Differential Equation Review

Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$

1. Find the values of the constants m and b, for which $y=mx+b$ is a solution to the differential equation.

Please respond on separate paper, following directions from your teacher.

Part D

1 point is earned for the value of m

1 point is earned for the value of b

Substituting $y=mx+b$ into the differential equation: $m = \frac{1}{2}x + (mx+b) - 1 = \left(m + \frac{1}{2}\right)x + (b-1)$
Then $0 = m + \frac{1}{2}$ and $m = b - 1 : m = -\frac{1}{2}$ and $b = \frac{1}{2}$

0	1	2

The student response earns all of the following points:

1 point is earned for the value of m

1 point is earned for the value of b

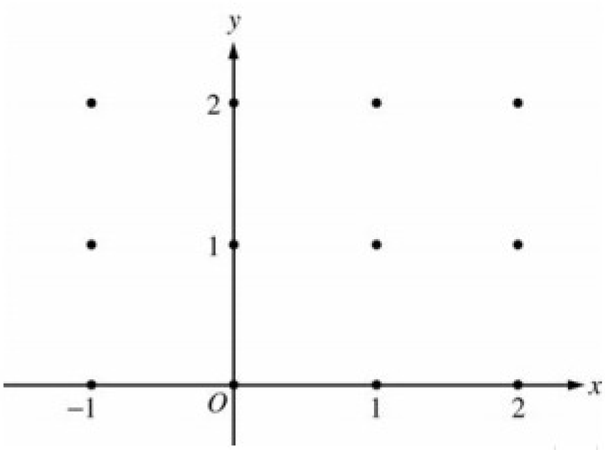
Substituting $y=mx+b$ into the differential equation:

$$m = \frac{1}{2}x + (mx+b) - 1 = \left(m + \frac{1}{2}\right)x + (b-1)$$
$$\text{Then } 0 = m + \frac{1}{2} \text{ and } m = b - 1 : m = -\frac{1}{2} \text{ and } b = \frac{1}{2}$$

Consider the differential equation $dy/dx=-xy^2/2$. Let $y=f(x)$ be the particular solution to this differential equation with the initial condition $f(-1)=2$.

Differential Equation Review

2. On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

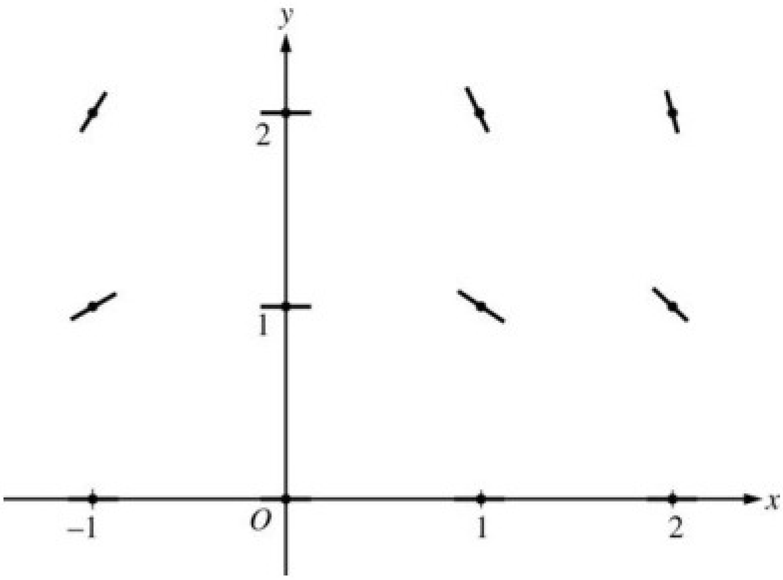


Please respond on separate paper, following directions from your teacher.

Part A

1 point is earned for zero slopes

1 point is earned for nonzero slopes



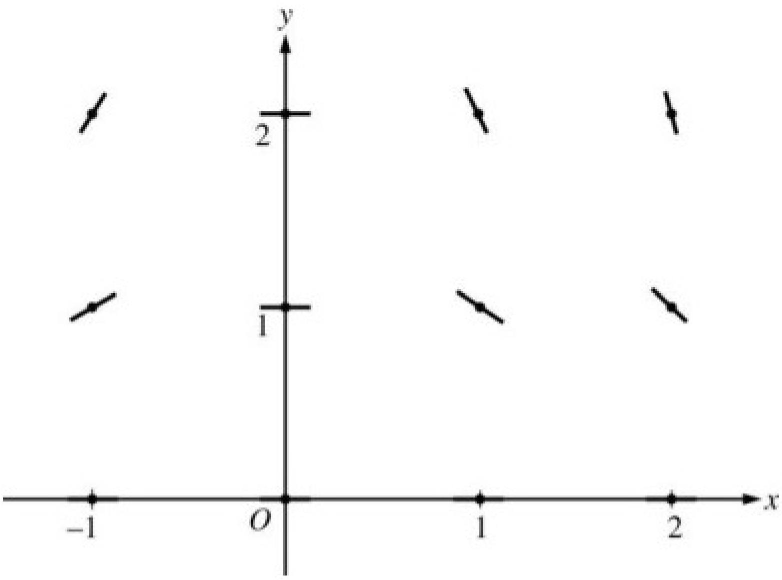
Differential Equation Review

0	1	2

The student response earns all of the following points:

1 point is earned for zero slopes

1 point is earned for nonzero slopes

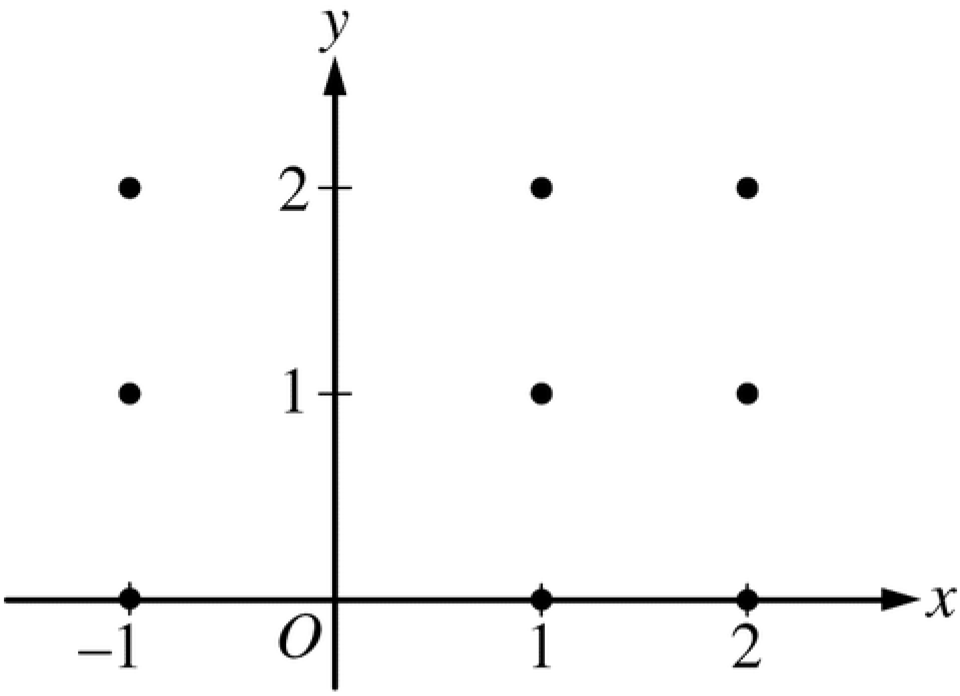


The following are related to this scenario:

Consider the differential equation $\frac{dy}{dx} = \left(1 - \frac{2}{x^2}\right)(y-1)$,where x≠0 . Let *y=f(x)* be the particular solution to the differential equation with initial condition *f*(1)=2 .

3. On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

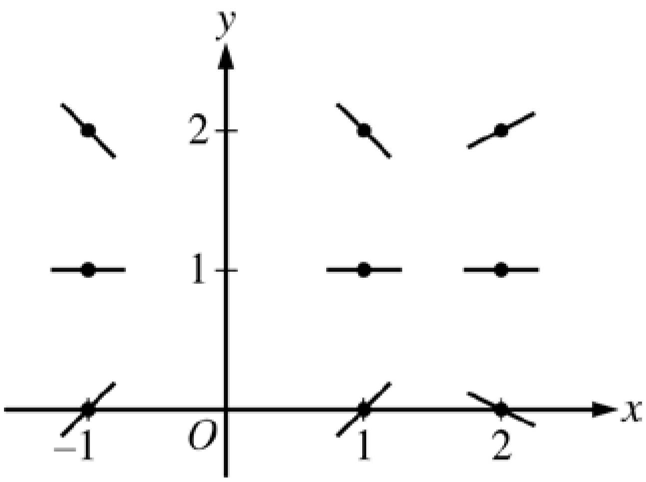
Differential Equation Review



Please respond on separate paper, following directions from your teacher.

Part B

The response can earn up to 2 points:



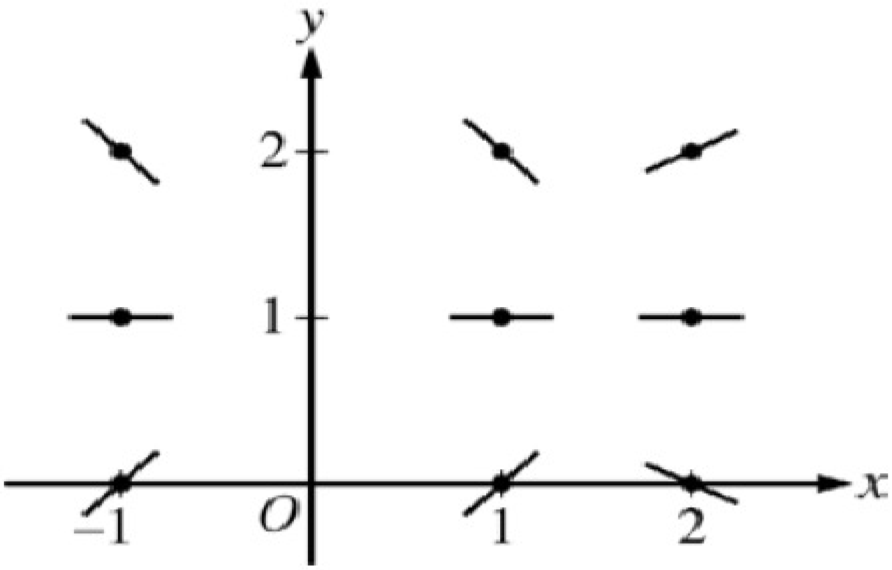
1 point: For zero slope at each point (x, y) where $y = 1$

1 point: For the remaining slopes

Differential Equation Review

0	1	2

The response earns both of the following points:



1 point: For zero slope at each point (x, y) where $y = 1$

1 point: For the remaining slopes

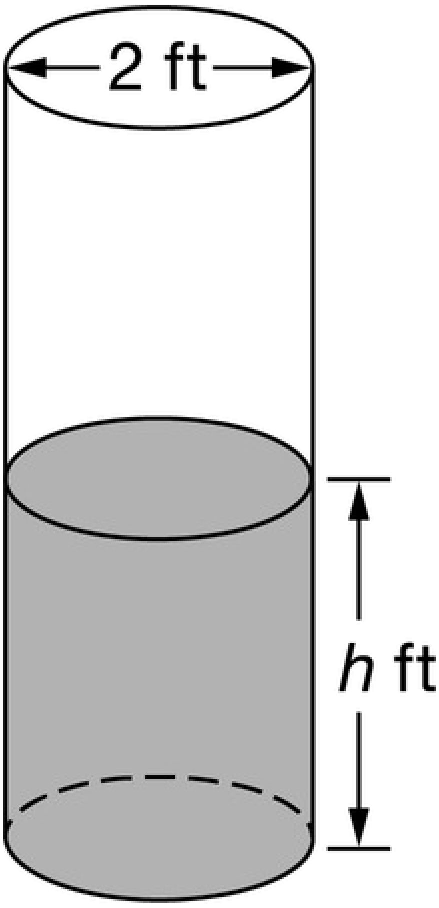
4. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Differential Equation Review



A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.

Please respond on separate paper, following directions from your teacher.

b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.

Differential Equation Review

Please respond on separate paper, following directions from your teacher.

c) At time $t = 0$ seconds, the height of the water is 5 feet. Use separation of variables to find an expression for h in terms of t.

Please respond on separate paper, following directions from your teacher.

Part A

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

The first point is earned for the derivative of the volume equation with respect to time. Responses that differentiate $V = \pi r^2 h$ correctly using the product rule also earn the first point.

0	1	2

The student response accurately includes **both** of the criteria below.

- ☐ $\frac{dV}{dt} = \pi \frac{dh}{dt}$
- ☐ answer with units

Solution:

$V = \pi r^2 h = \pi (1)^2 h = \pi h$

$\frac{dV}{dt}\Big|_{h=4} = \pi \frac{dh}{dt}\Big|_{h=4} = \pi \left(-\frac{1}{10}\sqrt{4}\right) = -\frac{\pi}{5}$ cubic feet per second

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

The first point is earned for the derivative of $-\frac{1}{10}\sqrt{h}$.

The second point is earned for $\frac{d^2h}{dt^2}$. Note that a response of $\frac{d^2h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt}$ earns both the first and second points.

0	1	2	3

The student response accurately includes **all three** of the criteria below.

- ☐ $\frac{d}{dh}\left(-\frac{1}{10}\sqrt{h}\right) = -\frac{1}{20\sqrt{h}}$
- ☐ $\frac{d^2h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt}$
- ☐ answer with explanation

Solution:

$$\frac{d^2h}{dt^2} = -\frac{1}{20\sqrt{h}} \cdot \frac{dh}{dt} = -\frac{1}{20\sqrt{h}} \cdot \left(-\frac{1}{10}\sqrt{h}\right) = \frac{1}{200}$$

Because $\frac{d^2h}{dt^2} = \frac{1}{200} > 0$ for $h > 0$, the rate of change of the height is increasing when the height of the water is **3** feet.

Part C

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

Zero out of 4 points are earned if no separation of variables.

At most 2 out of 4 points can be earned [1-1-0-0] if there is no constant of integration.

Both antiderivatives must be correct to earn the second point.

In order to earn the third point, at least one of the antiderivatives must be correct, and the h must be in a function that is not logarithmic or linear. In addition, the constant of integration must be included immediately following the antiderivatives.

The fourth point requires an expression for $h\ (t)$.

0	1	2	3	4

Differential Equation Review

The student response accurately includes **all four** of the criteria below.

- ☐ separation of variables
- ☐ antiderivatives
- ☐ constant of integration and uses initial condition
- ☐ $h(t)$

Solution:

$$\frac{dh}{\sqrt{h}} = -\frac{1}{10}\,dt$$
$$\int \frac{dh}{\sqrt{h}} = \int -\frac{1}{10}\,dt$$
$$2\sqrt{h} = -\frac{1}{10}t + C$$
$$2\sqrt{5} = -\frac{1}{10}\cdot 0 + C \Rightarrow C = 2\sqrt{5}$$
$$2\sqrt{h} = -\frac{1}{10}t + 2\sqrt{5}$$
$$h(t) = \left(-\frac{1}{20}t + \sqrt{5}\right)^2$$

A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1}x = \arctan x$)

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5. Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

Please respond on separate paper, following directions from your teacher.

Part D

1 point is earned for: $\int_0^2 v(t)\,dt$

1 point is earned for the handles initial condition

1 point is earned for the value of y(2)

1 point is earned for the answer with reason

$$y(2) = \int_0^2 v(t)\,dt = -1.360 or -1.361$$

The particle is moving away from the origin since $v(2) < 0$ and $y(2) < 0$.

0	1	2	3	4

The student response earns all of the following points:

1 point is earned for: $\int_0^2 v(t)\,dt$

1 point is earned for the handles initial condition

1 point is earned for the value of y(2)

1 point is earned for the answer with reason

$$y(2) = \int_0^2 v(t)\,dt = -1.360 or -1.361$$

The particle is moving away from the origin since $v(2) < 0$ and $y(2) < 0$.

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt}=\frac{1}{5}\left(100-B\right).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

6. Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

Please respond on separate paper, following directions from your teacher.

Differential Equation Review

Part C

1 point is earned for the separation of variables

1 point is earned for the antiderivatives

1 point is earned for the constant of integration

1 point is earned for using initial condition

1 point is earned for solving for B

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

$\frac{db}{dt} = \frac{1}{5}(100 - B)$

$$\int \frac{1}{100 - B} db = \int \frac{1}{5} dt$$

$-\ln|100-B| = \frac{1}{5} t + C$

Because $20 \leq B < 100$, $|100-B| = 100-B$.

$-\ln(100-20) = \frac{1}{5}(0) + C \Rightarrow -\ln(80) = C$

$100-B = 80\, e^{-t/5}$

$B(t) = 100 - 80e^{-t/5}, t \geq 0$

0	1	2	3	4	5

The student response earns all of the following points:

1 point is earned for the separation of variables

1 point is earned for the antiderivatives

1 point is earned for the constant of integration

1 point is earned for using initial condition

1 point is earned for solving for B

Differential Equation Review

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

$\frac{db}{dt} = \frac{1}{5}(100 - B)$

$$\int \frac{1}{100 - B} db = \int \frac{1}{5} dt$$

$-\ln|100-B| = \frac{1}{5}t + C$

Because $20 \le B < 100$, $|100-B| = 100-B$.

$-\ln(100-20) = \frac{1}{5}(0) + C$

$B(t) = 100 - 80e^{-t/5}$

$B(t) = 100 - 80e^{-t/5}\ t \ge 0$

Let f and g be functions that are differentiable for all real numbers x and that have the following properties.

i. $f'(x) = f(x) - g(x)$
ii. $g'(x) = g(x) - f(x)$
iii. $f(0) = 5$
iv. $g(0) = 1$

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7. Find $f(x)$ and $g(x)$. Show your work.

Please respond on separate paper, following directions from your teacher.

Part B

1 point is earned for the correct differential equation for one function

1 point is earned for correctly separating variables

1 point is earned for the correct antidifferentiation

1 point is earned for correctly evaluating constant

Differential Equation Review

1 point is earned for correctly finding g

1 point is earned for correctly finding f

Note: Max 1/6 if not working with DE for one function

$f(x) = 6 - g(x)$ so
$g'(x) = g(x) - 6 + g(x) = 2g(x) - 6$
$\frac{dy}{dx} = 2y - 6; \frac{dy}{2y-6} = dx$
$\frac{1}{2}\ln\left|2y - 6\right| = x + C$
$\ln|2y - 6| = 2x + K$
$\left|2y - 6\right| = e^{2x+K}$
$2y - 6 = Ae^{2x}$
$x = 0 = y = 1$ so $-4 = A$
$2y = -4e^{2x} + 6$
$y = 3 - 2e^{2x} = g(x)$
$f(x) = 6 - g(x) = 3 + 2e^{2x}$

OR

1 point is earned for the correct equation in f - g

1 point is earned for the correct solution with constant

1 point is earned for correctly evaluating constant

1 point is earned for correctly using $f(x) + g(x) = 6$

1 point is earned for correctly finding f

1 point is earned for correctly finding g

Differential Equation Review

$f(x) = 6 - g(x)$ so
$g'(x) = g(x) - 6 + g(x) = 2g(x) - 6$
$g'(x) - 2g(x) = -6$
$\frac{d}{dx}[g(x)e^{-2x}] = -6e^{-2x}$
$g(x)e^{-2x} = \int -6e^{-2x}\,dx$
$= 3e^{-2x} + C$
$g(x) = 3 + Ce^{2x}$
$1 = g(0) = 3 + C;\ C = -2$
$g(x) = 3 - 2e^{2x}$
$f(x) = 6 - g(x) = 3 + 2e^{2x}$

OR

Student attempts solution with infinite series

1 point is earned for the correct constant terms in *f* and *g*

1 point is earned for the correct linear terms in *f* and *g*

1 point is earned for the correct quadratic term in *f*

1 point is earned for the correct quadratic term in *g*

1 point is earned for the correct general term for *f* or *g*

1 point is earned for the correct series sum to 6

$f' - g' = 2f - 2g = 2(f - g)$, so
$f - g = Ae^{2x}$
$f(0) - g(0) = 4 = A$
$f(x) - g(x) = 4e^{2x}$
$f(x) + g(x) = 6$
$2\,f(x) = 6 + 4e^{2x}$
$f(x) = 3 + 2e^{2x}$
$g(x) = 6 - f(x) = 3 - 2e^{2x}$

0	1	2	3	4	5	6 ✓

The student response earns six of the following points:

Differential Equation Review

1 point is earned for the correct differential equation for one function

1 point is earned for correctly separating variables

1 point is earned for the correct antidifferentiation

1 point is earned for correctly evaluating constant

1 point is earned for correctly finding *g*

1 point is earned for correctly finding *f*

Note: Max 1/6 if not working with DE for one function

$f(x) = 6 - g(x)$ so
$g'(x) = g(x) - 6 + g(x) = 2g(x) - 6$
$\frac{dy}{dx} = 2y - 6; \frac{dy}{2y-6} = dx$
$\frac{1}{2}\ln\left|2y - 6\right| = x + C$
$\ln|2y - 6| = 2x + K$
$\left|2y - 6\right| = e^{2x+K}$
$2y - 6 = Ae^{2x}$
$x = 0 = y = 1$ so $-4 = A$
$2y = -4e^{2x} + 6$
$y = 3 - 2e^{2x} = g(x)$
$f(x) = 6 - g(x) = 3 + 2e^{2x}$

OR

1 point is earned for the correct equation in *f* - *g*

1 point is earned for the correct solution with constant

1 point is earned for correctly evaluating constant

1 point is earned for correctly using $f(x) + g(x) = 6$

1 point is earned for correctly finding *f*

1 point is earned for correctly finding *g*

Differential Equation Review

$f(x) = 6 - g(x)$ so
$g'(x) = g(x) - 6 + g(x) = 2g(x) - 6$
$g'(x) - 2g(x) = -6$
$\frac{d}{dx}[g(x)e^{-2x}] = -6e^{-2x}$
$g(x)e^{-2x} = \int -6e^{-2x}\,dx$
$= 3e^{-2x} + C$
$g(x) = 3 + Ce^{2x}$
$1 = g(0) = 3 + C;\ C = -2$
$g(x) = 3 - 2e^{2x}$
$f(x) = 6 - g(x) = 3 + 2e^{2x}$

OR

Student attempts solution with infinite series

1 point is earned for the correct constant terms in *f* and *g*

1 point is earned for the correct linear terms in *f* and *g*

1 point is earned for the correct quadratic term in *f*

1 point is earned for the correct quadratic term in *g*

1 point is earned for the correct general term for *f* or *g*

1 point is earned for the correct series sum to 6

$f' - g' = 2f - 2g = 2(f - g)$, so
$f - g = Ae^{2x}$
$f(0) - g(0) = 4 = A$
$f(x) - g(x) = 4e^{2x}$
$f(x) + g(x) = 6$
$2\,f(x) = 6 + 4e^{2x}$
$f(x) = 3 + 2e^{2x}$
$g(x) = 6 - f(x) = 3 - 2e^{2x}$

Consider the differential equation $\frac{dy}{dx} = x^4\,(y - 2)$.

8. Find the particular solution $y=f(x)$ to the given differential equation with the initial condition $f(0)=0$.

Please respond on separate paper, following directions from your teacher.

Part C

1 point is earned if separates variables

2 points are earned for the antiderivatives

1 point is earned for constant of integration

1 point is earned if uses initial condition

1 point is earned is solves for y, 0/1 if y is not exponential

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

Solution:

$$\frac{1}{y-2}dy = x^4 dx$$

$$\ln|y-2| = \frac{1}{5}x^5 + C$$

$$|y-2| = e^C e^{\frac{1}{5}x^5}$$

$$y-2 = Ke^{\frac{1}{5}x^5}, K = \pm e^C$$

$$-2 = Ke^0 = K$$

$$y = 2 - 2e^{\frac{1}{5}x^5}$$

0	1	2	3	4	5	6

The student response earns all of the following points:

- ☐ 1 point is earned if separates variables
- ☐ 2 points are earned for the antiderivatives
- ☐ 1 point is earned for constant of integration
- ☐ 1 point is earned if uses initial condition
- ☐ 1 point is earned is solves for y, 0/1 if *y* is not exponential

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

Solution:

$$\frac{1}{y-2}dy = x^4 dx$$

$$\ln|y-2| = \frac{1}{5}x^5 + C$$

$$|y-2| = e^C e^{\frac{1}{5}x^5}$$

$$y-2 = Ke^{\frac{1}{5}x^5}, K = \pm e^C$$

$$-2 = Ke^0 = K$$

$$y = 2 - 2e^{\frac{1}{5}x^5}$$

Consider the differential equation dy/ dx $=(y-1)^2\cos(\pi x)$.

9. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

Please respond on separate paper, following directions from your teacher.

Part C

1 point is earned for separating the variables.

2 points are earned for antiderivatives

1 point is earned for constant of integration

1 point is earned if uses initial condition

1 point is earned for answer

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note:0/6 if no separation of variables

$$\frac{1}{(y-1)^2}dy = \cos(\pi x)dx$$
$$-(y-1)^{-1} = \frac{1}{\pi}\sin(\pi x) + C$$
$$\frac{1}{1-y} = \frac{1}{\pi}\sin(\pi x) + C$$
$$1 = \frac{1}{\pi}\sin(\pi) + C = C$$
$$\frac{1}{1-y} = \frac{1}{\pi}\sin(\pi x) + 1$$
$$\frac{\pi}{1-y} = \sin(\pi x) + \pi$$
$$y = 1 - \frac{\pi}{\sin(\pi x)+\pi}\ for\ -\infty < x < \infty$$

0	1	2	3	4	5	6

The student response earns all of the following points:

1 point is earned for separating the variables.

2 points are earned for antiderivatives

1 point is earned for constant of integration

1 point is earned if uses initial condition

1 point is earned for answer

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note:0/6 if no separation of variables

Differential Equation Review

$$\frac{1}{(y-1)^2}dy = \cos(\pi x)dx$$
$$-(y-1)^{-1} = \frac{1}{\pi}\sin(\pi x) + C$$
$$\frac{1}{1-y} = \frac{1}{\pi}\sin(\pi x) + C$$
$$1 = \frac{1}{\pi}\sin(\pi) + C = C$$
$$\frac{1}{1-y} = \frac{1}{\pi}\sin(\pi x) + 1$$
$$\frac{\pi}{1-y} = \sin(\pi x) + \pi$$
$$y = 1 - \frac{\pi}{\sin(\pi x)+\pi}\ for\ -\infty < x < \infty$$

Let f be the function that is defined for all real numbers x and that has the following properties.

(i) $f''(x)=24x-18$

(ii) $f'(1)=-6$

(iii) $f(2)=0$

---

10. Write an expression for $f(x)$.

Please respond on separate paper, following directions from your teacher.

Part B

1 point is earned for antiderivative with or without C

1 point is earned for equating student's f(2) and zero

1 point is earned for solving the resulting equation

$$f(x) = 4x^3 - 9x^2 + C$$
$$f(2) = 0 = 32 - 36 + C$$
$$\therefore C = 4$$
$$\boxed{f(x) = 4x^3 - 9x^2 + 4}$$

0	1	2	3

The student response earns three of the following points:

1 point is earned for antiderivative with or without C

1 point is earned for equating student's f(2) and zero

1 point is earned for solving the resulting equation

$$f(x) = 4x^3 - 9x^2 + C$$
$$f(2) = 0 = 32 - 36 + C$$
$$\therefore C = 4$$
$$\boxed{f(x) = 4x^3 - 9x^2 + 4}$$

A particle moves along the x-axis in such a way that its acceleration at time t for $t \geq 0$ is given by $a(t)=4\cos(2t)$. At time $t = 0$, the velocity of the particle is $v(0)=1$ and its position is $x(0)=0$.

11. Write an equation for the position $x(t)$ of the particle.

Please respond on separate paper, following directions from your teacher.

Part B

1 point is earned for integrand

1 point is earned for $x(t) = -\cos 2t + t + C$or $x(t) = -\cos 2t + t$

1 point is earned for answer

Differential Equation Review

$$x(t) = \int 2\sin 2t + 1dt$$
$$x(t) = -\cos 2t + t + C$$
$$x(0) = 0 \Rightarrow C = 1$$
$$x(t) = -\cos 2t + t + 1$$

0	1	2	3

The student response earns three of the following points:

1 point is earned for integrand

1 point is earned for $x(t) = -\cos 2t + t + C$ or $x(t) = -\cos 2t + t$

1 point is earned for answer

$$x(t) = \int 2\sin 2t + 1dt$$
$$x(t) = -\cos 2t + t + C$$
$$x(0) = 0 \Rightarrow C = 1$$
$$x(t) = -\cos 2t + t + 1$$

The number of gallons, *P(t)*, of a pollutant in a lake changes at the rte *P′(t)* = $1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where *t* is measured in days. There are 50 gallons of the pollutant in the lake at time *t* = 0. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

12. Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

Please respond on separate paper, following directions from your teacher.

Part C

1 point is earned for the integrand

1 point is earned for the limits

Differential Equation Review

1 point is earned for the conclusion with reason based on integral of *P′(t)*

$$P(30.174) = 50 + \int_0^{30.174} \left(1 - 3e^{-0.2\sqrt{t}}\right) dt$$

= 35.104 < 40, so the lake is safe.

0	1	2	3

The student response earns all of the following points:

1 point is earned for the integrand

1 point is earned for the limits

1 point is earned for the conclusion with reason based on integral of *P′(t)*

$$P(30.174) = 50 + \int_0^{30.174} \left(1 - 3e^{-0.2\sqrt{t}}\right) dt$$

= 35.104 < 40, so the lake is safe.

For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time *t* days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time *t*=0.

13. To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

Please respond on separate paper, following directions from your teacher.

Part D

1 point is earned for the integral

1 point is earned for the answer

Differential Equation Review

1 point is earned for computing interior critical points

1 point is earned for completing analysis

$R(t)$=0 when t=0, t=2.5π, or t=7.5$\pi$$R(t)$>0 on 0<$t<2.5\pi$$R(t)$<0 on 2.5$\pi$<$t<7.5\pi$$R(t)$>0 on 7.5$\pi$<$t$<31The absolute maximum number of mosquitoes occurs at t=2.5π or at t=31.

$1000+\int_0^{2.5\pi} R(t)dt = 1039.357$There are 964 mosquitoes at t=31, so the maximum number of mosquitoes is 1039, to the nearest whole number.

0	1	2	3	4

The student response earns all of the following points:

1 point is earned for the integral

1 point is earned for the answer

1 point is earned for computing interior critical points

1 point is earned for completing analysis

$R(t)$=0 when t=0, t=2.5π, or t=7.5π

$R(t)$>0 on 0<t<2.5π

$R(t)$<0 on 2.5π<t<7.5π

$R(t)$>0 on 7.5π<t<31

The absolute maximum number of mosquitoes occurs at t=2.5π or at t=31.

$$1000+\int_0^{2.5\pi} R(t)dt = 1039.357,$$

There are 964 mosquitoes at t=31, so the maximum number of mosquitoes is 1039, to the nearest whole number.

Differential Equation Review

A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t)=\ln(t^2-3t+3)$. The particle is at position $x=8$ at time $t=0$.

14. Find the position of the particle at time $t=2$.

Please respond on separate paper, following directions from your teacher.

Part C

1 point is earned for $\int_0^2 \ln\left(u^2-3u+3\right)du$

1 point is earned if handles initial condition

1 point is earned for answer

$$s(t)=s(0)+\int_0^t \ln\left(u^2-3u+3\right)du$$
$$s(2)=8+\int_0^2 \ln\left(u^2-3u+3\right)du$$

=8.368 or 8.369

0	1	2	3

The student response earns all of the following points:

1 point is earned for $\int_0^2 \ln\left(u^2-3u+3\right)du$

1 point is earned if handles initial condition

1 point is earned for answer

$$s(t)=s(0)+\int_0^t \ln\left(u^2-3u+3\right)du$$
$$s(2)=8+\int_0^2 \ln\left(u^2-3u+3\right)du$$

=8.368 or 8.369

Differential Equation Review