Integration Applications Review

Name

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

1. Find the average value of f on the interval [-1, 1].

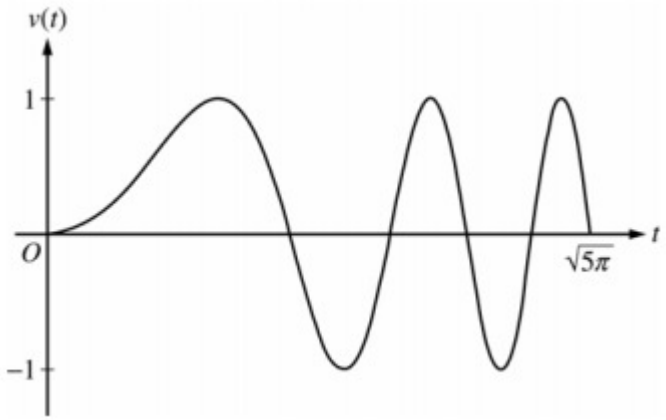
Please respond on separate paper, following directions from your teacher.

t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$, as shown in the table above, For $0 < t \le 2.4, v(t) > 0$.

2. For $0 \le t \le 4$ hours, Ruth's velocity can be modeled by the function g given by $g(t) = \frac{24t + 5\sin(6t)}{t + 0.7}$. According to the model, what was Ruth's average velocity during the time interval $0 \le t \le 4$?

Please respond on separate paper, following directions from your teacher.



A particle moves along the x-axis so that its velocity v at time $t \ge 0$ is given by $v(t) = \sin(t^2)$. The graph of v is

Integration Applications Review

shown above for <img src="/tmp/formula_5fca7cc0d3a807.45751234_1607105728.svg" style="vertical-align:middle">. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

3. Find the total distance traveled by the particle from time t=0 to t=3.

Please respond on separate paper, following directions from your teacher.

A toy train moves along a straight set up on a table. The position $x(t)$ of the train at the time t seconds is measured in centimeters from the center of the track. At time t=1 , the train is 6 centimeters to the left of the center, so $x(1)$=−6 . For $0 \le t \le 4$, the velocity of the train at the time t is given by $v(t)=3t^2-12$, where $v(t)$ is measured in centimeters per second.

4. For $0 \le t \le 4$, find $x(t)$.

Please respond on separate paper, following directions from your teacher.

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

5. How many gallons of water leak out of the tank from $t = 0$ to $t = 3$ minutes?

Please respond on separate paper, following directions from your teacher.

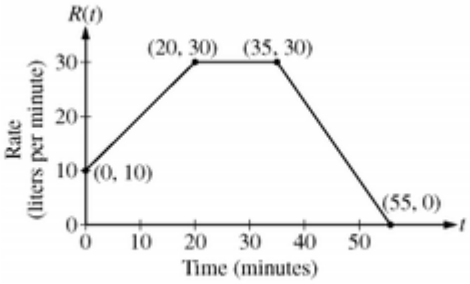
Integration Applications Review

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	–20	–30	–20	–14	–10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

6. Using appropriate units, explain the meaning of $\int_0^{30} a(t)dt$ in terms of the car's motion. Find the exact value of $\int_0^{30} a(t)dt$.

Please respond on separate paper, following directions from your teacher.



At time $t = 0$ minutes, a tank contains 100 liters of water. The piecewise-linear graph above shows the rate $R(t)$, in liters per minute, at which water is pumped into the tank during a 55-minute period.

7. At time $t = 10$ minutes, water begins draining from the tank at a rate modeled by the function D, where $D(t) = 10e^{(\sin t)/10}$ liters per minute. Water continues to drain at this rate until time $t = 55$ minutes. How many liters of water are in the tank at time $t = 55$ minutes?

Integration Applications Review

Please respond on separate paper, following directions from your teacher.

Let R be the region enclosed by the graphs of $y = \ln(x^2 + 1)$ and $y = \cos x$.

8. Find the area of R.

Please respond on separate paper, following directions from your teacher.

Let R be the region in the first quadrant under the graph of $y = \frac{x}{x^2+2}$ for $0 \le x \le \sqrt{6}$.

9. Find the area of R.

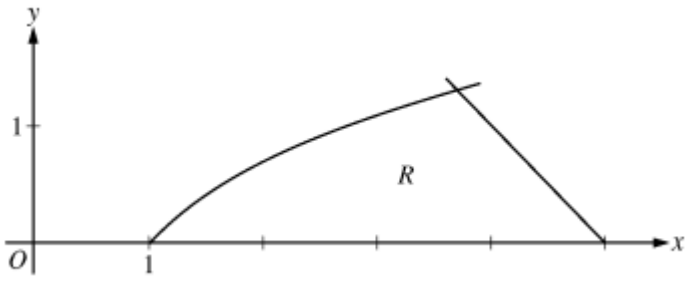
Please respond on separate paper, following directions from your teacher.

Consider the curve $y^2=4+x$ and chord AB joining the points $A(-4,0)$ and $B(0,2)$ on the curve.

10. Find the area of the region R enclosed by the curve and the chord AB.

Please respond on separate paper, following directions from your teacher.

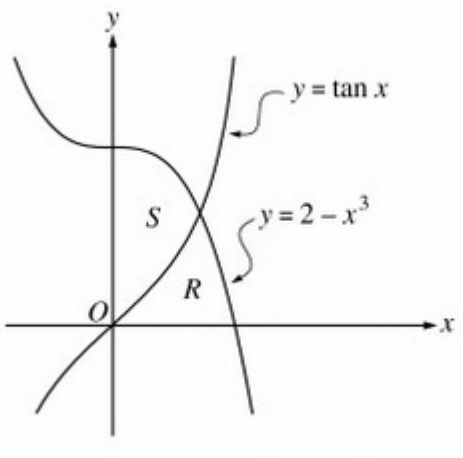
Integration Applications Review



Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y=\ln x$ and $y=5-x$, as shown in the figure above.

11. Find the area of R.

Please respond on separate paper, following directions from your teacher.

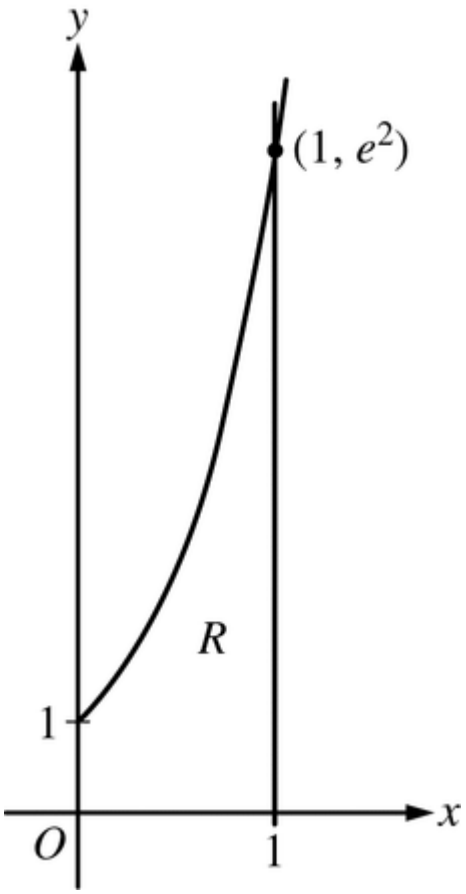


Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and y tan x.

12. Find the area of R.

Please respond on separate paper, following directions from your teacher.

Integration Applications Review



Let f(x)=e^{2x} . Let *R* be the region in the first quadrant bounded by the graph of y=f(x) and the vertical line x=1 , as shown in the figure above.

13. Region *R* forms the base of a solid whose cross sections perpendicular to the *y*-axis are squares. Write, but do not evaluate, and expression involving one or more integral that gives the volume of the solid.

Please respond on separate paper, following directions from your teacher.

Let *R* be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

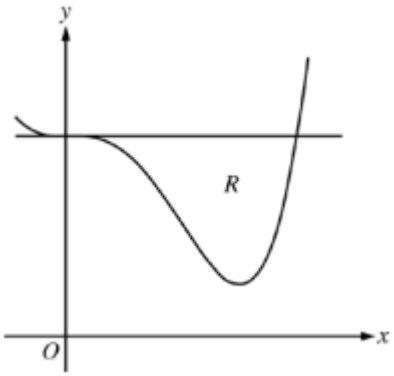
14. The region *R* is the base of a solid. For this solid, the cross sections perpendicular to the *y*-axis are squares. Find the volume of this solid.

Integration Applications Review

Please respond on separate paper, following directions from your teacher.

15. Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

Please respond on separate paper, following directions from your teacher.

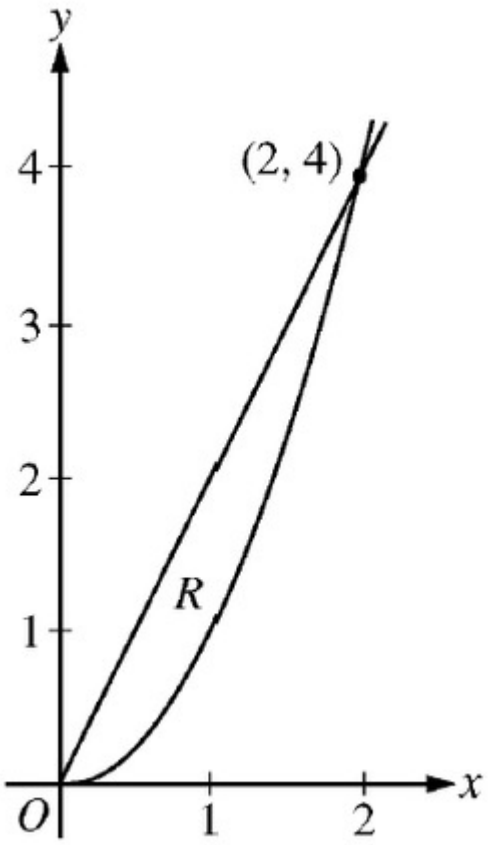


Let R be the region enclosed by the graph of $f(x)=x^4-2.3x^3+4$ and the horizontal line $y=4$, as shown in the figure above.

16. Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

Please respond on separate paper, following directions from your teacher.

Integration Applications Review



Let R be the region in the first quadrant enclosed by the graphs of y=2x and $y=x^2$, as shown in the figure above.

17. The region *R* is the base of a solid. For this solid, at each *x* the cross section perpendicular to the x-axis has area $A(x)=\sin(\pi/2x)$. Find the volume of the solid.

Please respond on separate paper, following directions from your teacher.

Let *R* be the region bounded by the *x*–axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

18. The vertical line $x = k$ divides the region *R* into two regions such that when these two regions are revolved about the *x*–axis, they generate solids with equal volumes. Find the value of *k*

Please respond on separate paper, following directions from your teacher.

Integration Applications Review

Let R be the region in the first quadrant enclosed by the hyperbola $x^2 - y^2 = 9$, the x-axis, and the line $x = 5$.

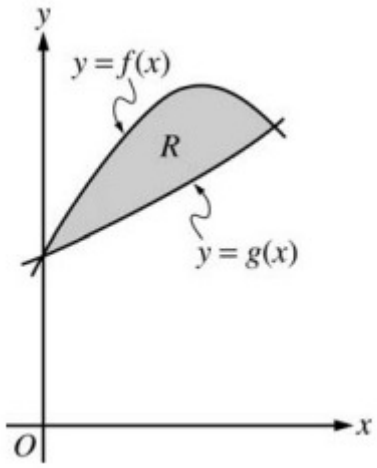
19. Find the volume of the solid generated by revolving R about the x-axis.

Please respond on separate paper, following directions from your teacher.

Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line x=10, and the x-axis.

20. Find the volume of the solid generated when R is revolved about the vertical line x=10.

Please respond on separate paper, following directions from your teacher.



Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

21. Find the volume of the solid generated when R is revolved about the x-axis.

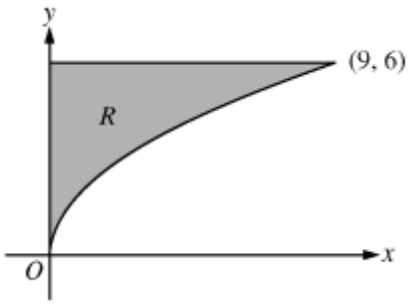
Integration Applications Review

Please respond on separate paper, following directions from your teacher.

Let R be the region enclosed by the graphs of $y=e^x$, $y=(x-1)^2$, and the line $x=1$.

22. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

Please respond on separate paper, following directions from your teacher.

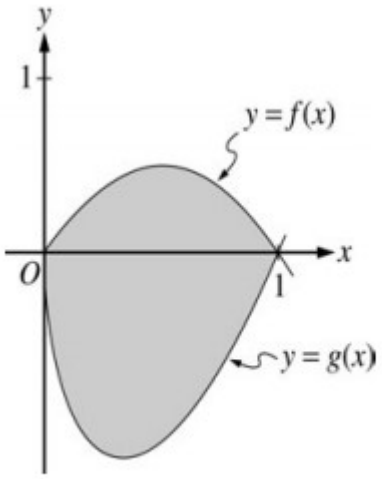


Let R be the region in the first quadrant bounded by the graph of $y = 2x$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

23. Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

Please respond on separate paper, following directions from your teacher.

Integration Applications Review



Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x - 1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

24. Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved above the horizontal line $y = 2$.

Please respond on separate paper, following directions from your teacher.