$89.999\ldots = 90$

Lesson 1 – Solving Sides of Triangles

Goal:

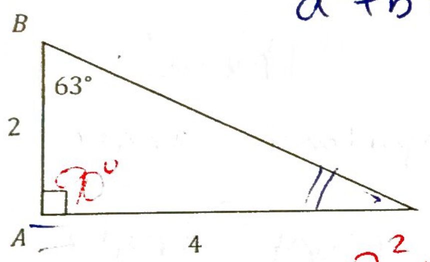
- Given a right-angle triangle with 1 side and 1 acute angle, you can determine the lengths of the other two sides. (Can be applied to contextualized problems)
- Understands that trig operators are NOT forms of multiplication, but a transformation of an angle that is related to a ratio of sides.

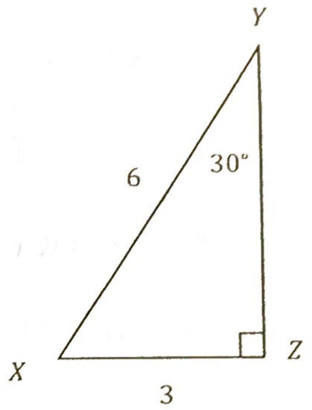
New Terminology:

- Acute Angle
- Opposite, Adjacent, Hypotenuse
- Sine, Cosine, Tangent
- Angle of Elevation/Inclination/Declination

Review: Determine the missing side lengths and angles of the triangles.

$a^2+b^2=c^2$





angle

$\angle C$ or $\angle ACB$

$= 180° - 90° - 63°$

$= 27°$

$2^2 + 4^2 = BC^2$

$4 + 16 = BC^2$

$BC = \sqrt{20}$

$= 4.5$

$\angle X = 180° - 90° - 30°$

$= 60°$

$3^2 + YZ^2 = 6^2$

$YZ^2 = 36 - 9$

$YZ = \sqrt{27} = 5.2$

What makes a right-angle triangle a Right-Angle Triangle?

☆ all angles add to 180°

☆ Pythagoras says $a^2+b^2=c^2$

☆ 1 angle is 90° $\Rightarrow$ implies The other 2 must add to 90°

what is the biggest the other angle could be?

We don't know, but it must be < 90

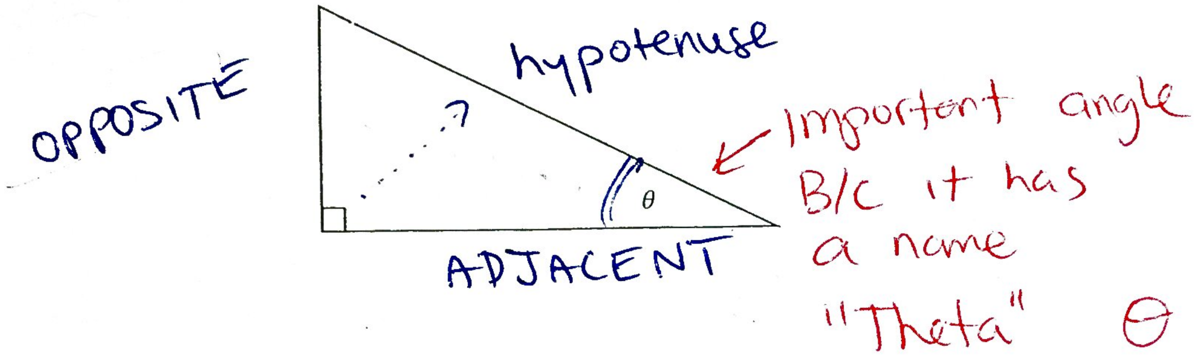
ACUTE ANGLE

You already know that if you know 2 sides of a right-angle triangle, the you know the other. We also know that if we know 1 of the acute angles then we know the other.

So:

1. To know all 3 sides, we only need 2 sides
2. To know all 2 acute angles, we only need 1 acute angle

But what if we know a mix of both?



Definitions:

- OPPOSITE side: is across from the important angle
- ADJACENT side: is touching the important angle.
- HYPOTENUSE side: is the longest side, its across from the right angle

Note that since we said the proportions and angle does not change depending on the scale of the triangle, we can define the relationship between the angle and the proportion (ratio) of sides.

1. $\frac{\text{opposite}}{\text{hypotenuse}} = \sin(\theta)$ sign of theta

stays the same stays the same

2. $\frac{\text{adjacent}}{\text{hypotenuse}} = \cos(\theta)$ co-sign of theta

3. $\frac{\text{opposite}}{\text{adjacent}} = \tan(\theta)$ tangent of theta

soh cah toa

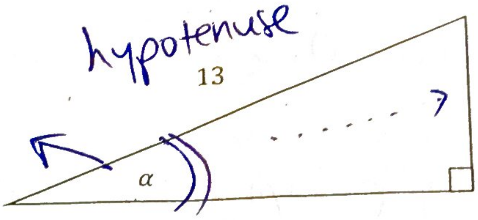
soh cah toa

Example: Determine the trig ratios of angle α in the following triangle:

hypotenuse

"alpha"
α

OPPOSITE



ADJACENT

$\sin(\alpha) = \frac{5}{13} = 0.38 = \frac{0.38}{1} = \frac{38}{100}$

opposite

hypotenuse

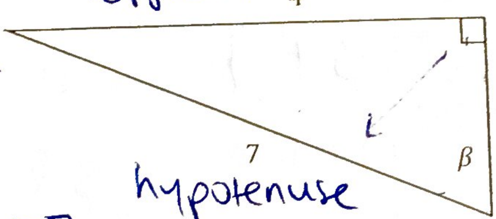
$\cos(\alpha) = \frac{12}{13} = 0.92$

$\tan(\alpha) = \frac{5}{12} = 0.42$

Practice: Determine the trig ratios of angle β in the following triangle:

OPPOSITE

soh cah toa



ADJACENT = a

"beta"

hypotenuse

$\sin(\beta) = \frac{4}{7} = 0.57$

$\cos(\beta) = \frac{\boxed{5.74}}{7} = 0.82$

$\tan(\beta) = \frac{4}{5.74} = 0.70$

$a^2 + 4^2 = 7^2$

$a^2 = 49 - 16$

$a = \sqrt{33} = 5.74$

"phi" φ

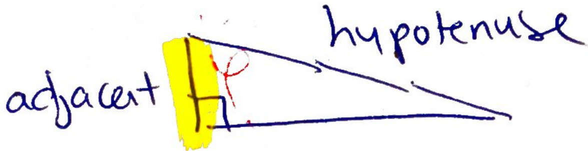
Discuss: What does $\cos\varphi = 0.34$ mean?

$$\frac{\text{adjacent}}{\text{hypotenuse}} = 0.34 \Rightarrow \text{adjacent} = 0.34 \times \text{hyp}$$

ratio

☆ φ is an angle

adjacent is $\sim \frac{1}{3}$ of hypotenuse



Once we understand what the trig operation represents, we can use it as the operation it is! Just like once you know the "add 1" operation you can apply that operation to any number you want.

Practice: Use the "add 1" operation on the numbers $-3, 0,$ and $\frac{4}{3}$.

$-3 \xrightarrow{\text{add 1}} -2 = -3+1$

$0 \xrightarrow{\text{add 1}} 1 = 0+1$

$\frac{4}{3} \xrightarrow{\text{add 1}} 7/3 = 4 \div 3 + 1$

Practice: Use the "sine" operation on the numbers $0^\circ, 32^\circ, 81^\circ, 90^\circ$

$0^\circ \xrightarrow{\text{sin that}} \sin(0^\circ) = 0 = \frac{\text{opposite}}{\text{hypotenuse}}$

$32^\circ \longrightarrow \sin(32^\circ) = 0.53$

$81^\circ \longrightarrow \sin(81^\circ) = 0.99$

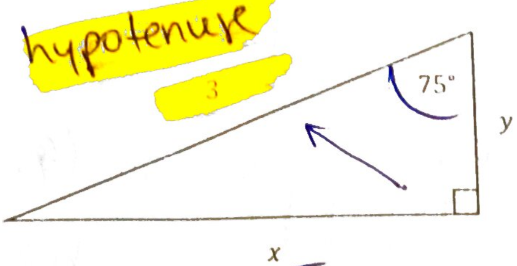
$90^\circ \longrightarrow \sin(90^\circ) = 1$

1 side and 1 angle

Example: Find the missing sides

$\boxed{\text{soh}}\ \boxed{\text{cah}}$ toa

hypotenuse



ADJACENT

OPPOSITE

Some #

$\sin(75°) = \frac{x}{3} = 0.97$

$x = 3 \times 0.97$

$\boxed{x = 2.90}$

$\cos(75°) = \frac{y}{3} = 0.26$

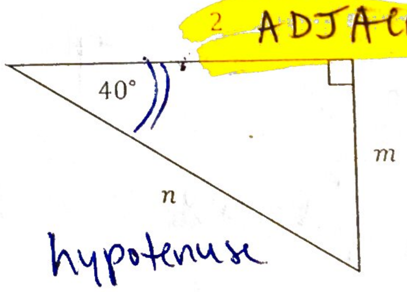
$y = 3 \times 0.26$

$\boxed{y = 0.78}$

Practice: Find the missing sides

soh $\boxed{\text{cah}}$ $\boxed{\text{toa}}$

ADJACENT



OPPOSITE

hypotenuse

$\cos(40°) = \left(\frac{2}{n} = \frac{0.77}{1}\right) \times n$

$\frac{2}{0.77} = \frac{0.77n}{0.77}$

$\boxed{2.61 = n}$

$\Downarrow$

$\frac{n}{2} = \frac{1}{0.77}$

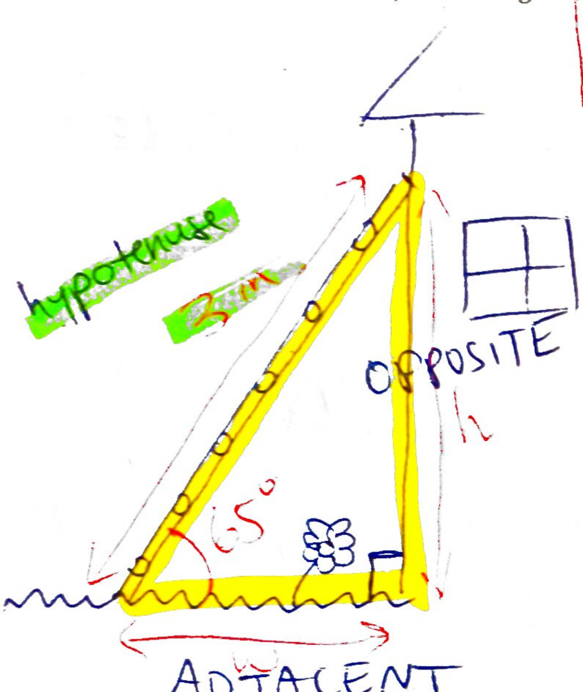
$\tan(40°) = \frac{m}{2} = 0.84$

$m = 2 \times 0.84$

$\boxed{m = 1.68}$

Example: A 3m ladder makes an **ANGLE OF INCLINATION** (elevation) of 65° with the ground as it rests on a wall. How far up the wall is the ladder and how far away from the ground is the base of the ladder?

soh | cah | toa



$$\sin(65^\circ) = \frac{h}{3} = 0.91$$

$$h = 3 \times 0.91 = \boxed{2.72\,m}$$

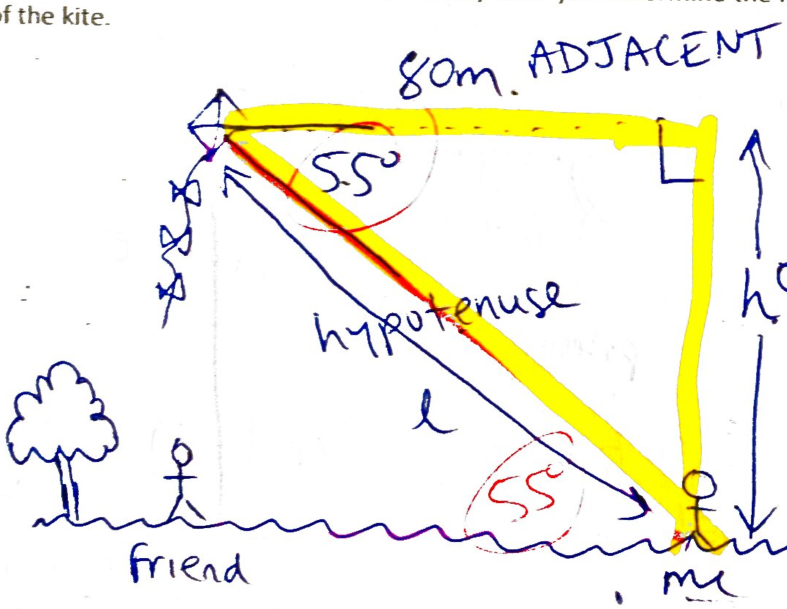
$$\cos(65^\circ) = \frac{w}{3} = 0.42$$

$$w = 0.42 \times 3$$

$$\boxed{= 1.27\,m}$$

Practice: A kite is flying in the wind and the **ANGLE OF DECLINATION** the kite makes with the ground is 55°. If your friend stands directly beneath the kite and is 80m away from you determine the length of string let out of the kite and the altitude of the kite.

soh
cah
toa



$$\cos(55^\circ) = \frac{80}{l}$$

$$\frac{l}{80} = \frac{1}{\cos(55^\circ)}$$

$$l = \frac{80}{\cos(55)}$$

$$= \boxed{139.5\,m}$$

$$\tan(55^\circ) = \frac{h}{80}$$

$$h = 80\tan(55) = \boxed{114.3\,m}$$

Assigned Problems: 3.1 page 107 – 111: # 1-3, 6a, 7b, 9, 10, 12, 15

14, 16

3.2 page 120 – 123: # 1, 2, 4, 6bd, 8, 9, 10, 14

11, 16

Key Ideas on page 107 and 119