Geometric Sequences and Series

KNOW	DO	UNDERSTAND
How to identify a sequence as geometric.	Build the equation for a geometric sequence and determine the sum of a geometric series	None yet
Vocab & Notation • Common ratio		

Definition: A **geometric sequence** is a sequence generated by multiplying the previous term by a fixed value.

$$a_{k+1} = a_k \cdot r$$

Definition: The **common ratio** is the ratio of consecutive terms in a geometric sequence.

$$\frac{a_{k+1}}{a_k} = r$$

Let's look and see how a geometric sequence is built by letting $a_1 = A$

Example: Determine sequences of the form $(a_k)_{k=1}^{\infty}$ and $(b_k)_{k=0}^{\infty}$ for the following patterns:

$(2, 6, 18, \cdots)$ $(100, -50, 25, \cdots)$

$\left(\frac{8}{9}, \frac{2}{9}, \frac{1}{18}, \cdots\right)$ $\left(-9, -25, -\frac{625}{9}, \cdots\right)$

** We can use our calculator to list the terms of a geometric sequence quickly using recursion.

Example: List the first 10 terms of the sequence where $a_1 = 10$ and $r = -\frac{4}{5}$

Practice: List the first 10 terms of the sequence where $a_1 = 4$ and $r = 1.06$

Now we would like to consider the geometric series, that is

$$S_n = \sum_{k=1}^{n} a_1 \cdot r^{k-1}$$

Oftentimes it is helpful to try and add or subtract copies of the summation to reduce it to something simpler.

Example: Determine the following sums

$$\sum_{k=1}^{10} 4 \cdot 3^{k-1}$$

$$\sum_{k=0}^{5} 3 \cdot \left(\frac{1}{2}\right)^{k}$$

Practice: Determine the following sums

$$\sum_{k=1}^{7} 8 \cdot \left(-\frac{2}{3}\right)^{k-1}$$

$$\sum_{k=0}^{6} \left(\frac{9}{4}\right)^{k}$$

Since we know the finite sum, we can consider what S_∞ would be:

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

Example: Determine the following sums

$$\sum_{k=1}^{\infty} 10 \cdot \left(-\frac{2}{5}\right)^{k-1}$$

$$\sum_{k=0}^{\infty} 7 \cdot \left(-\frac{3}{4}\right)^{k}$$

$$\sum_{k=2}^{\infty} 100 \cdot \left(\frac{1}{2}\right)^{k}$$

$$\sum_{k=0}^{\infty} 12 \cdot (1.01)^{k}$$

Practice Problems: Handout sections III, V(c,d)
Ryerson pdf: 1.3 # 1-7, 23
1.4 # 1-8, 16=19
1.5 # 1-7, 20