Lesson 12 – Arithmetic Sequences

Goal: • Can identify arithmetic sequences as a linear relationship. • Can build an equation for an arithmetic sequence and use it to find specific terms of the sequence.
New Terminology: • Arithmetic Sequence • Term • Common Difference

Discuss: Consider the pattern:

14, 19, 24, 29, ...

What comes next? What would the 32nd number in the pattern be? What position would 274 be in?

We call patterns like these **ARITHMETIC SEQUENCES** for two reasons:

1. Arithmetic because the change is constant addition

2. Sequence because it is an ordered list

Examples of arithmetic sequences:

a. {5, 5, 5, 5, 5, 5} constant add 0

b. {7, 3, -1, -5, -9, ...} constant add -4 (-4, -4)

c. {3.2, 4.4, 5.6, 6.8, 8.0, ...} constant add 1.2 (+1.2, +1.2)

The **COMMON DIFFERENCE** of the sequence is:

the constant addition (whatever that # is)

Discuss: What is the minimal information we need to define an arithmetic sequence?

→ common difference and first term

→ first two terms, the sequence is arithmetic

→ any term and first term, but we need the position, the sequence is arithmetic

→ any 3 consecutive terms

→ common difference and any term (with position)

→ any two terms (with position), the sequence is arithmetic

Example: The common difference is −12 and the 3rd term is 8. Find the first 5 terms of the arithmetic sequence.

$d = -12$ the common difference

$a_3 = 8$ ↑ the 3rd term

a_1 a_2 a_3 a_4 a_5

32, 20, 8, −4, −16

−12 −12 −12 −12

Practice: The common difference is 7 and the 4th term is 12. Find the first 5 terms of the arithmetic sequence.

$d = 7$ $a_4 = 12$

+7 +7 +7 +7

$-9, -2, 5, 12, 19$

a_1, a_2, a_3, a_4, a_5

Discuss: If the common difference of an arithmetic sequence is 2.7 and the first term is 3. What is the 117th term? This should be possible! build an equation (try) explain why it looks like it does in grade 9

dependent = start + change x independent

We want to generalize the sequence to solve this problem. So, let's use a let statement on two characteristic we know we need.

Let a_1 be the ~~start~~ first term

Let d be the common difference

Zeroth Term	First Term	Second Term	Third Term	...	n^{th} Term
	$a_1 = a_1 + 0d$	$a_2 = a_1 + 1d$	$a_3 = a_2 + d = a_1 + 2d$		$a_n = a_1 + (n-1)d$
a_0 ~~naught~~	$a_1 = a_0 + d$	$a_2 = a_0 + 2d$	$a_3 = a_0 + 3d$		$a_n = a_0 + nd$

Example: The common difference of an arithmetic sequence is $-\frac{12}{5}$ and the first term is 99. What term will be -109.8?

① $a_n = a_1 + (n-1)d$ ② $a_n = a_0 + nd$

$d = \frac{-12}{5}$ $a_1 = 99$ $a_n = -109.8$ $n = ?$

① $-109.8 = 99 + (n-1)(-12/5)$

$-208.8 = \frac{-12}{5}(n-1)$

$87 = n-1$

$\boxed{88 = n}$

② $a_0 = a_1 - d$

$= 99 - (-12/5)$

$= 101.4$

$-109.8 = 101.4 + n(-12/5)$

$-211.2 = \frac{-12}{5}n$

$\boxed{88 = n}$

Practice: The 77th term is 26 and the common difference of an arithmetic sequence is 3.7. What is the first term?

$a_{77} = 26$ $d = 3.7$ $a_1 = ?$

$a_n = a_1 + (n-1)d$

a_n: term value; n: position

$26 = a_1 + (77-1)\cdot 3.7$

$26 = a_1 + 281.2$

$\boxed{a_1 = -255.2}$

$a_n = a_0 + nd$

$26 = a_0 + 77 \cdot 3.7$

$26 = a_0 + 284.9$

$a_0 = -258.9$

$a_1 = a_0 + d = \boxed{-255.2}$

Practice: The 46th term is -823 and the first term of an arithmetic sequence is 42. What is the common difference?

$a_{46} = -823$ $a_1 = 42$ $d = ?$

I need to practice algebra

$a_n = a_1 + (n-1)d$

$\boxed{-823 = 42 + (46-1)d}$

$-823 = 42 + 45d$

$-865 = 45d$

$-19.\overline{22} = d = \frac{-865}{45} = \frac{-173}{9}$

Discuss: The 10th term of an arithmetic sequence is 64 and the 15th term is 99. What is the first term?

Example: The 23rd term of an arithmetic sequence is 92 and the 43rd term is 68. What is the first term?

$a_{23} = 92$ $a_{43} = 68$ 20 terms later

23rd term

$b_0 = 92$ $b_{20} = 68$

$b_n = b_0 + nd$ ★ relabel

$68 = 92 + 20d$

$-24 = 20d$

$-1.2 = d$

$a_n = a_1 + (n-1)d$

$92 = a_1 + (23-1)(-1.2)$

$92 = a_1 - 26.4$

$\boxed{118.4 = a_1}$

Practice: The 57th term of an arithmetic sequence is 57 and the 83rd term is 291. What is the first term?

$a_{57} = 57$ $a_{83} = 291$

$b_0 = 57$ $b_{26} = 291$

$b_n = b_0 + nd$ ☆ relabel

$291 = 57 + 26d$

$d = 9$

$a_n = a_1 + (n-1)d$

$57 = a_1 + (57-1)(9)$

$a_1 = -447$ $= a(1)$

$a(57) = 57$ $a(83) = 291$

KEY IDEAS:

- Sequences are ordered sets where there is a first, second, third and so on.
- Arithmetic sequences are built by adding the same number again and again (the **common difference**)
- You need two pieces of information for an arithmetic sequence:
  a. Any two terms and their position
  b. Any term and its position AND the common difference
- Arithmetic sequences are built from the formula

sequence term is a function of its position

$$a_n = a_1 + (n-1) \cdot d \quad = a(n)$$

OR

$$a_n = a_0 + n \cdot d \quad = a(n)$$

Where a_1 is the first term, d is the common difference and a_n is the n^{th} term. [Here a_0 is the term that would have come before the first term]

- This formula can be used to find any of the four variables: a_1, a_n, d, and n

Assigned Problems: Arithmetic Sequences Worksheet