Fundamental Theorem of Calculus: Part 1

Goal: • Understands why the integral is the antiderivative • Understands why the derivative of an integral is the integrand • Understands that the integral is the area under a curve
Terminology: • Fundamental Theorem of Calculus

Discussion: What is the value of the following limit?

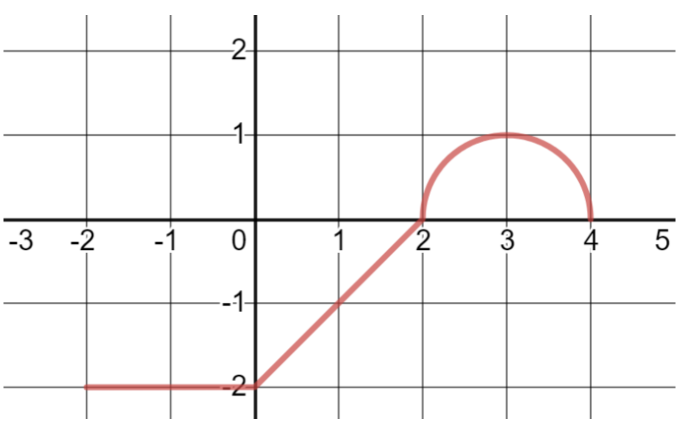
$$\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\cdot\Delta x$$

Where $f(x) = x - 1$ and $x_k = -1 + \frac{3k}{n}$ and $\Delta x = \frac{3}{n}$

Example: Consider the function f below, and define

$$F(t) = \int_{-2}^{t} f(x)dx$$

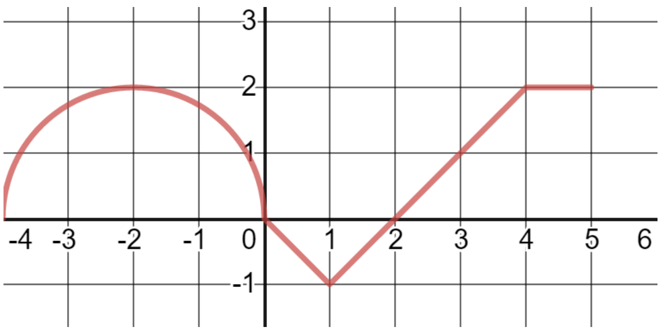
Determine the following values: $F(-2),\ F(0),\ F(2),\ F(4)$



Practice: Given the function f below and the function

$$F(t) = \int_{-4}^{t} f(x)dx$$

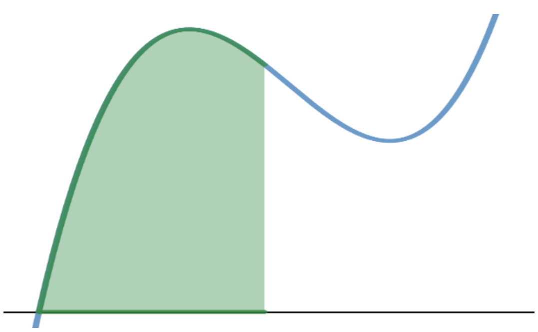
Determine the following values: $F(-4), F(0), F(2), F(5)$



In general, if we have some function, f, and define a new function

$$F(x) = \int_{a}^{x} f(t)dt$$

We can consider what happens when we have a small change in x, and then consider what happens when $\Delta x \to 0$.



This leads us to the first part of **Fundamental Theorem of Calculus:**

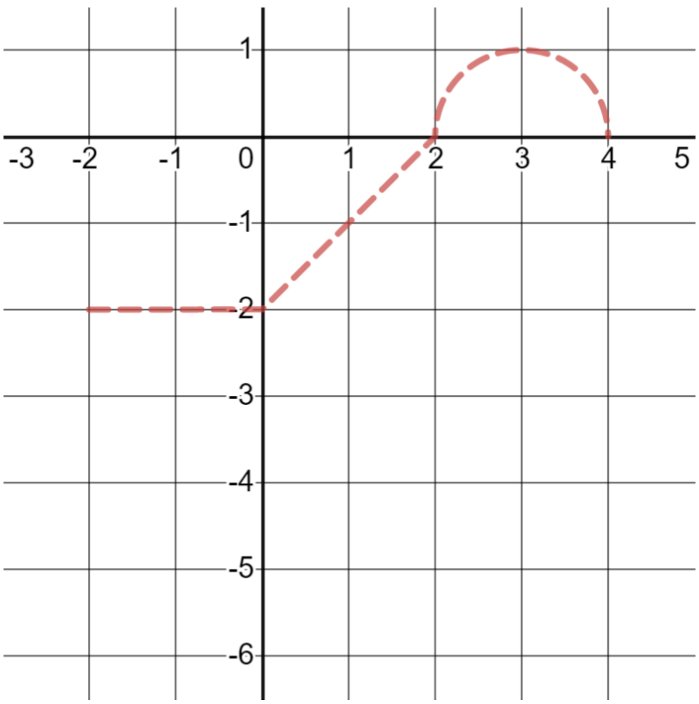
$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

This allows us to define the area under the curve or integral as the antiderivative of the function. It may not seem like much, but without this we would not have a physical meaning to the antiderivative.

Example: Given the function f below, and

$$F(x) = \int_{-2}^{x} f(t)dt$$

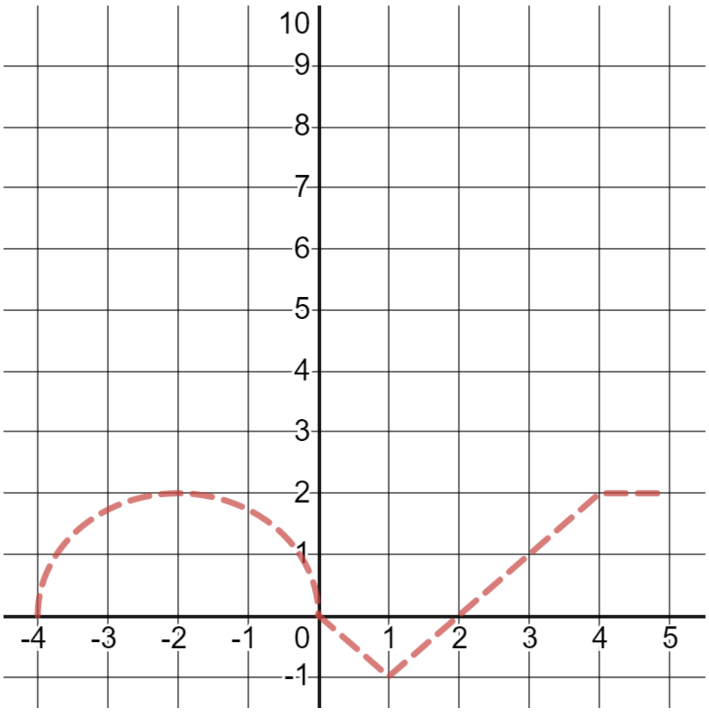
Determine where F has an extrema and inflection points.



Practice: Given the function f below and the function

$$F(x) = \int_{-4}^{x} f(t)dt$$

Determine where F has an extrema and inflection points



Practice Problems: Fundamental Theorem Practice Problems