Integration Applications Review

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

1. Find the average value of f on the interval [-1, 1].

Please respond on separate paper, following directions from your teacher.

Part C

The student response earns none of the following points:

1 point is earned for $\int_{-1}^{0} (1 - 2\sin x)dx \text{ and } \int_{0}^{1} e^{-4x}\,dx$

2 point is earned for antiderivatives

1 point is earned for answer

$$\int_{-1}^{1} f(x)dx = \int_{-1}^{0} f(x)dx + \int_{0}^{1} f(x)dx$$

$$= \int_{-1}^{0} (1 - 2\sin x)dx + \int_{0}^{1} e^{-4x}\,dx$$

$$= [x + 2\cos x]_{x=-1}^{x=0} + \left[-\tfrac{1}{4}e^{-4x}\right]_{x=0}^{x=1}$$

$$= (3 - 2\cos(-1)) + \left(-\tfrac{1}{4}e^{-4} + \tfrac{1}{4}\right)$$

$$\text{Average value} = \tfrac{1}{2}\int_{-1}^{1} f(x)dx$$

$$= \tfrac{13}{8} - \cos(-1) - \tfrac{1}{8}e^{-4}$$

0	1	2	3	4 ✓

The student response earns all of the following points:

Integration Applications Review

1 point is earned for $\int_{-1}^{0}(1-2\sin x)dx$ *and* $\int_{0}^{1}e^{-4x}dx$

2 point is earned for antiderivatives

1 point is earned for answer

$$\int_{-1}^{1}f(x)dx = \int_{-1}^{0}f(x)dx + \int_{0}^{1}f(x)dx$$

$$= \int_{-1}^{0}(1-2\sin x)dx + \int_{0}^{1}e^{-4x}dx$$

$$= [x+2\cos x]_{x=-1}^{x=0} + \left[-\tfrac{1}{4}e^{-4x}\right]_{x=0}^{x=1}$$

$$= (3-2\cos(-1)) + \left(-\tfrac{1}{4}e^{-4}+\tfrac{1}{4}\right)$$

$$\textit{Average value} = \tfrac{1}{2}\int_{-1}^{1}f(x)dx$$

$$= \tfrac{13}{8}-\cos(-1)-\tfrac{1}{8}e^{-4}$$

t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$, as shown in the table above, For $0<t\le 2.4, v(t)>0$.

2. For $0 \le t \le 4$ hours, Ruth's velocity can be modeled by the function g given by $g(t)=\frac{24t+5\sin(6t)}{t+0.7}$. According to the model, what was Ruth's average velocity during the time interval $0 \le t \le 4$?

Please respond on separate paper, following directions from your teacher.

Part C

Integration Applications Review

The response can earn up to 2 points:

1 point: For the correct integral

Average velocity = $\frac{1}{2.4}\int_0^{2.4} g(t)dt$

1 point: for the correct answer

Average velocity = 14.064 miles/hr

0	1	2

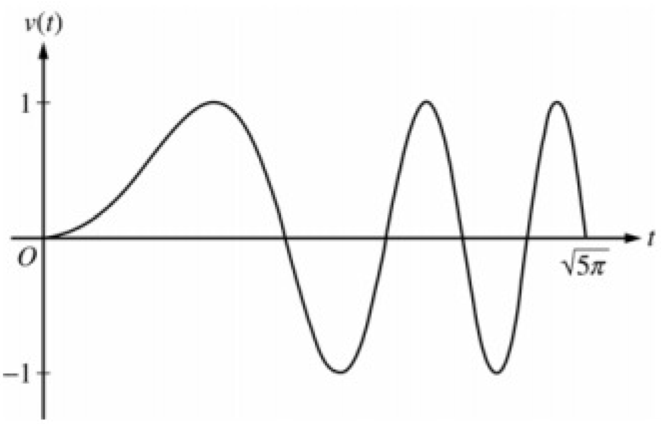
The response earns both of the following points:

1 point: For the correct integral

Average velocity = $\frac{1}{2.4}\int_0^{2.4} g(t)dt$

1 point: for the correct answer

Average velocity = 14.064 miles/hr



A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin(t^2)$. The graph of v is shown above for \(0\le t\le\sqrt{5\pi}\). The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

Integration Applications Review

3. Find the total distance traveled by the particle from time $t=0$ to $t=3$.

Please respond on separate paper, following directions from your teacher.

Part B

1 point is earned for the setup

1 point is earned for the answer

Distance == $\int_0^9 \left|v(t)\right| dt = 1.702$ **OR** For 0<t<3, v(t)=0 when t= π =1.77245 and t= 2π =2.50663 x(0)=5 x(π)=5+ ∫ 0 π v (t)dt=5.89483 x(2π)=5+ ∫ 0 2π v (t)dt=5.43041 x(3)=5+ ∫ 0 3 v (t)dt=5.77356 |x(π)−x(0)|+|x(2π)−x(π)|+|x(3)−x(2π)|=1.702

0	1	2

The student response earns all of the following points:

1 point is earned for the setup

1 point is earned for the answer

Distance= $\int_0^9 \left|v(t)\right| dt = 1.702$

OR

For 0<t<3, v(t)=0 when t= π =1.77245 and t= 2π =2.50663 x(0)=5 x(π)=5+ ∫ 0 π v (t)dt=5.89483 x(2π)=5+ ∫ 0 2π v (t)dt=5.43041 x(3)=5+ ∫ 0 3 v (t)dt=5.77356 |x(π)−x(0)|+|x(2π)−x(π)|+|x(3)−x(2π)|=1.702

A toy train moves along a straight set up on a table. The position $x(t)$ of the train at the time t seconds is measured in centimeters from the center of the track. At time $t=1$, the train is 6 centimeters to the left of the center, so $x(1)=-6$. For $0 \le t \le 4$, the velocity of the train at the time t is given by $v(t)=3t^2-12$, where $v(t)$ is measured in centimeters per second.

4. For $0 \leq t \leq 4$, find $x(t)$.

Please respond on separate paper, following directions from your teacher.

Part A

The response can earn up to 3 points:
1 point: For the correct integral
1 point: For the correct antiderivative
1 point: For the correct answer

$$x(t) = -6 + \int_1^t (3u^2 - 12)du$$
$$= -6 + \left[u^3 - 12u\right]_{u=1}^{u=t}$$
$$= t^3 - 12t + 5$$

0	1	2	3

The response earns all three of the following points:

1 point: For the correct integral
1 point: For the correct antiderivative
1 point: For the correct answer

$$x(t) = -6 + \int_1^t (3u^2 - 12)du$$
$$= -6 + \left[u^3 - 12u\right]_{u=1}^{u=t}$$
$$= t^3 - 12t + 5$$

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \leq t \leq 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

Integration Applications Review

5. How many gallons of water leak out of the tank from $t = 0$ to $t = 3$ minutes?

Please respond on separate paper, following directions from your teacher.

Part A

Method 1:

2 points are earned for the definite integral

1: limits

1: integrand

1 point is earned for the answer

- or -

Method 2:

1 point is earned for antiderivative with C

1 point is earned for solves for C using $L(0) = 0$

1 point is earned for the answer

Method 1: $\int_0^3 \sqrt{t+1}dt = \frac{2}{3}(t+1)^{3/2}\Big|_0^3 = \frac{14}{3}$

$-or-$

Method 2: $L(t)$ = gallons leaked in first t minutes

$$\frac{dL}{dt} = \sqrt{t+1}; L(t) = \frac{2}{3}(t+1)^{3/2} + C$$
$$L(0) = 0; C = -\frac{2}{3}$$
$$L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}; L(3)\,\frac{14}{3}$$

0	1	2	3	4	5	6

The student response earns all of the following points:

Integration Applications Review

Method 1:

2 points are earned for the definite integral

1: limits

1: integrand

1 point is earned for the answer

- or -

Method 2:

1 point is earned for antiderivative with C

1 point is earned for solves for C using $L(0) = 0$

1 point is earned for the answer

Method 1: $\int_0^3 \sqrt{t+1}dt = \frac{2}{3}(t+1)^{3/2}\Big|_0^3 = \frac{14}{3}$

$-or-$

Method 2: $L(t)$ = gallons leaked in first t minutes

$$\frac{dL}{dt} = \sqrt{t+1}; L(t) = \frac{2}{3}(t+1)^{3/2} + C$$
$$L(0) = 0; C = -\frac{2}{3}$$
$$L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}; L(3)\ \frac{14}{3}$$

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	−20	−30	−20	−14	−10	0	10
$a(t)$ (ft/sec^2)	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \le t \le 60$ seconds, the car's velocity v, measured in feet per second, and acceleration a, measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

Integration Applications Review

6. Using appropriate units, explain the meaning of $\int_0^{30} a(t)dt$ in terms of the car's motion. Find the exact value of $\int_0^{30} a(t)dt$.

Please respond on separate paper, following directions from your teacher.

Part B

1 point is earned for the explanation

1 point is earned for the value

$\int_0^{30} a(t)dt$is the car's change in velocity in ft/sec from *t*=0 sec to *t*=30 sec.

$$\int_0^{30} a(t)dt = \int_0^{30} v'(t)dt = v(30) - v(0)$$
$$= -14 - (-20) = 6ft/\text{sec}$$

0	1	2

The student response earns all of the following points:

1 point is earned for the explanation

1 point is earned for the value

$\int_0^{30} a(t)dt$is the car's change in velocity in ft/sec from *t*=0 sec to *t*=30 sec.

$$\int_0^{30} a(t)dt = \int_0^{30} v'(t)dt = v(30) - v(0)$$
$$= -14 - (-20) = 6ft/\text{sec}$$

Extra Point

1 point is earned for units in both (a) and (b)

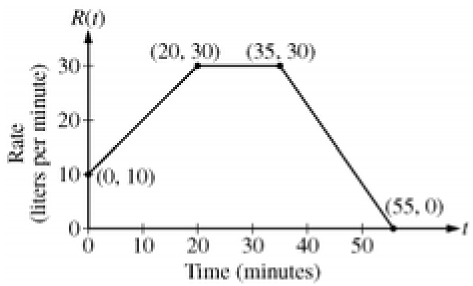
Units of ft in (a) and ft/sec in (b)

0	1

The student response earns one of the following points:

1 point is earned for units in both (a) and (b)

Units of ft in (a) and ft/sec in (b)



At time t = 0 minutes, a tank contains 100 liters of water. The piecewise-linear graph above shows the rate R(t), in liters per minute, at which water is pumped into the tank during a 55-minute period.

7. At time t = 10 minutes, water begins draining from the tank at a rate modeled by the function D, where $D(t) = 10e^{(\sin t)/10}$ liters per minute. Water continues to drain at this rate until time t = *55* minutes. How many liters of water are in the tank at time t = *55* minutes?

Please respond on separate paper, following directions from your teacher.

General

1 point is earned for: integral

1 point is earned for: expression for water in the tank

1 point is earned for: answer

$$Amt = 100 + 1150 - \int_0^{55} 10e^{(\sin t)/10}dt$$
$$= 1250 - 450.275371 = 799.725\ (or, 799.724)$$

0	1	2	3

The student response earns all of the following points:

1 point is earned for: integral

1 point is earned for: expression for water in the tank

1 point is earned for: answer

$$Amt = 100 + 1150 - \int_0^{55} 10e^{(\sin t)/10}dt$$
$$= 1250 - 450.275371 = 799.725\ (or, 799.724)$$

Let R be the region enclosed by the graphs of $y = \ln(x^2 + 1)$ and $y = \cos x$.

8. Find the area of R.

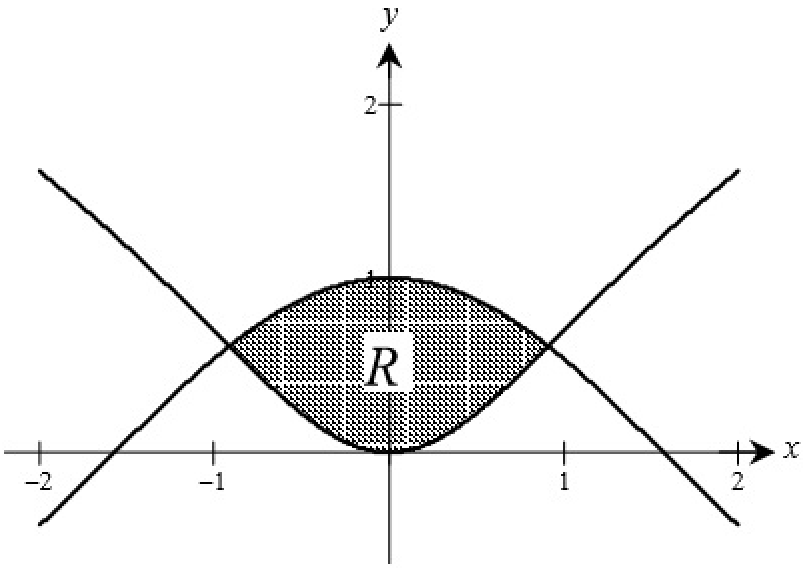
Please respond on separate paper, following directions from your teacher.

Part A

2 points are earned for the correct integral where 1 point is earned for the correct limits and 1 point is earned for the correct integrand

0/1 if integrand not $\cos x$ - $\ln(x^2 + 1)$ or $\ln(x^2 + 1)$ - $\cos x$

Integration Applications Review



$\ln(x+1) = \cos x$
$x = \pm 0.91586$
Let $B = 0.91586$

1 point is earned for the correct answer

$$\text{area} = \int_{-B}^{B} \left[\cos x - \ln(x^2+1)\right] dx$$
$$= 1.168$$

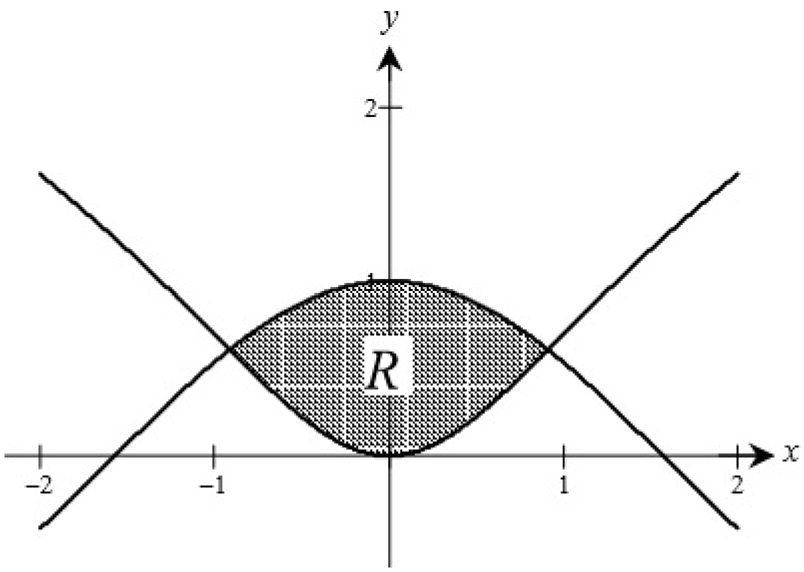
0	1	2	3

The student response earns three of the following points:

2 points are earned for the correct integral where 1 point is earned for the correct limits and 1 point is earned for the correct integrand

0/1 if integrand not $\cos x - \ln(x^2 + 1)$ or $\ln(x^2 + 1) - \cos x$

Integration Applications Review



$\ln(x+1) = \cos x$
$x = \pm 0.91586$
Let $B = 0.91586$

1 point is earned for the correct answer

$$\text{area} = \int_{-B}^{B} \left[\cos x - \ln(x^2+1)\right] dx$$
$$= 1.168$$

Let R be the region in the first quadrant under the graph of $y = \frac{x}{x^2+2}$ for $0 \le x \le \sqrt{6}$.

9. Find the area of R.

Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for correct integral

1 point is earned for antiderivative

1 point is earned for evaluation

$$A = \int_0^{\sqrt{6}} \frac{x}{x^2+2}\,dx$$
$$= \tfrac{1}{2}\, In(\mathrm{x}^2+2)\Big|_0^{\sqrt{6}}$$
$$= \tfrac{1}{2}\, In8 - \tfrac{1}{2}\, In2 = In2$$

0	1	2	3 ✓

The student response earns all of the following points:

1 point is earned for correct integral

1 point is earned for antiderivative

1 point is earned for evaluation

$$A = \int_0^{\sqrt{6}} \frac{x}{x^2+2}\,dx$$
$$= \tfrac{1}{2}\, In(\mathrm{x}^2+2)\Big|_0^{\sqrt{6}}$$
$$= \tfrac{1}{2}\, In8 - \tfrac{1}{2}\, In2 = In2$$

Consider the curve $y^2 = 4 + x$ and chord AB joining the points $A(-4,0)$ and $B(0,2)$ on the curve.

10. Find the area of the region R enclosed by the curve and the chord AB.

Please respond on separate paper, following directions from your teacher.

Part B

Method 1:

Integration Applications Review

1 point is earned for the correct bounds and constant

1 point is earned for the correct integrand

1 point is earned for the correct antidifferentiation and evaluation

OR

1 point is earned for the correct bounds and constant

1 point is earned for the correct subtraction of areas

1 point is earned for the correct answer

$$\int_{-4}^{0}\left[(\sqrt{4+x}-(\frac{1}{2}x+2)\right]dx=\frac{2}{3}(4+x)^{3/2}-\frac{1}{4}x^2-2x\bigg|_{-4}^{0}$$

$= \frac{2}{3}(4)^{3/2}-(-4+8)=\frac{16}{3}-4=\frac{4}{3}$

Method 2:

1 point is earned for the correct bounds and constant

1 point is earned for the correct integrand

1 point is earned for the correct antidifferentiation and evaluation

OR

1 point is earned for the correct bounds and constant

1 point is earned for the correct subtraction of areas

1 point is earned for the correct answer

$$\int_{0}^{2}\left[(2y-4)-(y^2-4)\right]dy=y^2-\frac{y^3}{3}\bigg|_{0}^{2}$$

$= 4-\frac{8}{3}=\frac{4}{3}$

Method 3:

1 point is earned for the correct bounds and constant

1 point is earned for the correct integrand

1 point is earned for the correct antidifferentiation and evaluation

OR

Integration Applications Review

1 point is earned for the correct bounds and constant

1 point is earned for the correct subtraction of areas

1 point is earned for the correct answer

$$\int_{-4}^{0} \sqrt{4+x}dx = \tfrac{16}{3}; \text{ Area of triangle} = 4$$

$$\text{Area of region} = \tfrac{16}{3} - 4 = \tfrac{4}{3}$$

Note: In (a), (b), and (c) any arithmetic error results in loss of last point.

0	1	2	3 ✓

The student response earns three of the following points:

Method 1:

1 point is earned for the correct bounds and constant

1 point is earned for the correct integrand

1 point is earned for the correct antidifferentiation and evaluation

OR

1 point is earned for the correct bounds and constant

1 point is earned for the correct subtraction of areas

1 point is earned for the correct answer

$$\int_{-4}^{0} \left[(\sqrt{4+x} - (\frac{1}{2}x+2)\right] dx = \frac{2}{3}(4+x)^{3/2} - \frac{1}{4}x^2 - 2x\Big|_{-4}^{0}$$

$$= \tfrac{2}{3}(4)^{3/2} - (-4+8) = \tfrac{16}{3} - 4 = \tfrac{4}{3}$$

Method 2:

1 point is earned for the correct bounds and constant

1 point is earned for the correct integrand

1 point is earned for the correct antidifferentiation and evaluation

OR

1 point is earned for the correct bounds and constant

1 point is earned for the correct subtraction of areas

1 point is earned for the correct answer

$$\int_0^2 \left[(2y-4)-(y^2-4)\right] dy = y^2 - \frac{y^3}{3}\bigg|_0^2$$
$$= 4 - \tfrac{8}{3} = \tfrac{4}{3}$$

Method 3:

1 point is earned for the correct bounds and constant

1 point is earned for the correct integrand

1 point is earned for the correct antidifferentiation and evaluation

OR

1 point is earned for the correct bounds and constant

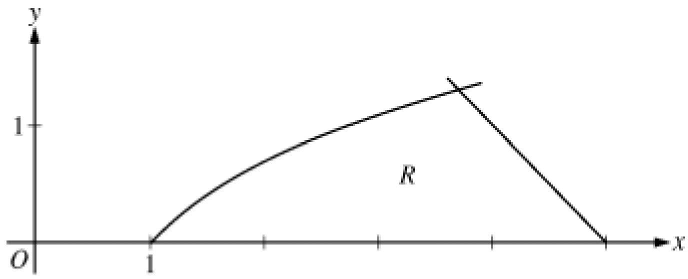
1 point is earned for the correct subtraction of areas

1 point is earned for the correct answer

$$\int\limits_{-4}^{0} \sqrt{4+x}dx = \tfrac{16}{3};\ \text{Area of triangle} = 4$$
$$\text{Area of region} = \tfrac{16}{3} - 4 = \tfrac{4}{3}$$

Note: In (a), (b), and (c) any arithmetic error results in loss of last point.

Integration Applications Review



Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y=\ln x$ and $y=5-x$, as shown in the figure above.

---

11. Find the area of R.

Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for the integrand

1 point is earned for the limits

1 point is earned for the answer

$\ln x = 5\text{-}x \Rightarrow x=3.69344$

Therefore, the graphs of $y=\ln x$ and $y=5-x$ intersect in the first quadrant at the point $(A, B)=(3.69344, 1.30656)$.

Area = $\int_0^B (5 - y - e^y)dy$

= 2.986 (or 2.985)

OR

Area = $\int_1^A \ln x dx + \int_A^5 (5 - x)^2$

=2.986 (or 2.985)

Integration Applications Review

0	1	2	3

The student response earns all of the following points:

1 point is earned for the integrand

1 point is earned for the limits

1 point is earned for the answer

$\ln x = 5\text{-}x \Rightarrow x = 3.69344$

Therefore, the graphs of y=ln x and y=5-x intersect in the first quadrant at the point (A, B)=(3.69344, 1.30656).

Area = $\int_0^B (5 - y - e^y)dy$

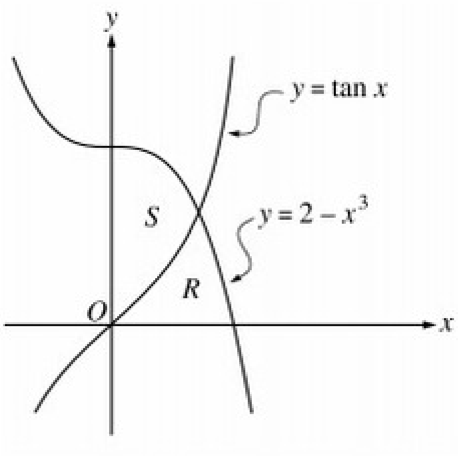
= 2.986 (or 2.985)

OR

Area = $\int_1^A \ln x dx + \int_A^5 (5 - x)^2$

=2.986 (or 2.985)

Integration Applications Review



Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y-axis and the graphs of $y = 2 - x^3$ and y tan x.

---

12. Find the area of R.

Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for the limits

1 point is earned for the integrand

1 point is earned for the answer

Points of intersection

$$2 - x^3 = \tan x, at\,(A, B) = (0.902155, 1.265751)$$

$$Area, R = \int_0^A \tan x dx + \int_A^{\sqrt[3]{2}} \left(2 - x^3\right) dx = 0.729$$

or

$$Area, R = \int_0^B \left((2 - x)^{1/3} - \tan^{-1} y\right) dy = 0.729$$

or

$$Area, R = \int_A^{\sqrt[3]{2}} \left(2 - x^3\right) dx - \int_0^A \left(2 - x^3 - \tan x\right) dx = 0.729$$

Integration Applications Review

0	1	2	3

The student response earns all of the following points:

1 point is earned for the limits

1 point is earned for the integrand

1 point is earned for the answer

Points of intersection

$$2 - x^3 = \tan x, at\,(A, B) = (0.902155, 1.265751)$$

$$Area, R = \int_0^A \tan x dx + \int_A^{\sqrt[3]{2}} \left(2 - x^3\right) dx = 0.729$$

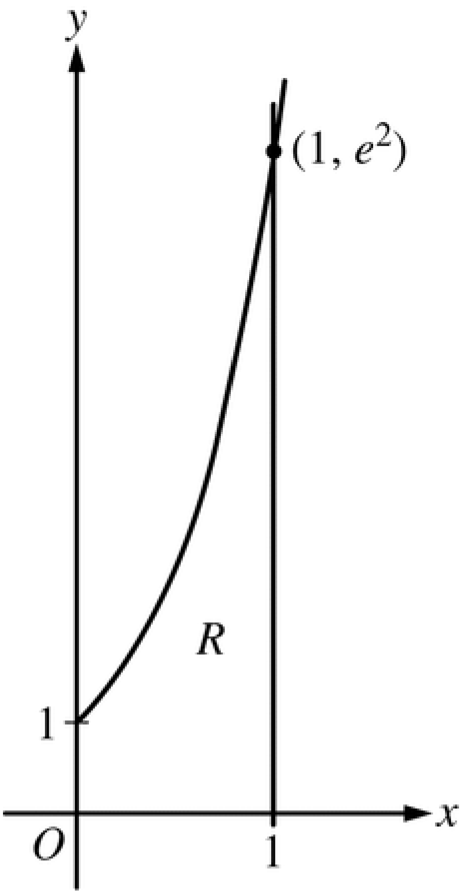
or

$$Area, R = \int_0^B \left((2 - x)^{1/3} - \tan^{-1} y\right) dy = 0.729$$

or

$$Area, R = \int_A^{\sqrt[3]{2}} \left(2 - x^3\right) dx - \int_0^A \left(2 - x^3 - \tan x\right) dx = 0.729$$

Integration Applications Review



Let $f(x)=e^{2x}$. Let *R* be the region in the first quadrant bounded by the graph of $y=f(x)$ and the vertical line $x=1$, as shown in the figure above.

13. Region *R* forms the base of a solid whose cross sections perpendicular to the *y*-axis are squares. Write, but do not evaluate, and expression involving one or more integral that gives the volume of the solid.

Please respond on separate paper, following directions from your teacher.

Part C

The response can earn up to 4 points:

2 points: For the integrand

1 point: For correct limits

1 point: For the correct answer

Integration Applications Review

Volume $= 1 + \int_1^{e^2} \left(1 - \frac{1}{2}\ln y\right)^2 dy$

0	1	2	3	4

The response earns all four of the following points:

2 points: For the integrand

1 point: For correct limits

1 point: For the correct answer

Volume $= 1 + \int_1^{e^2} \left(1 - \frac{1}{2}\ln y\right)^2 dy$

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

14. The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

Please respond on separate paper, following directions from your teacher.

Part C

1 point is earned for integrand

1 point is earned for limits and answer

$$\int_0^3 \left(3y - y^2\right)^2 dy = 8.1$$

Integration Applications Review

0	1	2

The student response earns all of the following points:

1 point is earned for integrand

1 point is earned for limits and answer

$$\int_0^3 \left(3y - y^2\right)^2 dy = 8.1$$

15. Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

Please respond on separate paper, following directions from your teacher.

Part B

1 point is earned for constant and limits

2 points are earned for integrand

1 point is earned for the answer

$$\pi \int_0^3 \left((3y+1)^2 - \left(y^2+1\right)^2\right) dy$$
$$= \frac{207\pi}{5} = 130.061 \text{ or } 130.062$$

0	1	2	3	4

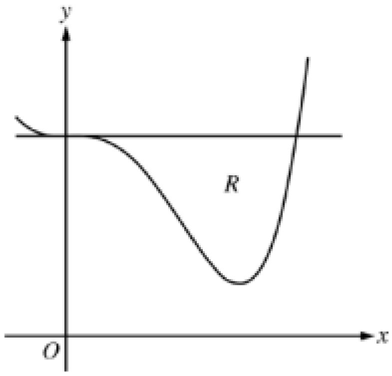
The student response earns all of the following points:

1 point is earned for constant and limits

2 points are earned for integrand

1 point is earned for the answer

$$\pi \int_0^3 \left((3y+1)^2 - \left(y^2+1\right)^2\right) dy$$
$$= \frac{207\pi}{5} = 130.061 \text{ or } 130.062$$



Let R be the region enclosed by the graph of $f(x)=x^4-2.3x^3+4$ and the horizontal line $y=4$, as shown in the figure above.

---

16. Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

Please respond on separate paper, following directions from your teacher.

Part B

2 points are earned for integrand

1 point is earned for answer

$$\text{Volume} = \int_0^{2.3} \frac{1}{2}(4-f(x))^2\, dx$$

=3.574 (or 3.573)

Integration Applications Review

0	1	2	3

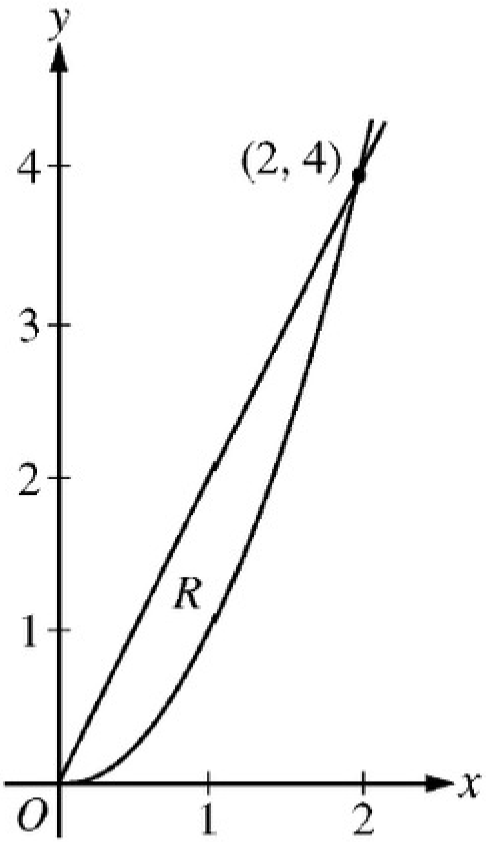
The student response earns all of the following points:

2 points are earned for integrand

1 point is earned for answer

Volume $= \int_{0}^{2.3} \frac{1}{2}(4\text{-}f(x))^2\,dx$

=3.574 (or 3.573)



Let R be the region in the first quadrant enclosed by the graphs of y=2x and y=x^2, as shown in the figure above.

Integration Applications Review

17. The region R is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x)$=sin(π/2x) . Find the volume of the solid.

Please respond on separate paper, following directions from your teacher.

Part B

1 point is earned for the integrand

1 point is earned for the anti derivative

1 point is earned for the answer

$$Volume = \int_0^2 \sin(\frac{\pi}{2}x)dx$$
$$= -\tfrac{2}{\pi}\cos(\tfrac{\pi}{2}x)\Big|_{x=0}^{x=2}$$
$$= \tfrac{4}{\pi}$$

0	1	2	3

The student response earns three of the following points:

1 point is earned for the integrand

1 point is earned for the anti derivative

1 point is earned for the answer

$$Volume = \int_0^2 \sin(\frac{\pi}{2}x)dx$$
$$= -\tfrac{2}{\pi}\cos(\tfrac{\pi}{2}x)\Big|_{x=0}^{x=2}$$
$$= \tfrac{4}{\pi}$$

Integration Applications Review

Let R be the region bounded by the x–axis, the graph of $y = \sqrt{x}$, and the line $x = 4$.

18. The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x–axis, they generate solids with equal volumes. Find the value of k

Please respond on separate paper, following directions from your teacher.

Part D

1 point is earned for equation in k

1 point is earned for the answer

$$\pi \int_0^k (\sqrt{x})^2 dx = 4\pi \qquad \text{or} \qquad \pi \int_0^k (\sqrt{x})^2 dx = \pi \int_k^4 (\sqrt{x})^2 dx$$

$$\pi \frac{k^2}{2} = 4\pi \qquad \pi \frac{k^2}{2} = 8\pi - \pi \frac{k^2}{2}$$

$$k = \sqrt{8} \quad \text{or} \quad 2.828$$

0	1	2

The student response earns two of the following points:

1 point is earned for equation in k

1 point is earned for the answer

$$\pi \int_0^k (\sqrt{x})^2 dx = 4\pi \qquad \text{or} \qquad \pi \int_0^k (\sqrt{x})^2 dx = \pi \int_k^4 (\sqrt{x})^2 dx$$

$$\pi \frac{k^2}{2} = 4\pi \qquad \pi \frac{k^2}{2} = 8\pi - \pi \frac{k^2}{2}$$

$$k = \sqrt{8} \quad \text{or} \quad 2.828$$

Integration Applications Review

Let R be the region in the first quadrant enclosed by the hyperbola $x^2 - y^2 = 9$, the x-axis, and the line $x = 5$.

19. Find the volume of the solid generated by revolving R about the <u>*x-axis.*</u>

Please respond on separate paper, following directions from your teacher.

Part A

2 points are earned for a correct integrand

1 point is earned for appropriate limits and kπ

1 point is earned for correct antiderivative

1 point is earned for substitution and/or evaluation

Discs:

$$V = \pi \int_5^3 \left(x^2 - 9\right)\ dx$$

$$= \pi \left[\tfrac{1}{3}x^3 - 9x\right]_3^5$$

$$= \pi \left[\left(\tfrac{125}{3} - 45\right) - (9 - 27)\right] = \tfrac{44}{3}\pi$$

Or

Shells:

$$V = 2\pi \int_0^4 \left(5 - \sqrt{9 + y^2}\right)\ ydy$$

$$= 2\pi \left[\tfrac{5}{2}\text{y}^2 \text{-} \tfrac{1}{3}\left(9 + y^2\right)^{\frac{3}{2}}\right]_0^4$$

$$= 2\pi \left(40 - \tfrac{125}{3} + \tfrac{27}{3}\right) = \tfrac{44}{3}\pi$$

0	1	2	3	4	5

The student response earns all of the following points:

2 points are earned for a correct integrand

1 point is earned for appropriate limits and kπ

1 point is earned for correct antiderivative

1 point is earned for substitution and/or evaluation

Discs:

$$V = \pi \int_5^3 \left(x^2 - 9\right)\, dx$$

$$= \pi \left[\tfrac{1}{3}x^3 - 9x\right]_3^5$$

$$= \pi \left[\left(\tfrac{125}{3} - 45\right) - (9 - 27)\right] = \tfrac{44}{3}\pi$$

Or

Shells:

$$V = 2\pi \int_0^4 \left(5 - \sqrt{9 + y^2}\right)\, ydy$$

$$= 2\pi \left[\tfrac{5}{2}\mathrm{y}^2 \text{-} \tfrac{1}{3}\left(9 + y^2\right)^{\frac{3}{2}}\right]_0^4$$

$$= 2\pi \left(40 - \tfrac{125}{3} + \tfrac{27}{3}\right) = \tfrac{44}{3}\pi$$

Let R be the region enclosed by the graph of $y = \sqrt{x - 1}$, the vertical line x=10, and the x-axis.

20. Find the volume of the solid generated when R is revolved about the vertical line x=10.

Please respond on separate paper, following directions from your teacher.

Part C

1 point is earned for the limits and constant

Integration Applications Review

1 point is earned for the integrand

1 point is earned for the answer

Volume $=\pi \int_{0}^{3} (10-(y^2+1))^2\,dy = 407.150$

0	1	2	3

The student response earns all of the following points:

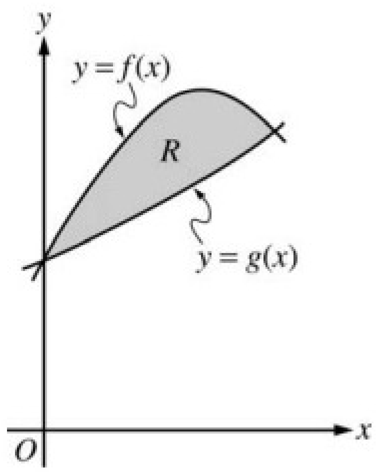
1 point is earned for the limits and constant

1 point is earned for the integrand

1 point is earned for the answer

Volume $= \pi \int_{0}^{3} (10-(y^2+1))^2\,dy$

$=407.150$



Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of *f* and *g* as shown in the figure above.

21. Find the volume of the solid generated when *R* is revolved about the *x*-axis.

Please respond on separate paper, following directions from your teacher.

Part B

The student response earns none of the following points:

2 point is earned for the integrand

(-1) for each error

Note: 0/2 if integral not of form C ∫ a b *((R2(x)-(g2(x))dx*

1 point is earned for the answer

$$Volume = \pi \int_0^s \left((f(x))^2 - (g(x))^2 \right) dx$$

$$= \pi \int_0^s \left((1+\sin(2x))^2 - \left(e^{x/2}\right)^2 \right) dx$$

=4.266 or 4.267

0	1	2

The student response earns none of the following points:

2 point is earned for the integrand

(-1) for each error

Note: 0/2 if integral not of form C ∫ a b *((R2(x)-(g2(x))dx*

1 point is earned for the answer

Integration Applications Review

$$Volume = \pi \int_0^s \left((f(x))^2 - (g(x))^2 \right) dx$$

$$= \pi \int_0^s \left((1 + \sin(2x))^2 - \left(e^{x/2} \right)^2 \right) dx$$

=4.266 or 4.267

Let *R* be the region enclosed by the graphs of $y=e^x$,$y=(x-1)^2$, and the line $x=1$.

22. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid generated when *R* is revolved about the <u>*y*-axis</u>.

Please respond on separate paper, following directions from your teacher.

Part C

2 points are earned for Definite integral

2 points are **deducted** for f,g, or u error

1 point is **deducted** for limit error, K error and integrand reversal

$$V = 2\pi \int_1^0 x \left[e^x - (x-1)^2 \right] dx$$

or

$$V = \pi \int_1^0 1 - \left(1 - \sqrt{y}\right)^2 dy + \pi \int_e^1 1 - (\ln y)^2 dy$$

Integration Applications Review

0	1	2

The student response earns none of the following points:

2 points are earned for Definite integral

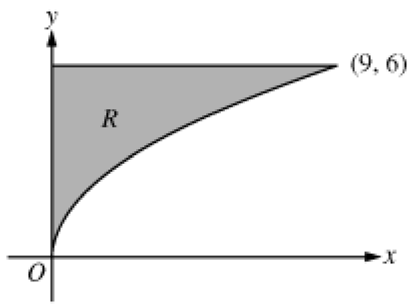
2 points are **deducted** for f,g, or u error

1 point is **deducted** for limit error, K error and integrand reversal

$$V = 2\pi \int_1^0 x\left[e^x - (x-1)^2\right] dx$$

or

$$V = \pi \int_1^0 1 - \left(1-\sqrt{y}\right)^2 dy + \pi \int_e^1 1 - (\ln y)^2 dy$$



Let R be the region in the first quadrant bounded by the graph of $y = 2x$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

23. Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

Please respond on separate paper, following directions from your teacher.

Integration Applications Review

Part B

2 point is earned for integrand

1 point is earned for limits and constant

$$Volume = \pi \int_0^9 \left((7 - 2\sqrt{x})^2 - (7-6)^2 \right) dx$$

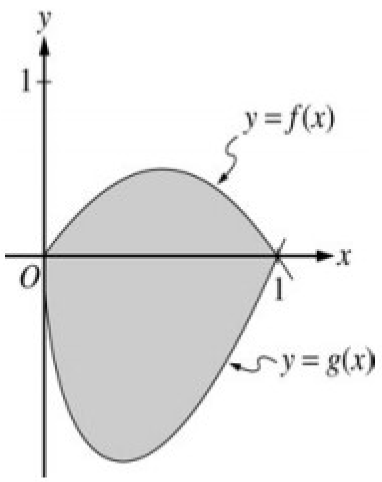
0	1	2	3

The student response earns all of the following points:

2 point is earned for integrand

1 point is earned for limits and constant

$$Volume = \pi \int_0^9 \left((7 - 2\sqrt{x})^2 - (7-6)^2 \right) dx$$



Let f and g be the functions given by $f(x) = 2x(1 - x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and

g are shown in the figure above.

24. Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved above the horizontal line $y = 2$.

Please respond on separate paper, following directions from your teacher.

Part B

1 point is earned for the limits and constant

2 points are earned for integrand

< -1 > each error

1 point is earned for the answer

$$Volume = \pi \int_0^1 \left((2 - g(x))^2 - (2 - f(x))^2 \right) dx$$
$$= \pi \int_0^1 \left((2 - 3(x-1)\sqrt{x})^2 - (2 - 2x(1-x))^2 \right) dx$$
$$= 16.179$$

0	1	2	3	4

The student response earns all of the following points:

1 point is earned for the limits and constant

2 points are earned for integrand

< -1 > each error

1 point is earned for the answer

$$Volume = \pi \int_0^1 \left((2 - g(x))^2 - (2 - f(x))^2 \right) dx$$
$$= \pi \int_0^1 \left((2 - 3(x-1)\sqrt{x})^2 - (2 - 2x(1-x))^2 \right) dx$$
$$= 16.179$$

Integration Applications Review