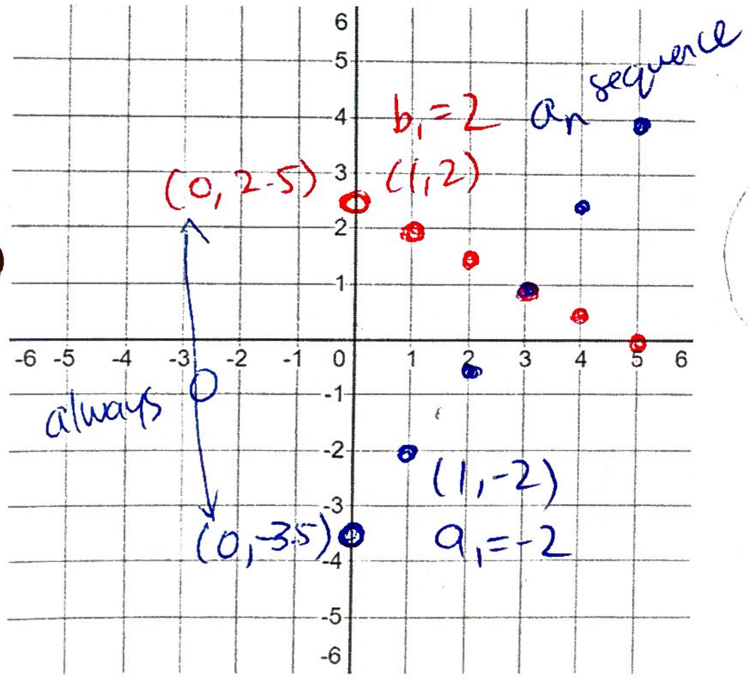
Lesson 13 – Slope and Linear Equations

Goal:
- Can describe the slope of a line given as a graph, set of ordered pairs, or equation
- Can use multiple definitions of slope
- Can build the equation of a line in slope-intercept form

New Terminology:
- Slope
- Intercept
- Slope-Intercept Form

Discuss: Consider the arithmetic sequence with a common difference of 1.5 and the 3rd term is 1. Determine the first 5 terms of the sequence and plot them on the grid.

Plot a second sequence that still has a 3rd term of 1, but the common difference is -0.5



$d = 1.5 \quad a_3 = 1$

$a_n = a_1 + (n-1)d$

-2	-0.5	1	2.5	4
a_1	a_2	a_3	a_4	a_5

+1.5 +1.5 +1.5 +1.5

2	1.5	1	0.5	0
b_1	b_2	b_3	b_4	b_5

−0.5 −0.5 −0.5 −0.5

a_n value $\equiv y$ dependent

position $\equiv x$ independent

$x \in \mathbb{N}$ NOT reals!

Arithmetic sequences are just discrete lines (we allow $x \in \mathbb{R}$)

Remember with our formula for arithmetic sequences we had two major parts to the equation:

$$a_n = a_0 + n \cdot d$$

In function notation we could write this as:

$a_n = a(n)$

Which shows that n is the position value is a function of its position

and $a(n)$ is the value

While a_0 and d are special constants.

capital delt

The common difference, d, is now called SLOPE and defined as:

1.) slope $= \frac{\text{change in dependent}}{\text{change in independent}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$

2.) slope $= \frac{\text{rise}}{\text{run}}$ useful for graph

3) slope = the constant addition for 1 change in position

The zeroth term, a_0, is now called the Y-INTERCEPT and defined as:

→ The point where the line crosses the y-axis (the x-value is 0)

$a(n) = a_0 + nd \Rightarrow a(0) = a_0 + \cancel{0 \cdot d}^{0}$

if $x=0$

$= a_0 \rightarrow y$ is a_0

"first"

Practice: Determine the common difference of an arithmetic sequence if $a_4 = 8$ and $a_{10} = 6$. 'second'

difference of values $a_{10} - a_4 = 6 - 8 = -2$

end - start

difference of position $10 - 4 = 6$

values have ↓

position ↑

$d = \text{value}/\text{position} = -2/6$

$= -\frac{1}{3}$ value/position $= \frac{\text{difference in values}}{\text{difference in position}}$

Discuss: Determine the slope of a line that passes through the points (3,4) and (12,20). [How is this like finding the common difference of an arithmetic sequence?]

$a_3 = 4$ and $a_{12} = 20$

position value

x_1 y_1
(3,4)
(12,20)
x_2 y_2

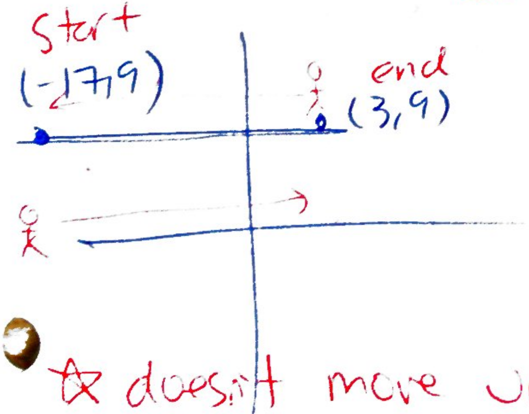
$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{\overset{\text{end}}{y_2} - \overset{\text{start}}{y_1}}{\underset{\text{end}}{x_2} - \underset{\text{start}}{x_1}} = \frac{20-4}{12-3}$$

$$= \frac{16}{9}$$

Practice: Determine the slope of a line that passes through the points $(-3,2)$ and $(5,-8)$.

Discuss: Determine the slope of the line that passes through the points (3,9) and $(-17,9)$. AND determine the slope of the line that passes through the points $(-2,-4)$ and $(-2,5)$.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{+0}{+20} = \underline{\underline{0}}$$



★ doesn't move up or down

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$(-2,-4)$ $(-2,5)$
x_2 y_2 x_1 y_1

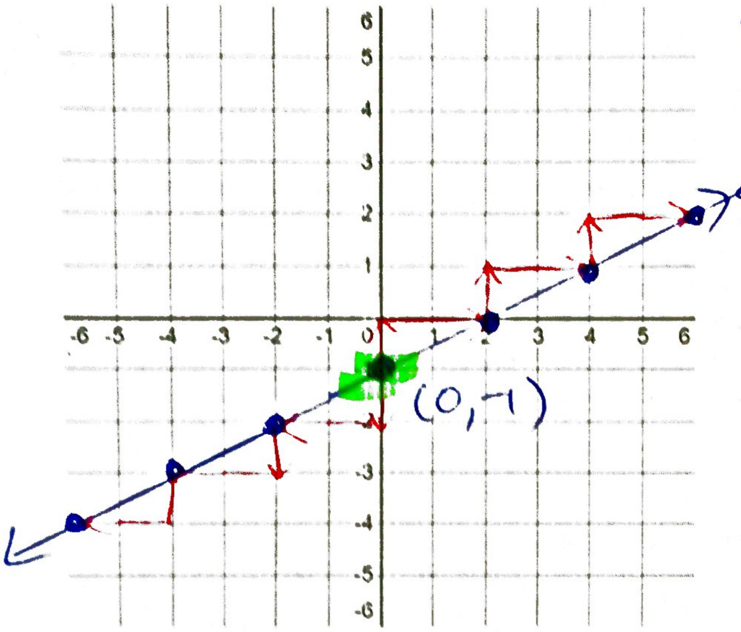
$$\text{slope} = \frac{-4-5}{-2+2} = \frac{-9}{0}$$

$= \text{undefined}$

★ what shape does this make?

Once we are comfortable with the slope of a line, we can describe the *y*-intercept and then graph the line.

Practice: Graph the line with a slope of $\frac{1}{2}$ and *y*-intercept of −1.



y-intercept = −1

⇒ pass through (0,−1)

Slope = $\frac{1}{2}$ = $\frac{\text{rise}}{\text{run}}$ = $\frac{-1}{-2}$

up 1 and to the right 2

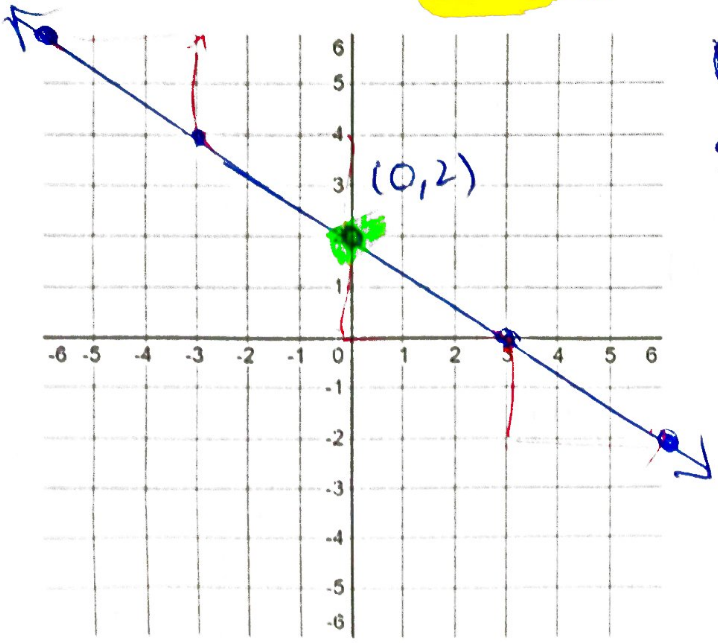
down 1 and to the left 2

$f(x) = mx + b$

⇒ $\boxed{f(x) = \frac{1}{2}x - 1}$

★ moves up (+ve slope)

Practice: Graph the line with a slope of $-\frac{2}{3}$ and *y*-intercept of 2.



pass through (0,2)

Slope = $-\frac{2}{3}$ = $\frac{-2}{3}$ = $\frac{2}{-3}$ = $\frac{\text{rise}}{\text{run}}$

down 2 and right 3

up 2 and loft 3

$f(x) = mx + b$

⇒ $\boxed{f(x) = \frac{-2}{3}x + 2}$

★ moves down (−ve slope)

All that's left is to put it together in an equation form. But we already have a beautiful equation from our arithmetic sequence.

$$a(n) = d \cdot n + a_0$$

The standard convention is for the slope to be: m (change the common differace)

And the y-intercept to be: b (change zeorth term)

So, our linear equation in **SLOPE-INTERCEPT FORM** is:

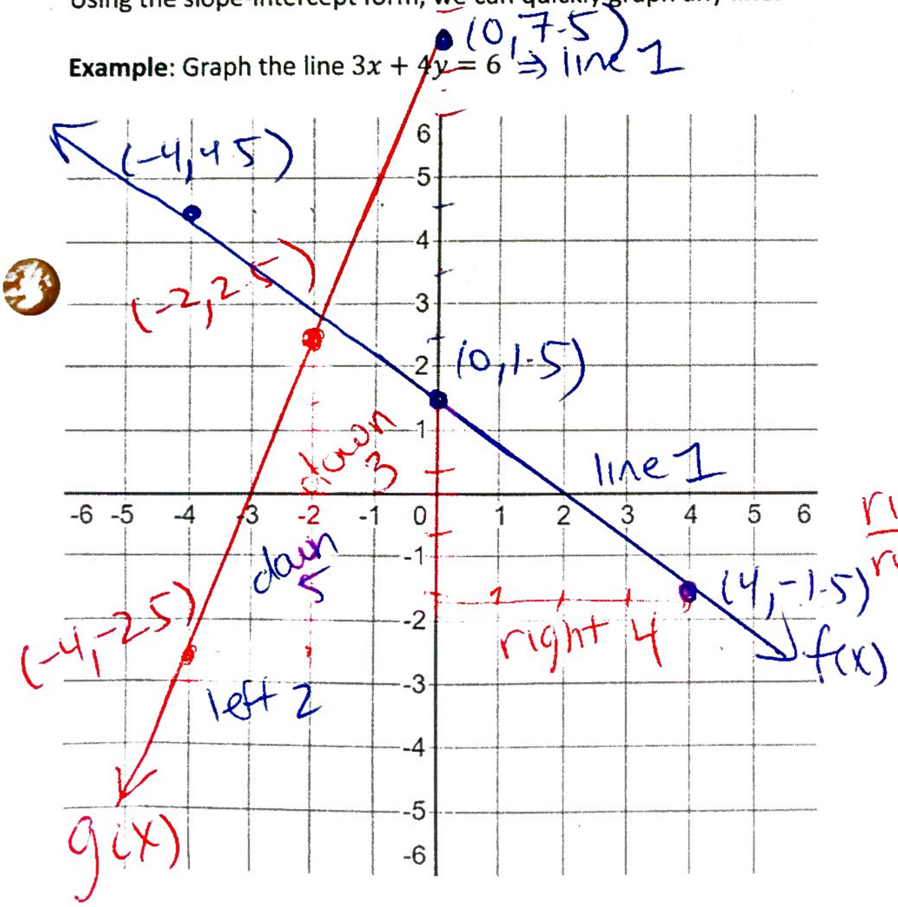
$$y = f(x) = mx + b$$

isolated

Let's go back and determine the equations to the lines described!

Using the slope-intercept form, we can quickly graph any line.

Example: Graph the line $3x + 4y = 6$ ⇒ line 1



$3x + 4y = 6$

$-3x \quad -3x$

$4y = -3x + 6$

$\frac{4y}{4} = \frac{-3x}{4} + \frac{6}{4}$

$$\boxed{y = f(x) = \frac{-3}{4}x + \frac{3}{2}}$$

$\frac{\text{rise}}{\text{run}}$ = slope

y-int.

$2.5 = \text{slope} = \frac{5}{2} = \frac{-5}{-2}$

down 5 left 2

Practice: On the same grid, graph and label the line $5x - 2y + 15 = 0$

$+2y \quad +2y$

$\frac{5x + 15}{2} = \frac{2y}{2}$

$2.5x + 7.5 = y = g(x)$

slope

y-int

Finally, we want to be able to make the equation to lines given their characteristics. We use the basic idea that every linear function will have the form:

$$f(x) = mx + b$$

And that the y-intercept is the point $(0, b)$. always on the line

Example: Find the equation to the line that has a slope of $\frac{1}{3}$ and passes through the point $(4, 6)$ use to find b

$f(x) = \frac{1}{3}x + b \leftarrow$ unknown

if $x=4$ then $f(4)=6$

$f(4) = \boxed{6 = \frac{1}{3} \cdot 4 + b}$

$-4/3 \quad -4/3$

$14/3 = b \qquad \Rightarrow \boxed{f(x) = \frac{1}{3}x + 14/3}$

Practice: Find the equation to the line that has a y-intercept of -3 and passes through the point $(2, 5)$.

$f(x) = mx + b \qquad f(2)=5 \qquad (0,-3)$

$f(x) = mx - 3$

$5 = m \cdot 2 - 3$

$8 = 2m \Rightarrow m = 4$

OR

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-5}{0-2} = \frac{-8}{-2} = 4$

$\boxed{f(x) = 4x - 3}$

Discuss: Determine the equation of the line that passes through $(6, 5)$ and $(-3, 8)$.

ELFS

Assigned Problems: 6.5 page 325 – 328 # 1-5, 10, 18

🌶 12, 14, 16

7.1 page 349 – 356 # 1-3, 5-10, 12, 13, 19-21, 24

🌶 15, 18, 23 (ghost pepper)

Key Ideas on page 324 and 349