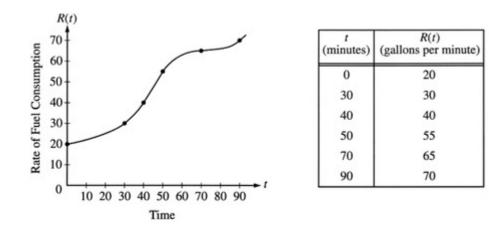
AP Calculus AB

Integration Review: Week 1



The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twicedifferentiable and strictly increasing function *R* of time *t*. The graph of *R* and a table of selected values of R(t), for the time interval $0 \le t \le 90$ minutes, are shown above.

1. Approximate the value of $\int_0^{90} R(t) dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t) dt$? Explain your reasoning.

Please respond on separate paper, following directions from your teacher.

Part C

1 point is earned for the correct value of left Riemann sum

$$\int_{0}^{90} \mathrm{R(t)dt} pprox (30)(20) + (10)(30) + (10)(40) + (20)(55) + (20)(65) = 3700$$

1 point is earned for answer "less" with reason

Yes, this approximation is less because the graph of R is increasing on the interval.

		\checkmark
0	1	2

The student response earns two of the following points:

1 point is earned for the correct value of left Riemann sum



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Integration Review: Week 1

$$\int_{0}^{90} \mathrm{R(t)dt} \approx (30)(20) + (10)(30) + (10)(40) + (20)(55) + (20)(65) = 3700$$

1 point is earned for answer "less" with reason

Yes, this approximation is less because the graph of R is increasing on the interval.

t (hours)	0	0.4	0.8	1.2	1.6	2.0	2.4
v(t) (miles per hour)	0	11.8	9.5	17.2	16.3	16.8	20.1

Ruth rode her bicycle on a straight trail. She recorded her velocity v(t), in miles per hour, for selected values of t over the interval $0 \le t \le 2.4$, as shown in the table above, For $0 < t \le 2.4$, v(t) > 0.

2. Using correct units, interpret the meaning of
$$\int_{0}^{2.4} v(t) dt$$
 in the context of the problem. Approximate $\int_{0}^{2.4} v(t) dt$ using a midpoint Riemann sum with three subintervals of equal length and values from the

table.

Please respond on separate paper, following directions from your teacher.

Part B

The response can earn up to 3 points:

1 point: For the correct interpretation

$$\int_0^{2.4} v(t) dt$$
 is the total distance Ruth, traveled, in miles, from time $t = 0$ to time $t = 2.4$ hours

1 point: For use of the midpoint Riemann sum

1 point: For the correct approximation

$$\int_{0}^{2.4} v(t) dt pprox (0.8)(11.8) + (0.8)(17.2) + (0.8)(16.8) = 36.64 ext{ miles}$$

			\checkmark
0	1	2	3

The response earns all three of the following points:

1 point: For the correct interpretation

 $\int_0^{2.4} v(t) dt$ is the total distance Ruth, traveled, in miles, from time t = 0 to time t = 2.4 hours.

1 point: For use of the midpoint Riemann sum

1 point: For the correct approximation

$$\int_{0}^{2.4} v(t) dt pprox (0.8)(11.8) + (0.8)(17.2) + (0.8)(16.8) = 36.64 ext{ miles}$$

A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where *a*, *b*, and *k* are constants. The function *f* has a local minimum at x = -1, and the graph *f* has a point of inflection at x = -2.

3. If
$$\int_0^1 f(x) dx = 32$$
, what is the value of k?

Please respond on separate paper, following directions from your teacher.

Part B

2 points are earned for antidifferentiation < -1 > each error

1 point is earned for the expression in k

1 point is earned for k

$$\int_{0}^{1} \left(4x^{3} + 24x^{2} + 36x + k \right) dx$$

= $x^{4} + 8x^{3} + 18x^{2} + kx |_{x=0}^{x=1} = 27 + k$
 $27 + k = 32$
 $k = 5$

0		2	
0	2	3	4
•		-	

The student response earns all of the following points:

2 points are earned for antidifferentiation < -1 > each error

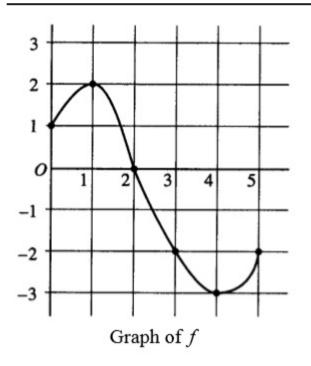
1 point is earned for the expression in k

1 point is earned for k

$$\int_{0}^{1} \left(4x^{3} + 24x^{2} + 36x + k\right) dx$$

= $x^{4} + 8x^{3} + 18x^{2} + kx|_{x=0}^{x=1} = 27 + k$
 $27 + k = 32$
 $k = 5$





Let f be a function whose domain is the closed interval [0,5]. The graph of f is shown above.

Let
$$h(x) = \int_{0}^{\frac{x}{2}+3} f(t)dt$$
.

4. Find h' (2).

Please respond on separate paper, following directions from your teacher.

Part B

2 points are earned for the correct answer for $h'(x) = f(\frac{x}{2} + 3) \cdot \frac{1}{2}$

< -1 > no chain rule

1 point is earned for the correct answer, using f(4)

 $egin{aligned} h'(x) &= f(rac{x}{2}+3) \cdot rac{1}{2} \ h'(2) &= f(4) \cdot rac{1}{2} = -rac{3}{2} \end{aligned}$

			\checkmark
0	1	2	3

The student response earns three of the following points:

2 points are earned for the correct answer for $h'(x) = f(\frac{x}{2} + 3) \cdot \frac{1}{2}$

< -1 > no chain rule

1 point is earned for the correct answer, using f(4)

 $h'(x) = f(rac{x}{2}+3) \cdot rac{1}{2}$ $h'(2) = f(4) \cdot rac{1}{2} = -rac{3}{2}$

5. Find the domain of *h*.

Please respond on separate paper, following directions from your teacher.

Part A

2 points are earned for the correct endpoints, where one point is earned for the first endpoint and one point is earned for the second endpoint

< -1 > uses strict inequality

 $0 \le \frac{x}{2} + 3 \le 5$ $-6 \le x \le 4$

		\checkmark
0	1	2

The student response earns two of the following points:

2 points are earned for the correct endpoints, where one point is earned for the first endpoint and one point is earned for the second endpoint

< -1 > uses strict inequality



$$0 \le rac{x}{2} + 3 \le 5$$

 $-6 \le x \le 4$

6. At what x is h(x) a minimum? Show the analysis that leads to your conclusion.

Please respond on separate paper, following directions from your teacher.

Part C

1 point is earned for correctly identifying the minimum is not in interior of domain

h' is positive, then negative, so minimum is at an endpoint

1 point is earned for correctly stating h(-6) = 0

$$h(-6)=\int_0^0 f(t)dt=0$$

1 point is earned for correctly stating h(4) is negative

$$h(4)=\int_0^5 f(t)dt<0$$

1 point is earned for the correct answer, x = 4

Since the area below the axis is greater than the area above the axis therefore minimum at x = 4

Note: Domain [0,5], max 1/4, 1 - 0 - 0 - 0

No interior critical point in student's domain, max 1/4, 0 - 0 - 0 - 1

				\checkmark
0	1	2	3	4

The student response earns four of the following points:

1 point is earned for correctly identifying the minimum is not in interior of domain

h' is positive, then negative, so minimum is at an endpoint

1 point is earned for correctly stating h(-6) = 0

$$h(-6)=\int_0^0 f(t)dt=0$$

1 point is earned for correctly stating h(4) is negative

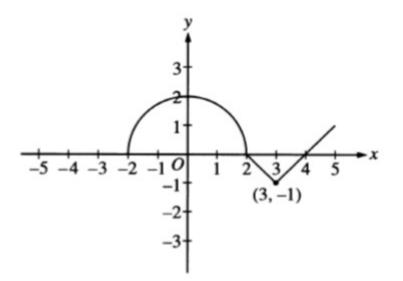
$$h(4)=\int_0^5 f(t)dt<0$$

1 point is earned for the correct answer, x = 4

Since the area below the axis is greater than the area above the axis therefore minimum at x = 4

Note: Domain [0,5], max 1/4, 1 - 0 - 0 - 0

No interior critical point in student's domain, max 1/4, 0 - 0 - 0 - 1



The graph of the function f consists of a semicircle and two line segments as shown above. Let g be the function given by $g(x) = \int_0^x f(t) dt$.

7. Find g(3).

Please respond on separate paper, following directions from your teacher.

Part A

2 points are earned for the correct answer

< -1 > each incorrect area

< -1 > error in summing

$$egin{aligned} g(3) &= \int_0^3 f(t) dt \ &= rac{1}{4} \cdot \pi \cdot 2^2 - rac{1}{2} = \pi - rac{1}{2} \end{aligned}$$

		•
0	1	2

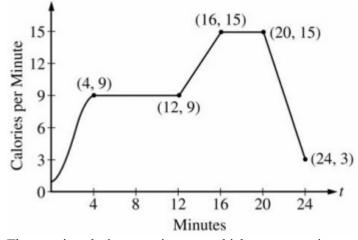
The student response earns two of the following points:

2 points are earned for the correct answer

< -1 > each incorrect area

< -1 > error in summing

$$g(3) = \int_0^3 f(t) dt \ = rac{1}{4} \cdot \pi \cdot 2^2 - rac{1}{2} = \pi - rac{1}{2}$$



The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the



 $function (f). In the figure above, (f(left(t)right)=-(frac {1} {4}t^3+(frac {3} {2}t^2+1)) for (0(le t)le4) and (f) is piecewise linear for (4)le t)le24).$

8. Find the total number of calories burned over the time interval $6 \le t \le 18$ minutes.

Please respond on separate paper, following directions from your teacher.

Part C

1 point is earned for the method

1 point is earned for the answer

$$\int_0^{18} f(t)dt = 6(9) + \frac{1}{2}(4)(9+15) + 2(15) = 132 \text{ calories}$$

		•
0	1	2

The student response earns all of the following points:

1 point is earned for the method

1 point is earned for the answer

$$\int_0^{18} f(t)dt = 6(9) + \frac{1}{2}(4)(9+15) + 2(15) = 132 \text{ calories}$$

Let f be a function such that f''(x)=6x+8.

9. Find f(x) if the graph of f is tangent to the line 3x - y = 2 at the point (0, -2).

Please respond on separate paper, following directions from your teacher.



Part A

1 point is earned for f'(0)=3

1 point is earned for finding C

1 point is earned for using f to find f with constant

1 point is earned for using (0,-2) to find d

 $f'(x) = 3x^{2} + 8x + C$ f'(0) = 3 C = 3 $f(x) = x^{3} + 4x^{2} + 3x + d$ d = -2 $f(x) = x^{3} + 4x^{2} + 3x - 2$

					\checkmark	
0	1	2	3	4	5	

The student response earns five of the following points:

1 point is earned for f'(x) with C

1 point is earned for f'(0)=3

1 point is earned for finding C

1 point is earned for using f to find f with constant

1 point is earned for using (0,-2) to find d

$$f'(x) = 3x^{2} + 8x + C$$

$$f'(0) = 3$$

$$C = 3$$

$$f(x) = x^{3} + 4x^{2} + 3x + d$$

$$d = -2$$

$$f(x) = x^{3} + 4x^{2} + 3x - 2$$

1



The functions f and g are given by
$$f(x) = \int_0^{3x} \sqrt{4 + t^2} dt \, and \, g(x) = f(\sin x)$$

10. Find f(x) and g'(x).

Please respond on separate paper, following directions from your teacher.

Part A

2 points are earned for f(x)

2 points are earned for g'(x)

$$f'(x) = 3\sqrt{4 + (3x)^2}$$

 $g'(x) = f'(\sin x) \cdot \cos x$
 $= 3\sqrt{4 + (3\sin x)^2} \cdot \cos x$

0 1	2	3	4
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The student response earns all of the following points:

2 points are earned for f(x)

2 points are earned for g'(x)

$$f'(x) = 3\sqrt{4 + (3x)^2}$$

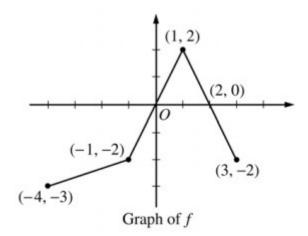
 $g'(x) = f'(\sin x) \cdot \cos x$
 $= 3\sqrt{4 + (3\sin x)^2} \cdot \cos x$



AP OClegeBoard

AP Calculus AB

Integration Review: Week 1



The graph of the function f above consists of three line segments.

11. Let g be the function given by $g(x) = \int_{-4}^{x} f(t) dt$. For each of g(-1), g'(-1), and g''(-1), find the value or state that it does not exist.

Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for g(-1)

1 point is earned for g'(-1)

1 point is earned for g"(-1)

$$g(-1) = \int_{-4}^{-1} f(t)dt = -rac{1}{2}(3)(5) = -rac{15}{2}g'(-1) = f(-1) = -2$$

g''(-1) does not exist because f is not differentiable at x=-1.

			\checkmark
0	1	2	3

The student response earns all of the following points:

1 point is earned for g(-1)



1 point is earned for g'(-1)

1 point is earned for g"(-1)

$$g(-1) = \int_{-4}^{-1} f(t)dt = -rac{1}{2}(3)(5) = -rac{15}{2}$$

g'(-1)=f(-1)=-2

- g''(-1) does not exist because f is not differentiable at x=-1.
- 12. Let *h* be the function given by $h(x) = \int_x^3 f(t) dt$. Find all values of *x* in the closed interval $-4 \le x \le 3$ for which h(x)=0.

Please respond on separate paper, following directions from your teacher.

Part C

2 point is earned for the correct values

(-1) each missing or extra value

x=-1,1,3

0	1	2
Ŭ	_	_

The student response earns all of the following points:

2 point is earned for the correct values

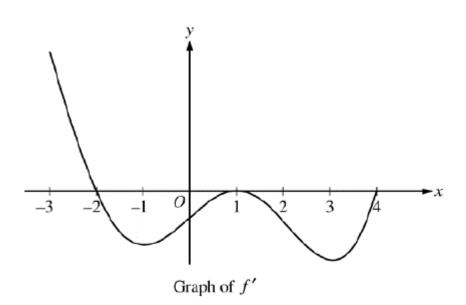
(-1) each missing or extra value

x=-1,1,3



AP Calculus AB

Integration Review: Week 1



The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval [-3, 4]. The graph of f' has horizontal tangents at x = -1, x = 1, and x = 3. The areas of the regions bounded by the *x*-axis and the graph of f' on the intervals [-2, 1] and [1, 4] are 9 and 12, respectively.

13. Given that f(1) = 3, write an expression for f(x) that involves an integral. Find f(4) and f(-2).

Please respond on separate paper, following directions from your teacher.

Part D

1 point is earned for the integrand

1 point is earned for expressing f(x)

1 point is earned for f(4) and f(-2)

$$f(x) = 3 + \int\limits_{1}^{x} f'(t) dt$$

$$f(4) = 3 + \int_{1}^{4} f'(t)dt = 3 + (-12) = -9$$



$$f(-2) = 3 + \int_{1}^{-2} f(t)dt = 3 - \int_{-2}^{1} f(t)dt$$

= 3-(-9) = 12

0	1	2	2
0	1	2	3

The student response earns all of the following points:

1 point is earned for the integrand

1 point is earned for expressing f(x)

1 point is earned for f(4) and f(-2)

$$f(x) = 3 + \int\limits_{1}^{x} f'(t)dt$$

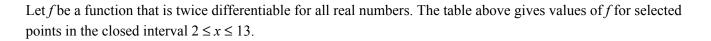
A

$$f(4)=3+\int\limits_{1}^{4}f'(t)dt=3+(-12)=-9$$

$$f(-2) = 3 + \int_{1}^{-2} f(t)dt = 3 - \int_{-2}^{1} f(t)dt$$

$$= 3 - (-9) = 12$$

x	2	3	5	8	13
f(x)	1	4	-2	3	6





13

Integration Review: Week 1

14.

Use a left Riemann sum with subintervals indicated by the data in the table to approximate $\int f(x)dx$.

Show the work that leads to your answer.

Please respond on separate paper, following directions from your teacher.

Part C

1 point is earned for left Riemann sum

1 point is earned for the answer

$$\int\limits_{2}^{13} f(x) dx pprox f(2)(3-2) + f(3)(5-3) + f(5)(8-5) + f(8)(13-8) = 18$$



The student response earns two of the following points:

1 point is earned for left Riemann sum

1 point is earned for the answer

$$\int\limits_{2}^{13} f(x) dx pprox f(2)(3-2) + f(3)(5-3) + f(5)(8-5) + f(8)(13-8) = 18$$

t (days)	0	10	22	30
W'(t) (GL per day)	0.6	0.7	1.0	0.5



The twice-differentiable function W models the volume of water in a reservoir at time t, where W(t) is measured in gigaliters (GL) and t is measured in days. The table above gives values of W'(t) sampled at various times during the time interval $0 \le t \le 30$ days. At time t = 30, the reservoir contains 125 gigaliters of water.

15. Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_{0}^{30} W'(t) dt$. Use this approximation to estimate the volume of water W(t), in gigaliters, in the reservoir at time t = 0. Show the computations that lead to your answer.

Please respond on separate paper, following directions from your teacher.

Part B

One point is earned for the left Riemann sum

One point is earned for the approximation

One point is earned for the answer

$$\int_{0}^{30} W'(t) dt pprox (10)(0.6) + (12)(0.7) + (8)(1.0) = 22.4$$

$$W(0)=W(30)-\int_{0}^{30}W'(t)dt=125-22.4=102.6$$

			\checkmark
0	1	2	3

The student earns all of the following points:

One point is earned for the left Riemann sum

One point is earned for the approximation

One point is earned for the answer

$$\int_{0}^{30} W'(t) dt pprox (10)(0.6) + (12)(0.7) + (8)(1.0) = 22.4$$
 $W(0) = W(30) - \int_{0}^{30} W'(t) dt = 125 - 22.4 = 102.6$

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2 \sin(\sqrt{t+10})$ for $0 \le t \le 120$ minutes.

16. Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

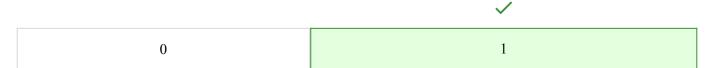
Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for trapezoidal approximation

$$\frac{(0+7)}{2} \cdot 8 + \frac{(7+8)}{2} \cdot 6 + \frac{(8+2)}{2} \cdot 8 + \frac{(2+0)}{2} \cdot 2$$

=115 ft²



The student response earns one of the following points:

1 point is earned for trapezoidal approximation

$$\frac{(0+7)}{2} \cdot 8 + \frac{(7+8)}{2} \cdot 6 + \frac{(8+2)}{2} \cdot 8 + \frac{(2+0)}{2} \cdot 2$$

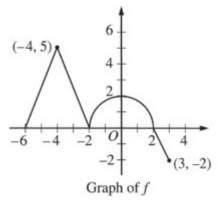
=115 ft²





AP Calculus AB

Integration Review: Week 1



The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by $g(x) = \int_{-2}^{x} f(t) dt$.

17. Find *g*(-6) and *g*(3).

Please respond on separate paper, following directions from your teacher.

Part A

One point is earned for g(-6)

One point is earned for g(3)

$$g(-6) = \int_{-2}^{-6} f(t)dt = -\int_{-6}^{-2} f(t)dt = -\frac{1}{2}.4.5 = -10$$
$$g(3) = \int_{-2}^{3} f(t)dt = \frac{1}{2}\pi.2^{2} - \frac{1}{2}.1.2 = 2\pi - 1$$

		\checkmark
0	1	2

The student earns all of the following points:

One point is earned for g(-6)



One point is earned for g(3)

$$g(-6) = \int_{-2}^{-6} f(t)dt = -\int_{-6}^{-2} f(t)dt = -\frac{1}{2}.4.5 = -10$$
$$g(3) = \int_{-2}^{3} f(t) = \frac{1}{2}\pi.2^{2} - \frac{1}{2}.1.2 = 2\pi - 1$$

18. Find g'(0).

Please respond on separate paper, following directions from your teacher.

Part B

One point is earned for g'(0)

g'(0) = f(0) = 2

	\checkmark
0	1

The student earns all of the following points:

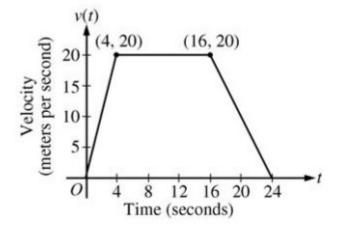
One point is earned for g'(0)

g'(0) = f(0) = 2

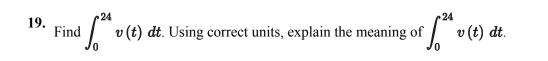


AP Calculus AB

Integration Review: Week 1



A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity v(t), in meters per second, is modeled by the piecewise-linear function defined by the graph above.



Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for the value

1 point is earned for the meaning with units

$$\int_{0}^{24} v(t) dt = \frac{1}{2}(4) (20) + (12) (20) + \frac{1}{2}(8) (20) = 360$$

The car travels 360 meters in these 24 seconds.

		\checkmark
0	1	2

The student response earns all of the following points:

1 point is earned for the value



1 point is earned for the meaning with units

$$\int_{0}^{24} v(t) dt = \frac{1}{2}(4) (20) + (12) (20) + \frac{1}{2}(8) (20) = 360$$

The car travels 360 meters in these 24 seconds.

t (minutes)	0	4	9	15	20
W(t) (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time *t* is modeled by a strictly increasing, twice-differentiable function *W*, where W(t) is measured in degrees Fahrenheit and *t* is measured in minutes. At time *t*=0, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time *t*=0. Values of W(t) at selected times *t* for the first 20 minutes are given in the table above.

20. For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20} \int_0^{20} W(t) dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20} \int_0^{20} W(t) dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

Please respond on separate paper, following directions from your teacher.

Part C

1 point is earned for the left Riemann sum

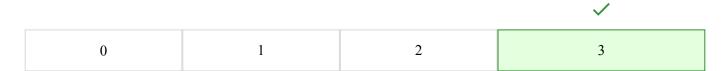
1 point is earned for the approximation

1 point is earned for underestimate with reason

$$\frac{1}{20} \int_{0}^{20} W(t) dt \approx \frac{1}{20} (4.W(0) + 5.W(4) + 6.W(9) + 5.W(15))$$
$$\frac{1}{20} (4.55.0 + 5.57.1 + 6.61.8 + 5.67.9)$$
$$\frac{1}{20} .1215.8 = 60.79$$



This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.



The student response earns all of the following points:

1 point is earned for the left Riemann sum

1 point is earned for the approximation

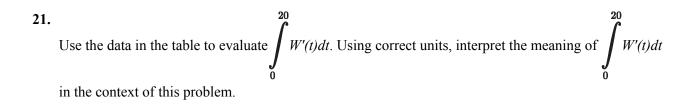
1 point is earned for underestimate with reason

$$rac{1}{20}\int\limits_{0}^{20}W(t)dtpproxrac{1}{20}(4.W(0)+5.W(4)+6.W(9)+5.W(15))$$

 $\frac{1}{20}(4.55.0 + 5.57.1 + 6.61.8 + 5.67.9)$

$$\frac{1}{20}.1215.8 = 60.79$$

This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.



Please respond on separate paper, following directions from your teacher.

Part B

1 point is earned for the value

1 point is earned for the interpretation with units

$$\int\limits_{0}^{20}W'(t)dt=W(20)-W(0)=71.0-55.0=16$$

The water has warmed by 16 $^{\circ}$ F over the interval from *t*=0 to *t*=20 minutes.

		\checkmark
0	1	2

The student response earns all of the following points:

1 point is earned for the value

1 point is earned for the interpretation with units

$$\int\limits_{0}^{20}W'(t)dt=W(20)-W(0)=71.0-55.0=16$$

The water has warmed by 16 $^{\circ}$ F over the interval from *t*=0 to *t*=20 minutes.

22. For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t} \cos(0.06t)$. Based on the model, what is the temperature of the water at time t=25?

Please respond on separate paper, following directions from your teacher.

Part D

1 point is earned for the integrand

1 point is earned for the answer

$$W(t)=71.0+\int_{20}^{25}W'(t)dt$$

=71.0+2.043155 = 73.043



		·
0	1	2

The student response earns all of the following points:

1 point is earned for the integrand

1 point is earned for the answer

$$W(t) = 71.0 + \int_{20}^{25} W'(t) dt$$

=71.0+2.043155 = 73.043

23. Use the data in the table to estimate W'(12). Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for the estimate

1 point is earned for the interpretation with units

$$W'(12) pprox rac{W(15) - W(9)}{15 - 9} = rac{67.9 - 61.8}{6}$$

=1.017 (or 1.016)

The water temperature is increasing at a rate of approximately 1.017° F per minute at time

t = 12 minutes.

		\checkmark	
0	1	2	

The student response earns all of the following points:

1 point is earned for the estimate

1 point is earned for the interpretation with units

$$W'(12) pprox rac{W(15) - W(9)}{15 - 9} = rac{67.9 - 61.8}{6}$$

=1.017 (or 1.016)

The water temperature is increasing at a rate of approximately 1.017 oF per minute at time t = 12 minutes.