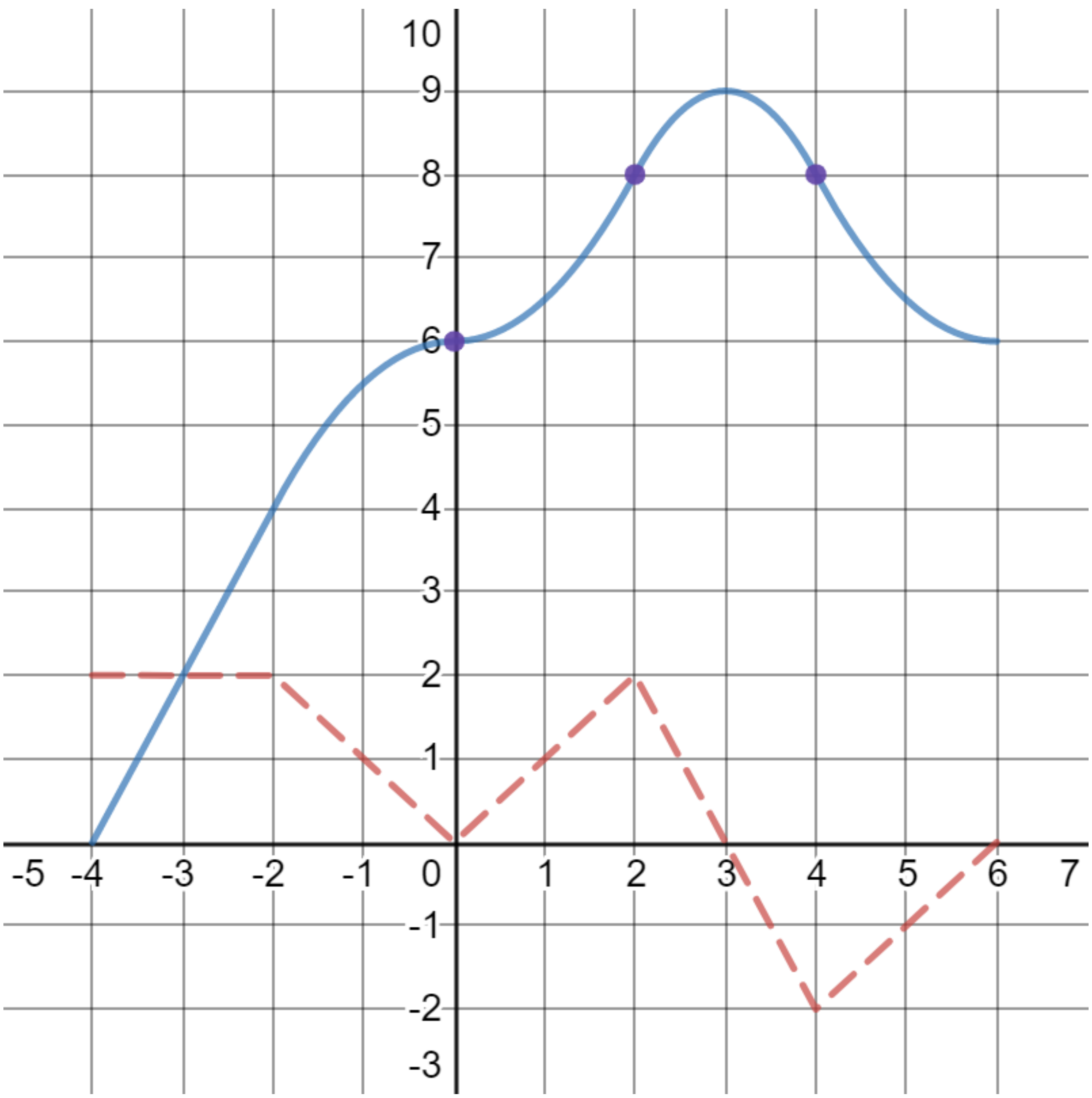


3.

$$h(x) = \int_{-4}^{x} f(t)dt$$

x	-4	-2	0	3	6
$h(x)$	0	4	6	9	6

Maximums of h	Minimums of h	Inflection Points of h
Local max at $x = 3$ where $h'(x) = f(x)$ goes positive to negative. Since $h(3) > 0$ and $h(3) > h(6)$, we have our absolute max at $x = 3$ too	No pure local minimum so we never go from negative to positive. The absolute min will be the lowest endpoint which is $h(-3)$	We have $h'' = f'$ and the slope changes sign at $x = 0, 2,$ and 4



4.

$$k(x) = \int_{-3}^{x} f(t)dt$$

x	-3	1	2	3	5
$k(x)$	0	$8 - 2\pi$	$6.5 - 2\pi$	$8 - 2\pi$	$6 - 2\pi$

Maximums of k	Minimums of k	Inflection Points of k
Local max at $x = 1$ and 3 where $k'(x) = f(x)$ goes positive to negative. Since $k(1) = k(3) > k(5)$ and 0 we have our absolute max is shared at $x = 1$ and 3	Local min at $x = 2$ where $k' = f$ goes from negative to positive. We have $k(2) > 0 > k(5)$ so the absolute min is at $x = 5$	When does $k'' = f'$ change sign? At $x = -1$ and $x = 3$ Notice that there is an inflection point at a local maximum.