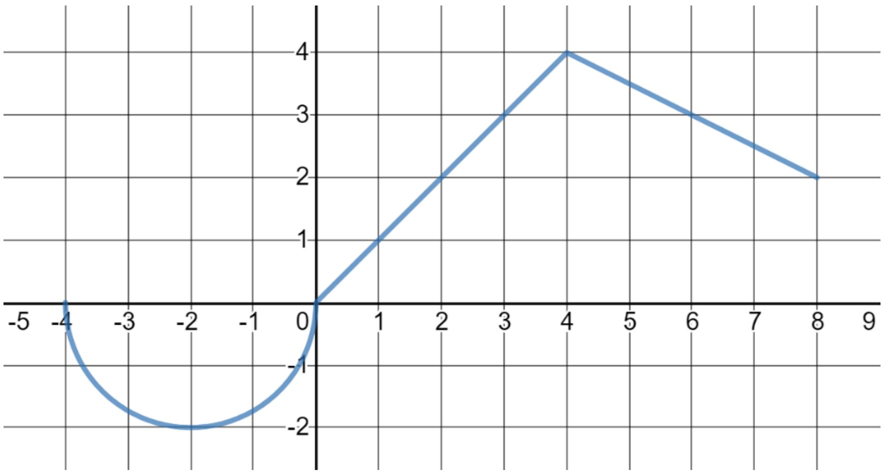
Fundamental Theorem of Calculus: Part 1

Goal:
- Understands why the integral is the antiderivative
- Understands why the derivative of an integral is the integrand
- Can analyze functions defined as integrals

Terminology:
- Antiderivative
- Fundamental Theorem of Calculus

Review: Find the average value of f below on the interval $[-4, 8]$



Consider the function

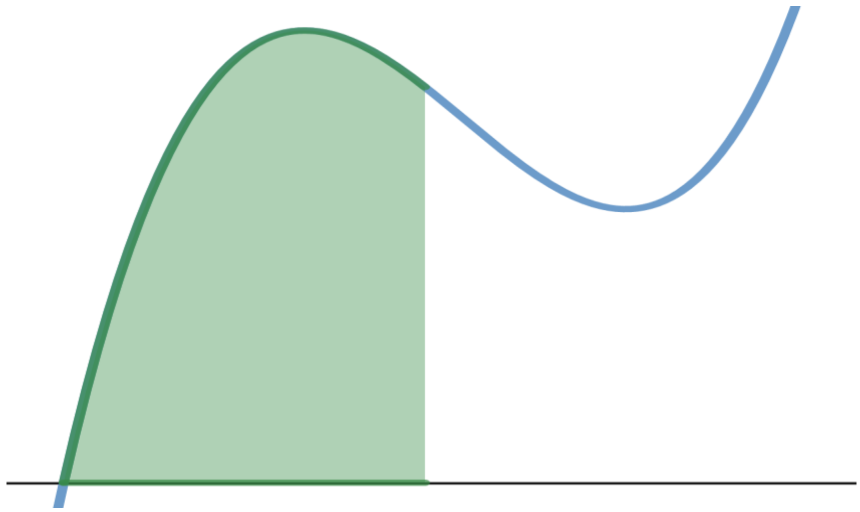
$$F(t) = \int_{-4}^{t} f(x)dx$$

Where f is given above. Determine the following values: $F(-4),\ F(0),\ F(4),\ F(8)$

In general, if we have some function, g, and define a new function

$$G(x) = \int_a^x g(t)dt$$

We can consider what happens when we have a small change in x, and then consider what happens when $\Delta x \to 0$.



This leads us to the first part of **Fundamental Theorem of Calculus:**

$$\frac{d}{dx}\int_a^x g(t)dt = g(x)$$

Example: Determine $h'(2)$, given that

$$h(x) = \int_0^{x^2} \sin z \, dz$$

Example: Determine a function $y(x)$ such that

$$\frac{dy}{dx} = \sqrt{\tan x}$$

Note: Finding such a function without an integral is possible and is in fact:

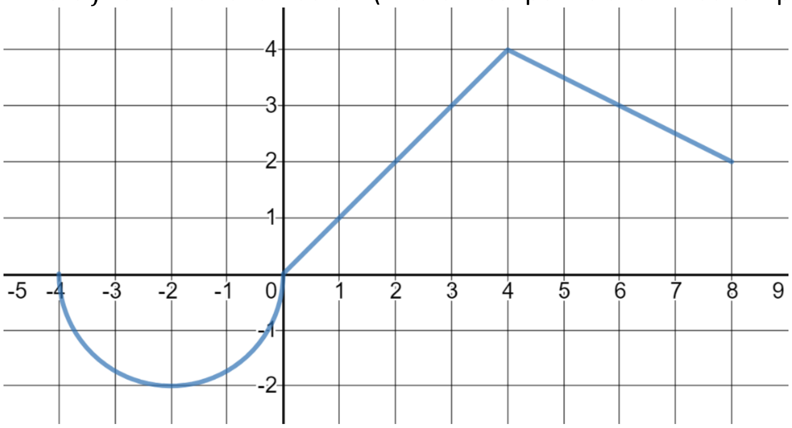
$$y = \frac{1}{\sqrt{2}}\left[\arctan\left(\sqrt{2\tan x} - 1\right) + \arctan\left(\sqrt{2\tan x} + 1\right) + \frac{1}{2}\ln\left(\frac{\tan x - \sqrt{2\tan x} + 1}{\tan x + \sqrt{2\tan x} + 1}\right)\right]$$

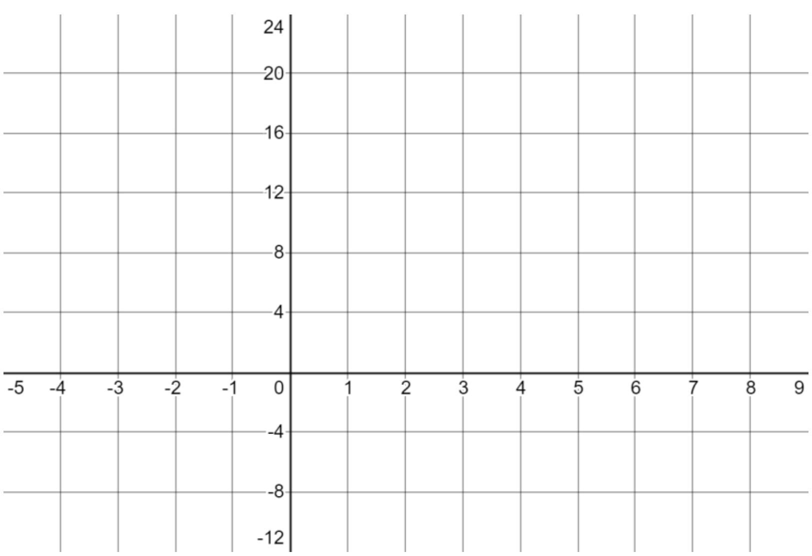
But arguably, the simple integral does a good job and we can graph it and do computations with it thanks to computers.

Practice: Accurately sketch the curve

$$F(x) = \int_{-4}^{x} f(u)du$$

Where f is the function below (Find critical points and inflection points)





Practice Problems: 5.4 # 37-46, 48-50, 53-56, 60 # 64
Look Ahead: How can the Fundamental Theorem be used to evaluate $\int_a^b f(x)dx$?

In Class Evidence

For the following, find dy/dx

39. $\int_0^{\sqrt{x}} \sin t^2 \, dt$

42. $\int_{\sin x}^{\cos x} t^2 dt$

49. Find the linearization of

$$f(x) = 2 + \int_0^x \frac{10}{1+t} dt$$

At $x = 0$

56. Let

$$f(t) = \begin{cases} \dfrac{\sin t}{t}, & t \neq 0 \\ 1, & -t = 0 \end{cases}$$

And define

$$\mathrm{Si}(x) = \int_0^x f(t)dt$$

a. Show that $\mathrm{Si}(x)$ is odd.
b. What is $\mathrm{Si}(0)$?
c. Find the values of x where $\mathrm{Si}(x)$ has a local extreme value
d. Graph $\mathrm{Si}(x)$ using your calculator