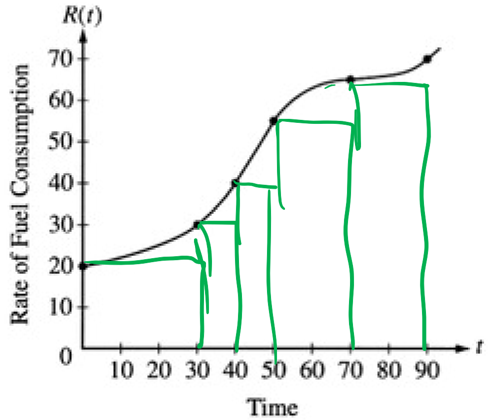
Integration Review: Week 1

Name



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes, are shown above.

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1. Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

2/2

Since $R(t)$ is strictly increasing using LRAM will under estimate the true value

Please respond on separate paper, following directions from your teacher.

$$\int_0^{90} R(t)\,dt \approx 30\cdot R(0) + 10\left(R(30)+R(40)\right) + 20\left(R(50)+R(70)\right)$$

$= 3700$ gallons

t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
$v(t)$ (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

Ruth rode her bicycle on a straight trail. She recorded her velocity $v(t)$, in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$, as shown in the table above, For $0 < t \le 2.4, v(t) > 0$.

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2. Using correct units, interpret the meaning of $\int_0^{2.4} v(t)\,dt$ in the context of the problem. Approximate

This is the displacement in miles Ruth moved in the first 2.4 hrs if $v(t) \ge 0$ $\forall$ t then this is the dist. travelled.

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$\frac{3}{3}$

$\int_0^{2.4} v(t)\,dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the table.

$\Delta x = \frac{2.4-0}{3} = 0.8 \Rightarrow \int_0^{2.4} v(t)\,dt \approx 0.8\left(v(0.4)+v(1.2)+v(2)\right)$

Please respond on separate paper, following directions from your teacher.

$= 36.64$ miles

A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

$f'(x) = 12x^2 + 2ax + b$

$f''(x) = 24x + 2a$

where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph f has a point of inflection at $x = -2$.

$f'(-1) = 0$ $\quad f''(-1) > 0$ $\quad f''(-2) = 0$

$12 + 48(-1) + b = 0$ $\quad 0 = -48 + 2a$

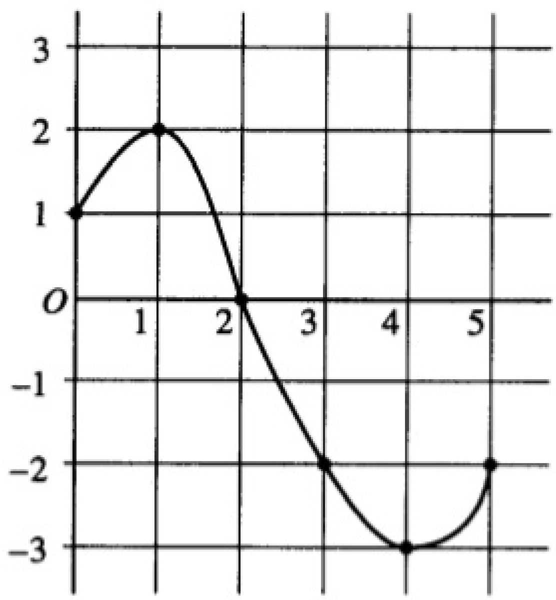
$b = 36$ $\quad a = +24$

3. If $\int_0^1 f(x)\,dx = 32$, what is the value of k?

Please respond on separate paper, following directions from your teacher.

$32 = \left[x^4 + \frac{ax^3}{3} + \frac{bx^2}{2} + kx\right]_0^1 = 1 + \frac{a}{3} + \frac{b}{2} + k = 32$

$1 + 8 + 18 + k = 32 \Rightarrow \boxed{k=5}$



Graph of f

Hard to mark...

assuming 4/4

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Let f be a function whose domain is the closed interval $[0,5]$. The graph of f is shown above.

Let $h(x) = \int_0^{\frac{x}{2}+3} f(t)\,dt$.

2/2

4. Find $h'(2)$.

$h'(x) = f(\frac{x}{2}+3)\cdot\frac{1}{2} \Rightarrow h'(2) = \frac{1}{2}f(4) = -\frac{3}{2}$

Please respond on separate paper, following directions from your teacher.

2/2

5. Find the domain of h.

$\frac{x}{2}+3 \in [0,5] \Rightarrow 0 \le \frac{x}{2}+3 \le 5$

Please respond on separate paper, following directions from your teacher.

$-3 \le \frac{x}{2} \le 2$

$\boxed{-6 \le x \le 4}$

6. At what x is $h(x)$ a minimum? Show the analysis that leads to your conclusion.

Please respond on separate paper, following directions from your teacher.

$h'(x) = \frac{1}{2}f(\frac{x}{2}+3)$ find crit. points $h'(x) = 0$ or $\nexists$ → never

$\Rightarrow \frac{1}{2}f(\frac{x}{2}+3) = 0$

$\Rightarrow \frac{x}{2}+3 = 2 \Rightarrow x = -2$

4/4

$h''(x) = \frac{1}{4}f'(\frac{x}{2}+3)$

$h''(-2) = \frac{1}{4}f'(2) < 0$

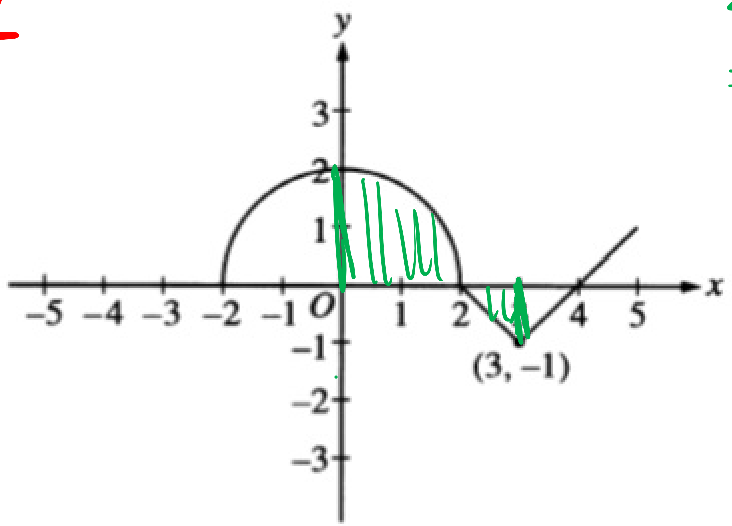
So $x=-2$ is max

Test endpoints

$h(-6) = 0$

$h(4) = \int_0^5 f(t)\,dt < 0$ by inspection.

$\boxed{x=4}$ is the min $\Leftarrow$



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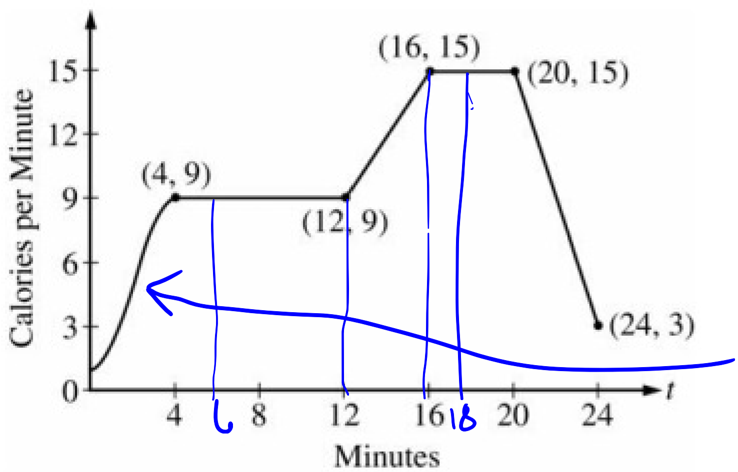
The graph of the function f consists of a semicircle and two line segments as shown above. Let g be the function given by $g(x) = \int_0^x f(t)dt$.

7. Find $g(3)$.

$\int_0^3 f(t)\,dt = \frac{1}{4}\pi \cdot 4 - \frac{1}{2} = \pi - \frac{1}{2}$

$\frac{2}{2}$

Please respond on separate paper, following directions from your teacher.



$f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1 \quad 0 \le t \le 4$

The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function f. In the figure above, $f\left(t\right)=-\frac{1}{4}t^3+\frac{3}{2}t^2+1$ for $0\le t\le4$ and f is piecewise linear for $4\le t\le24$.

8. Find the total number of calories burned over the time interval $6 \le t \le 18$ minutes.

$\frac{2}{2}$

Please respond on separate paper, following directions from your teacher.

$\int_6^{18} f(t)\,dt = 6 \cdot 9 + \frac{1}{2}(15+9) \cdot 4 + 2 \cdot 15 = 132$ calories

Let f be a function such that $f''(x)=6x+8$.

9. Find $f(x)$ if the graph of f is tangent to the line $3x - y = 2$ at the point $(0, -2)$. ⇒ pass thru $(0,-2)$

$y = 3x - 2 \rightarrow$ slope $f'(0) = 3$

$\frac{4}{4}$

Please respond on separate paper, following directions from your teacher.

$f''(x) = 6x + 8 \Rightarrow f'(x) = 3x^2 + 8x + C \Rightarrow f(x) = x^3 + 4x^2 + 3x + k$

$\Rightarrow f'(0) = C = 3$

$f(0) = -2 = k$

$\boxed{f(x) = x^3 + 4x^2 + 3x - 2}$

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The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4+t^2}\,dt$ and $g(x) = f(\sin x)$

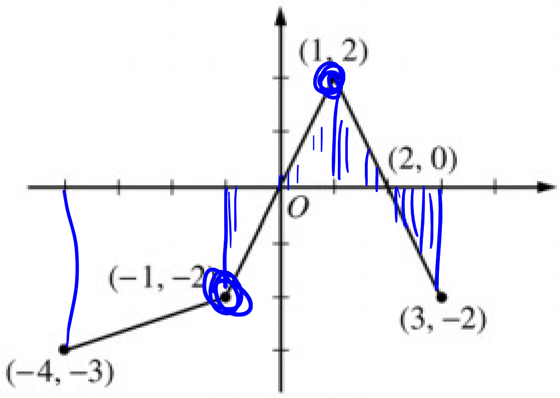
10. Find $f'(x)$ and $g'(x)$.

$f'(x) = 3\sqrt{4+9x^2}$, $g'(x) = f'(\sin x)\cdot\cos x$

$= 3\cos x\sqrt{4+9\sin^2 x}$

$\frac{4}{4}$

Please respond on separate paper, following directions from your teacher.



Graph of f

The graph of the function f above consists of three line segments.

11. Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

$g(-1) = \int_{-4}^{-1} f(t)dt = \frac{1}{2}(-2+-3)\cdot 3 = \boxed{-3.75 = g(-1)}$

$\frac{3}{3}$

Please respond on separate paper, following directions from your teacher.

$g'(x) = f(x)$; $g''(x) = f'(x) \Rightarrow \boxed{g'(-1) = -2}$

corner ← $\boxed{g''(-1) \text{ DNE}}$

12. Let h be the function given by $h(x) = \int_x^3 f(t)\,dt$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x)=0$.

Please respond on separate paper, following directions from your teacher.

$\frac{1}{2}$

$h(x) = \int_x^3 f(t)dt$ when $x = -1$ and $x = 1$ The area above the x-axis = area below

missed $x=3$ trivial case

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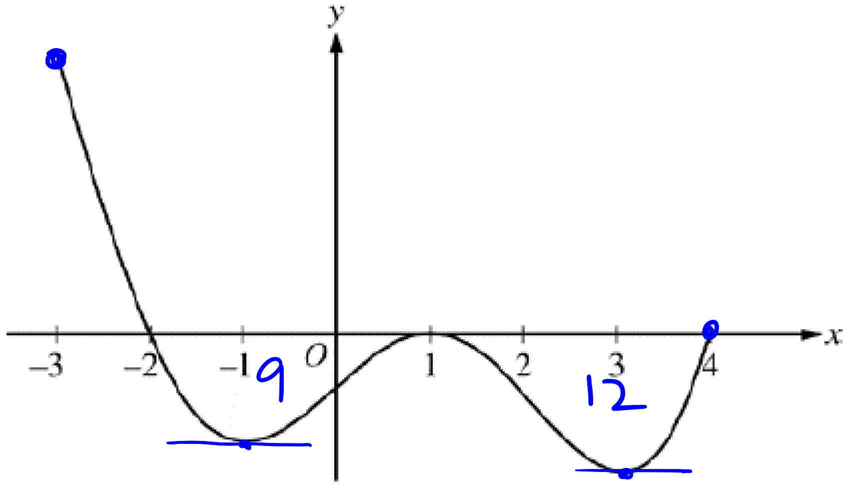

Graph of f'

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval [–3, 4]. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals [–2, 1] and [1, 4] are 9 and 12, respectively.

13. Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

$\frac{3}{3}$

Please respond on separate paper, following directions from your teacher.

$f(x) = \int_1^x f'(t)\,dt + 3$; $f(4) = \int_1^4 f'(t)\,dt + 3 = -12 + 3 = \boxed{-9 = f(4)}$

$f(-2) = \int_1^{-2} f'(t)\,dt$

$= 9 + 3 = \boxed{12 = f(-2)}$

x	2	3	5	8	13
$f(x)$	1	4	–2	3	6

Let f be a function that is twice differentiable for all real numbers. The table above gives values of f for selected points in the closed interval $2 \le x \le 13$.

14.

$\frac{2}{2}$

Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)dx$.

Show the work that leads to your answer.

$\int_2^{13} f(x)dx = 1f(2) + 2f(3) + 3f(5) + 5f(8) = 18$

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Please respond on separate paper, following directions from your teacher.

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

15. Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

$\frac{3}{3}$

Please respond on separate paper, following directions from your teacher.

$\int_0^{30} W'(t)\,dt \approx 10(0.6) + 12(0.7) + 8(1) = 22.4 = W(30) - W(0)$

$\Rightarrow W(0) = 125 - 22.4 = 102.6$ GL

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

16. Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

$\frac{1}{1}$

$A \approx \left(\frac{0+7}{2}\right)8 + \left(\frac{8+7}{2}\right)6 + \left(\frac{8+2}{2}\right)8 + \left(\frac{0+2}{2}\right)2 = 115$ feet^2

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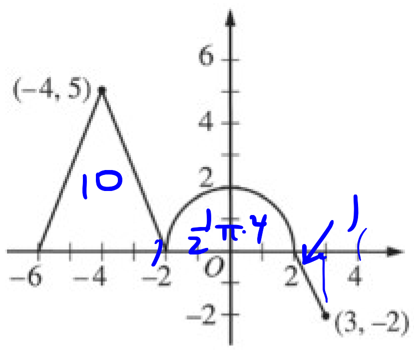

Graph of f

The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t)dt$.

$\frac{2}{2}$

17. Find $g(-6)$ and $g(3)$. $g(-6) = \int_{-2}^{-6} f(t)\,dt = -10$ $g(3) = \int_{-2}^{3} f(t)dt = 2\pi - 1$

Please respond on separate paper, following directions from your teacher.

18. Find $g'(0)$. $g'(x) = f(x) \Rightarrow g'(0) = f(0) = 2$

$\frac{1}{1}$

Please respond on separate paper, following directions from your teacher.

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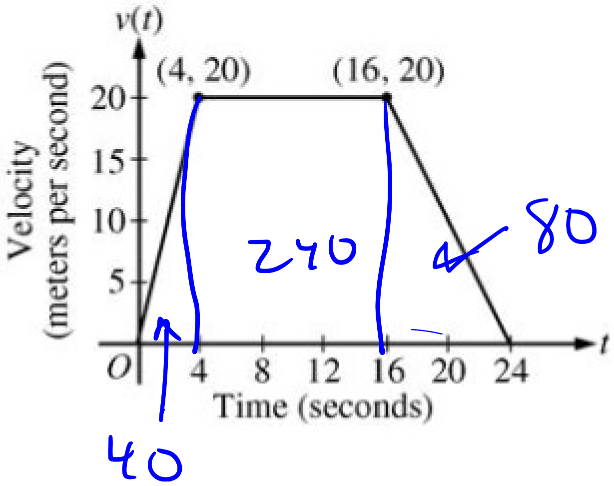

A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity *v(t)*, in meters per second, is modeled by the piecewise-linear function defined by the graph above.

2/2

19. Find $\int_0^{24} v(t)\, dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\, dt$.

Please respond on separate paper, following directions from your teacher.

$\int_0^{24} v(t)dt = 360\text{ m}$

The distance travelled in m between t=0 and t=24 seconds

	4	5	6	5	
t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit) °F	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time t=0, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time t=0. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

3/3

20. For $0 \leq t \leq 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\, dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\, dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$\text{avg } W = \frac{1}{20}\left[4(55) + 5(57.1) + 6(61.8) + 5(67.9)\right] = 60.79°F$

Since its being heated $W'(t) > 0$ and so

This will underestimate the value of $\int_0^{20} W(t)\,dt$

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Please respond on separate paper, following directions from your teacher.

21. Use the data in the table to evaluate $\int_0^{20} W'(t)dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)dt$ in the context of this problem.

°F/min

1/2

$\int_0^{20} W'(t)dt = W(20) - W(0) = 16°F$

Please respond on separate paper, following directions from your teacher.

This is how much the water was $\boxed{\text{heated in 20 minutes}}$

maybe too vague from t=0 to t=20 min

22. For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t=25$?

2/2

Please respond on separate paper, following directions from your teacher.

$\frac{dW}{dt} = 0.4\sqrt{t}\cos(0.06t)$

$W = \int_{20}^{t} 0.4\sqrt{x}\cos(0.06x)\,dx + 71$

needs a graphing calculator

23. Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

2/2

Please respond on separate paper, following directions from your teacher.

$W(25) = 2.043 + 71 = \underline{73.043°F}$

$W'(12) \approx \frac{67.9 - 61.8}{15 - 9} = 1.017$ °F/min

At $t = 12$ min the water is changing by about 1°F per minute.