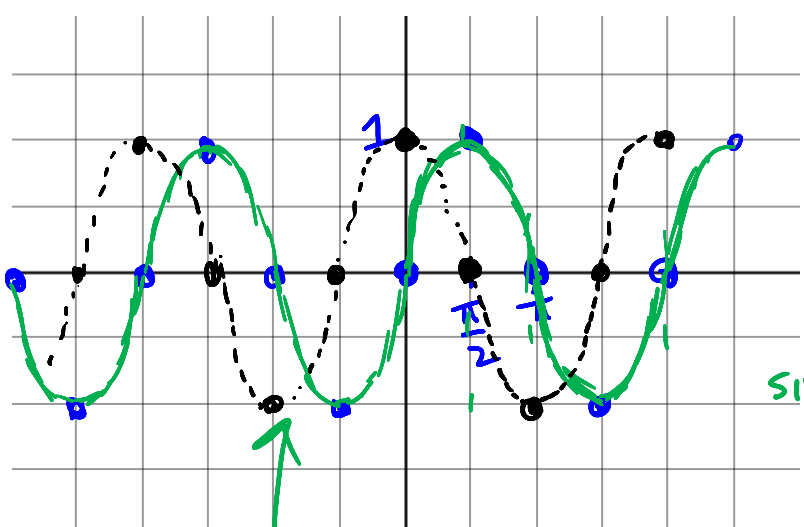
Graphing Sine and Cosine

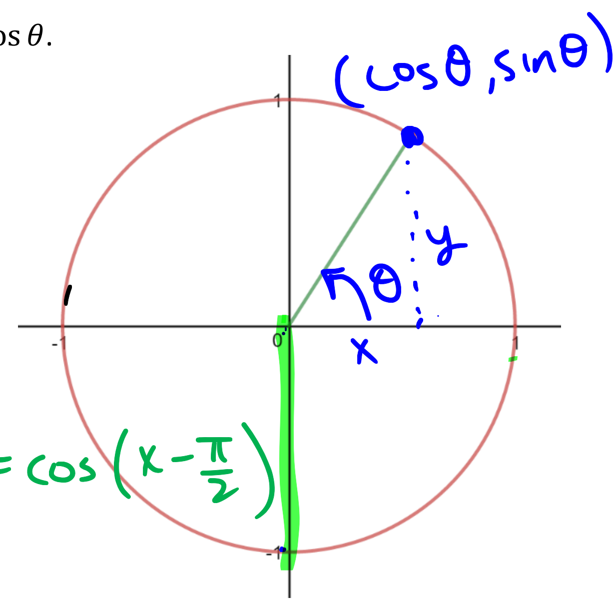
KNOW	DO	UNDERSTAND
How to identify the amplitude and period of a trig function. What a sinusoidal function looks like.	Can graph a trig function from the equation or characteristics accurately. Can build the equation of a trig function from the graph or characteristics accurately.	*Transformation:* Can explain how certain characteristics are or are not affected by a transformation. *Function Characteristics:* How the amplitude relates to the max/min values, midline as the average, period as the frequency, and shift as the start.

Vocab & Notation

- Amplitude
- Period
- Midline
- Phase Shift
- Sinusoidal function

Using a unit circle, graph the angle θ and the values of $\sin\theta$ and $\cos\theta$.





$\sin x = \cos\left(x - \frac{\pi}{2}\right)$

$\sin\theta$ tracks y values

There are 4 areas of symmetry for 1 rotation.

Definition: Functions that repeat after a certain amount of time are called **periodic functions** (periodic meaning occurring at regular intervals). Periodic functions that have this "wave" shape are called **sinusoidal functions**.

shift horiz.

We want to analyze this curve so that we can graph functions of the form:

$f(x) = \sin(x)$

$$a \cdot \overset{\cos}{\cancel{\sin}}(b(x - c)) + d$$

$x \to \boxed{\text{sine}} \to y$

ratio o/h

shift vert.

vertical stretch reflec.

horiz. stretch reflec.

Definition: The **midline** is the average value of the function.

For $\sin\theta$ it is 0 and only the vertical shift (d) changes this

Definition: The **amplitude** is the distance from the midline to the maximum or minimum, or equivalently, half the distance between the max and min.

for $\sin\theta$ the amplitude is 1 (b/c max is 1)

only changed by vertical stretch (a) ★ never negative

Definition: The **period** is the length of one complete cycle of a periodic function. Not necessarily how long it takes to repeat itself, but how long it takes to repeat the pattern.

for $\sin\theta$, $T = 2\pi$ (1 rotation)

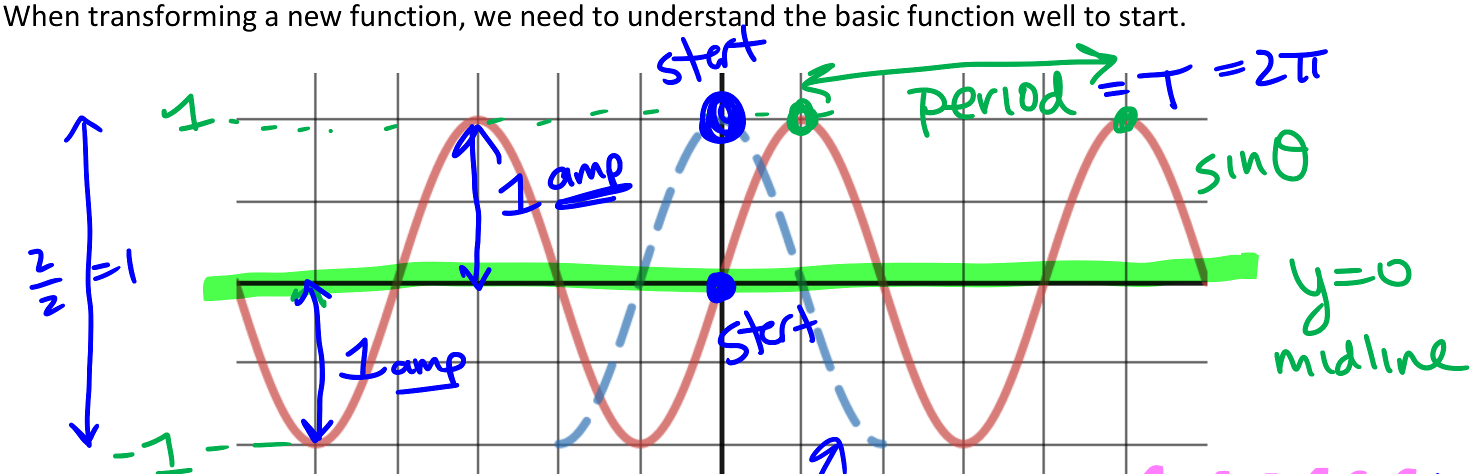
only change by horiz. stretch (b), $T_{new} = \frac{2\pi}{b}$

Definition: The **phase shift** is where the starting point of $\theta = 0$ got moved to.

$\sin\theta$ starts on the midline and goes up

$\cos\theta$ starts at the top and goes down (changed by c)

When transforming a new function, we need to understand the basic function well to start.

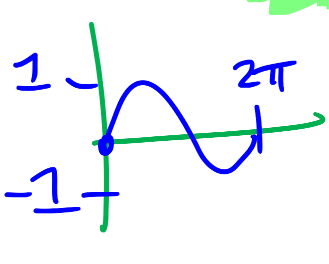


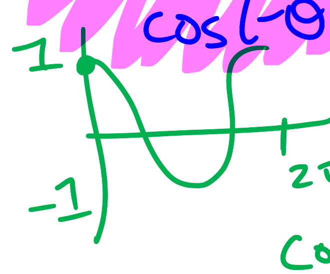
★ $\sin\theta$ is odd

$\sin(-\theta) = -\sin\theta$

$\cos\theta$ is even!

$\cos(-\theta) = \cos\theta$

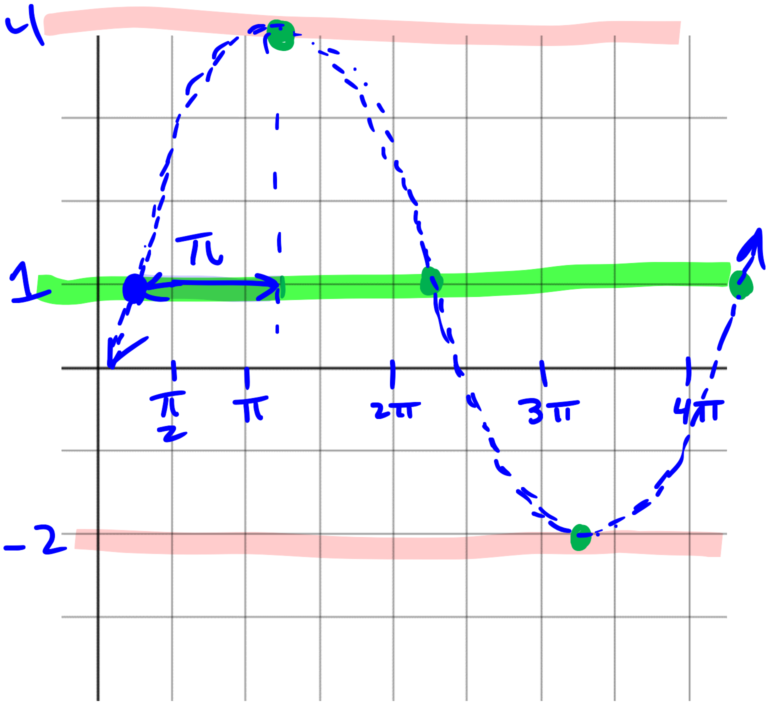




Example: Graph $f(\theta) = 3\sin\left(\frac{1}{2}\left(\theta - \frac{\pi}{4}\right)\right) + 1$

shift

- Identify the midline from the vertical displacement
- Use the amplitude to find the max and min lines
- Use the phase shift to identify the starting point
- Split the period into quarters.



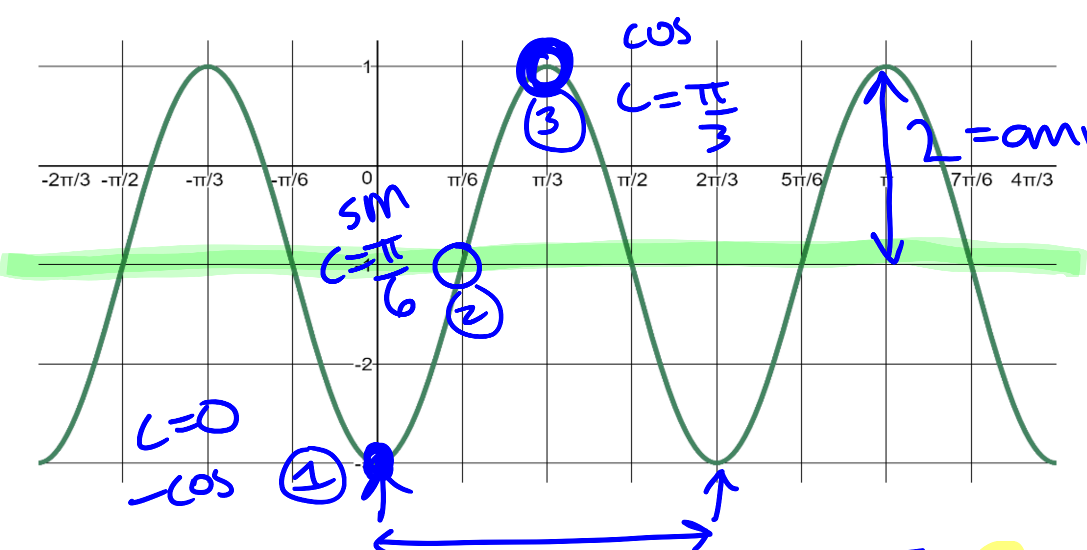
midline is $y = 1$

amplitude is 3

$T = \frac{2\pi}{1/2} = 4\pi$

shift right $\frac{\pi}{4}$

Example: Determine 3 different equations that could describe the following function.



2 = amp = a

midline = -1 = d

$T = \frac{2\pi}{b}$

$b = \frac{2\pi}{T}$

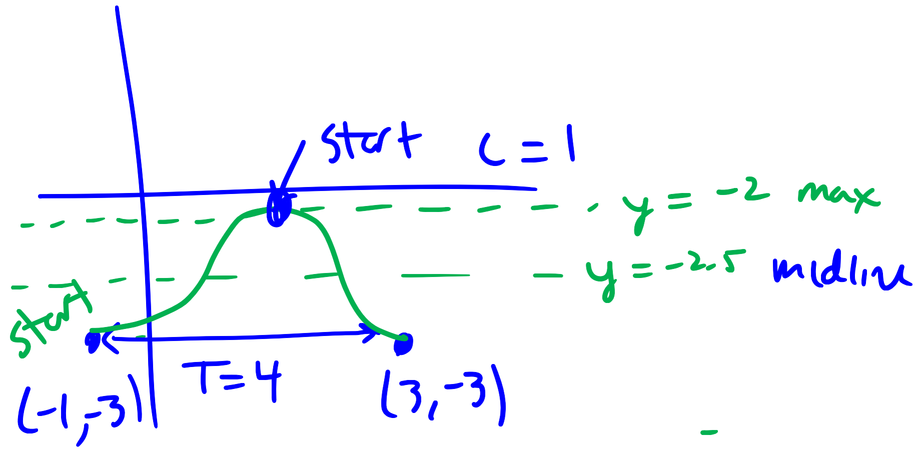
$T = \frac{2\pi}{3} \longrightarrow b = \frac{2\pi}{2\pi/3} = 3$

① $y = -2\cos(3\theta) - 1$

② $y = 2\sin\left(3\left(\theta - \frac{\pi}{6}\right)\right) - 1$

$y = 2\cos\left(3\left(\theta - \frac{\pi}{3}\right)\right) - 1$

Example: Determine two equations (one sine, one cosine) that could describe a sinusoidal function that has two minimums at $(-1, -3)$ and $(3, -3)$ and has an amplitude of 0.5.

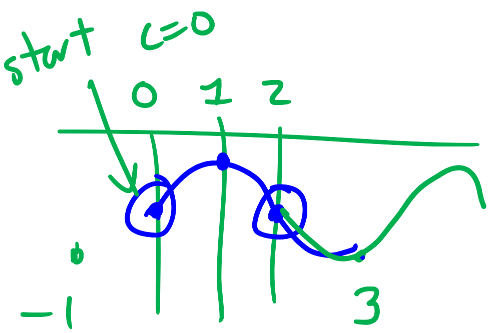


$d = -2.5$ $a = 0.5$ $T = 4 \Rightarrow b = \frac{2\pi}{4} = \frac{\pi}{2}$

$c = -1$; $-\cos$

$$y_1 = -0.5\cos\left(\frac{\pi}{2}(\theta + 1)\right) - 2.5$$

$$y_2 = 0.5\cos\left(\frac{\pi}{2}(\theta - 1)\right) - 2.5$$



$$y_3 = 0.5\sin\left(\frac{\pi}{2}\theta\right) - 2.5$$

$$y_4 = -0.5\sin\left(\frac{\pi}{2}(\theta - 2)\right) - 2.5$$

Practice Problems: 5.1 page 233 – 236 # 1-12, 20-22
5.2 page 250 – 254 # 1-7, 10-19