Fundamental Theorem of Calculus: Part 2

Goal:
- Understands how to evaluate a definite integral for basic functions on \([a, b]\)
- Understands how to derive the second part of Fundamental Theorem

Terminology:
- Total Area

Reminder:
- Test on Tuesday Feb 4

Recall that last time we looked at:

\[
\int_{a}^{b} g(t) dt = ???
\]

\[
\Delta G(x) = g(c) \cdot \Delta x
\]

So that \( \frac{\Delta G(x)}{\Delta x} = g(c) \) and as \( \Delta x \to 0 \) we have that
\( c \to x \) so that

\[
\lim_{\Delta x \to 0} \frac{\Delta G(x)}{\Delta x} = \lim_{\Delta x \to 0} g(c)
\]

\[
\Rightarrow \frac{dG}{dx} = g(x)
\]

And we saw \( G \) is the antiderivative of \( g \) since it is some function that if we differentiate we get

\[
\frac{d}{dx} G(x) = g(x)
\]

So the question remains, how do we evaluate a discrete integral with antiderivates?
Example: Evaluate the following
\[ \int_0^4 (t^2 - 4t + 1)dt \]

Practice: Evaluate the following (\(n\) is a constant)
\[ \int_0^1 (x^n + \sqrt{x})dx \]
Example: Find the total area between the $x$-axis and the curve $x^2 - 2x$ on the interval $[0, 3]$

Practice: Find the total area between the $x$-axis and the curve $x^3 - 4x^2 + 3x$ on the interval $[0, 3]$

Practice Problems: 5.3 # 17-20
5.4 # 1-13, 15-18, 25-28, 51

# 14
In Class Evidence

1. 
\[ \int_{0.5}^{2} \left( 2 - \frac{1}{x} \right) dx \]

2. 
\[ \int_{2}^{1} 3^x dx \]

3. 
\[ \int_{0}^{\pi/3} 4 \sec x \tan x \, dx \]
17. Find the total area of the region between the curve \( y = x^3 - 3x^2 + 2x \) and the \( x \)-axis on the interval \([0, 2]\).

28. Find the area between the line \( y = 0.5 \) and the curve \( y = \sin x \) on the interval \( \left[ \frac{\pi}{6}, \frac{5\pi}{6} \right] \).