Integral Properties and Mean Value Theorem

Goal:
• Can use limit properties with Riemann sums and can justify integral properties using area. • Can find the average value of a function
Terminology:
• Mean Value Theorem for Integrals

Integral Properties

Property	Proof or Justification
$\int_a^b f(x)dx = -\int_b^a f(x)dx$	
$\int_a^a f(x)dx = 0$	
$\int_a^b kf(x)dx = k\int_a^b f(x)dx$	

$\int_a^b \big(f(x) \pm g(x)\big)dx$ $= \int_a^b f(x)dx \pm \int_a^b f(x)dx$	
$\int_a^c f(x)dx$ $= \int_a^b f(x)dx + \int_b^c f(x)dx$	
If $f(x) \leq g(x)$ on $[a, b]$ then $\int_a^b f(x)dx \leq \int_a^b g(x)dx$	

$\min f \cdot (b-a) \leq \int_a^b f(x)dx$ $\int_a^b f(x)dx \leq \max f \cdot (b-a)$	
$avg(f) = \frac{1}{b-a}\int_a^b f(x)dx$	

The **Mean Value Theorem** for integrals states the following:
If f is continuous on $[a, b]$, then there exists some point $c \in [a, b]$ such that

$$f(c) = avg(f) = \frac{1}{b-a}\int_a^b f(x)dx$$

Proof:

Practice Problems: 5.3 # 1-6, 25-28 (use fnInt), 29-31, 33, 34, 36 # 39
Look Ahead: How is the area related to a rate of change?

In Class Evidence

1. Given that $\int_1^2 f(x)dx = -4$, $\int_1^5 f(x)dx = 6$ and $\int_1^5 g(x)dx = 8$ find the following integrals

d. $\int_2^5 f(x)dx$ f. $\int_1^5 \left(4f(x) - g(x)\right)dx$ h. $\int_5^1 f(t)dt$

27. Use fnInt to find the average value of the function $y = -3x^2 - 1$ on the interval $[0, 1]$.

Use geometry to find the average value of the following functions

29. On the interval $[-4, 2]$

$$f(x) = \begin{cases} x + 4, & -4 \le x \le -1 \\ 2 - x, & -1 < x \le 2 \end{cases}$$

30. $f(t) = 1 - \sqrt{1 - t^2}$ on $[-1, 1]$

31. $f(\theta) = \sin\theta$ on $[0, 2\pi]$

36. Show that $\int_0^1 \sqrt{x + 8}dx \in [2\sqrt{2}, 3]$