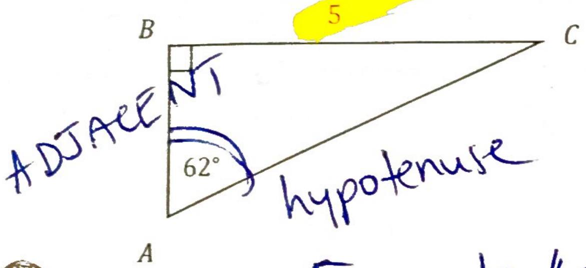
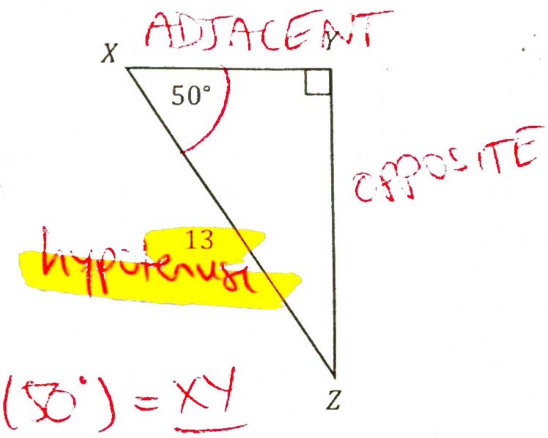
Lesson 2 – Solving Angles of Triangles

Goal:
• Given a right-angle triangle with 2 sides, you can determine the measure of the two acute angles. (Can be applied to contextualized problems) • Understands that trig operators have a reverse operator (just like add/subtract) that will transform a ratio of sides to an angle between 0° and 90°.
New Terminology: • Inverse

Review: Determine the two missing side lengths of each triangle. Use cosine, sine, and tangent all at least once.

soh cah toa





$\sin(62°) = \frac{5}{AC}$

$AC \cdot \sin(62°) = 5$

$AC = \frac{5}{\sin(62°)}$

$\boxed{= 5.7}$

$\tan(62°) = \frac{5}{AB}$

$AB = \frac{5}{\tan 62°}$

$\boxed{= 2.7}$

$\cos(50°) = \frac{XY}{13}$

$XY = 13 \cos 50°$

$\boxed{= 8.4}$

$\sin(50°) = \frac{YZ}{13}$

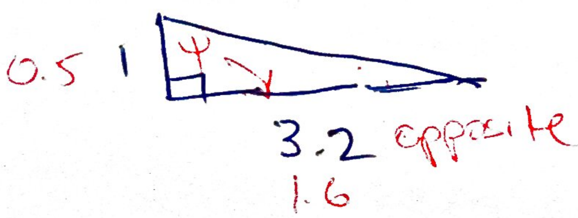
$YZ = 13 \sin 50° = \boxed{10.0}$

Review: In your own words what does $\tan \psi = 3.2$ mean?

"psi"

opposite is 3.2 x as large as adjacent

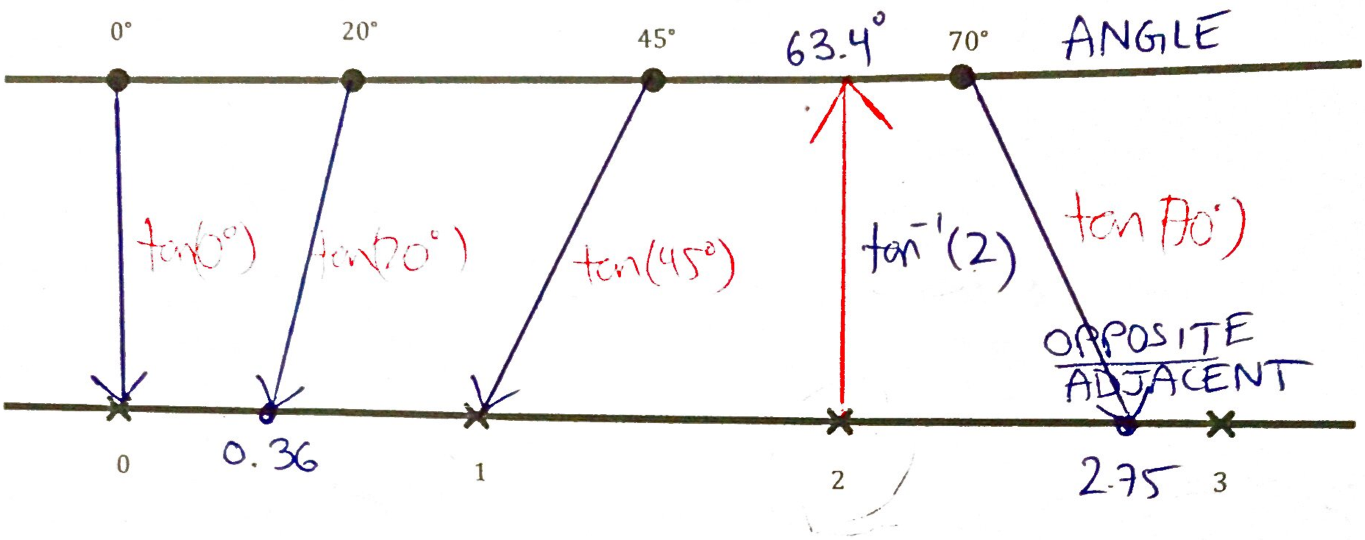
$\frac{\text{opposite}}{\text{adjacent}} = \frac{3.2}{1} = \tan \psi$



Chapter 3 Trigonometry

Remember that yesterday we looked at how we can relate the angle and the ratios (proportions) of sides to each other using our trig operators.

Practice: Use the "tangent" operator on the following numbers 0°, 20°, 45°, 70° and illustrate the connection



We can easily find the ratio of a given angle, but what if we want to go backward and determine the angle of a given ratio?

Answer: use inverse trig operators

$\cancel{\tan}^{-1}(\cancel{\tan}\,20^\circ) = \tan^{-1}(0.36)$

inverse

$20^\circ = \tan^{-1}(0.36)$

$= \arctan(0.36)$

Example: What does the following represent?

$\arcsin\left(\frac{7}{10}\right)$ ← opposite ← hypotenuse

$\cos^{-1}\left(\frac{3}{8}\right)$ ← adjacent ← hypotenuse

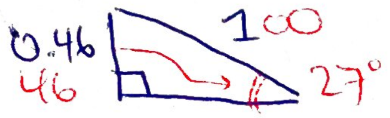
$\sin^{-1}(7/10) = 44.4^\circ$

$\cos^{-1}(3/8) = 68^\circ$

Practice: What does the following represent?

$\sin^{-1}(0.46) = 27°$

$\frac{0.46}{1} = \frac{\text{opposite}}{\cancel{\text{adjacent}}\ \text{hypotenuse}} = \frac{46}{100}$

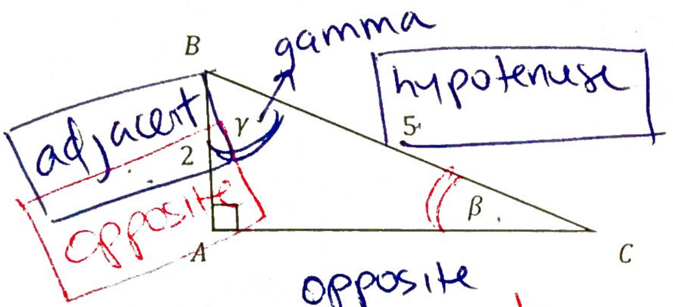


arctan(2.3)

$\frac{2.3}{1} = \frac{\text{opposite}}{\text{adjacent}} = \frac{23}{10}$

$\tan^{-1}(2.3) = 66°$

Example: Solve for the acute angles in the following triangle



opposite
adjacent

$\boxed{\text{soh}}\ \boxed{\text{cah}}$ toa

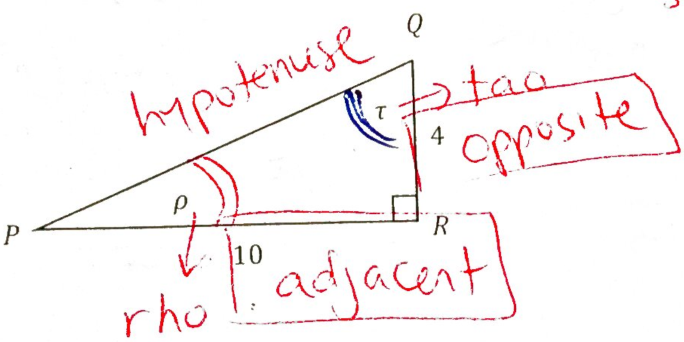
$\cos\gamma = \frac{2}{5} \Rightarrow \boxed{\gamma = \cos^{-1}(2/5) = 66.4°}$

$\sin\beta = 2/5 \Rightarrow \boxed{\beta = \arcsin(2/5) = 23.6°}$

☆ $\beta = 90° - 66.4° = 23.6$ ✓

Practice: Solve for the acute angles in the following triangle

soh cah $\boxed{\text{toa}}$



$\tan\rho = \frac{4}{10}$

$\rho = \tan^{-1}\left(\frac{4}{10}\right)$

$\boxed{= 21.8°}$

$\tau = 90° - 21.8°$

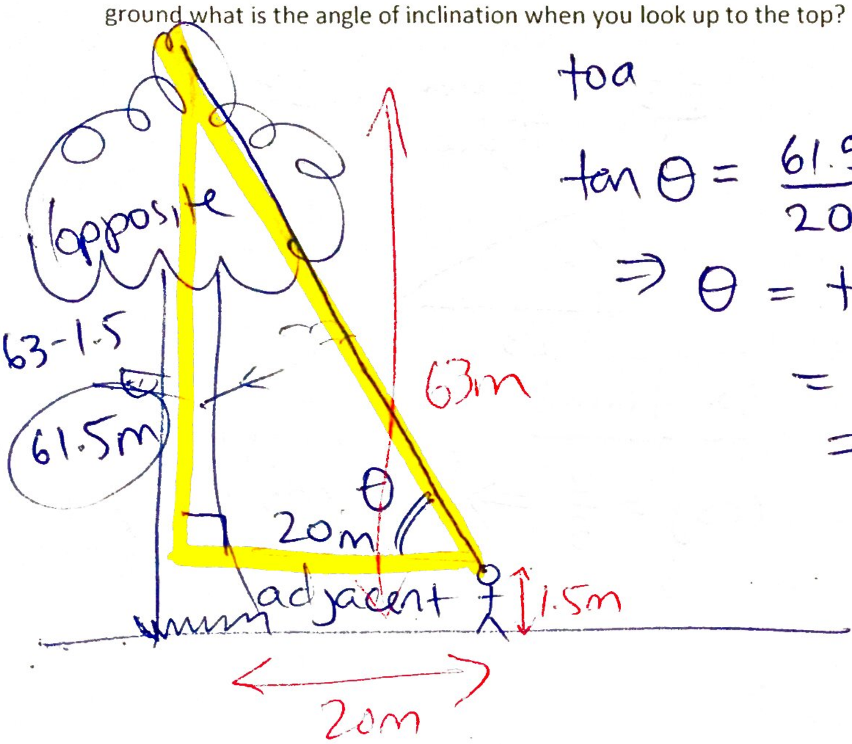
$\boxed{= 68.2°}$

$\tan\tau = \frac{10}{4}$

$\tau = \arctan\left(\frac{10}{4}\right)$

$= 68.2°$

Example: A tree stands 63 m tall and you are standing 20 m from the base of the tree. If your eyes are 1.5 m above the ground what is the angle of inclination when you look up to the top?



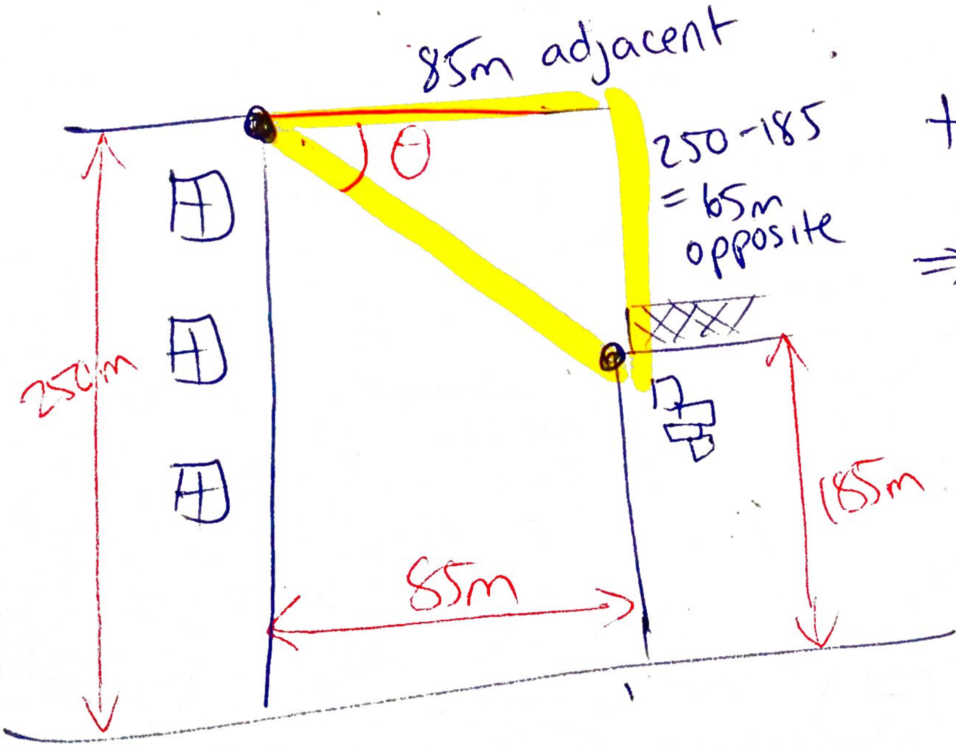
toa

$$\tan\theta = \frac{61.5}{20}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{61.5}{20}\right)$$

$$= \underline{\underline{72^\circ}}$$

Practice: Two buildings are separated by 85 m. The shorter building is 185 m tall and the taller building is 250 m tall. What is the angle of declination from the tall building to the short building?

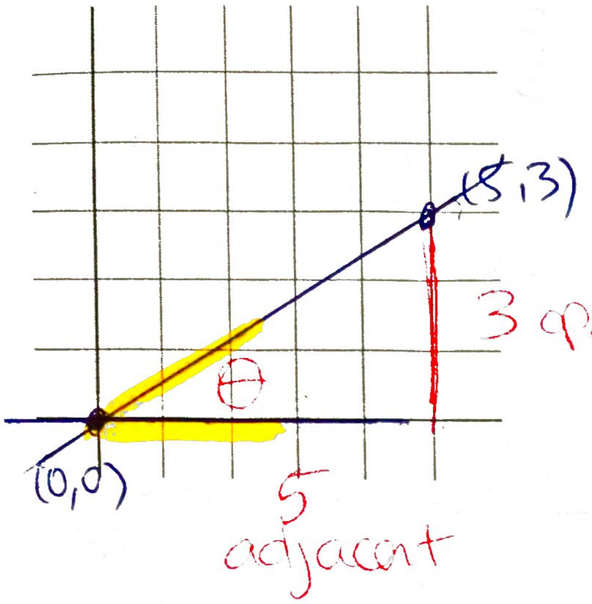


$$\tan\theta = \frac{65}{85}$$

$$\Rightarrow \theta = \arctan\left(\frac{65}{85}\right)$$

$$= \underline{\underline{37.4^\circ}}$$

Discuss: Determine the angle between the line through (0,0) and (5,3) and the x-axis?



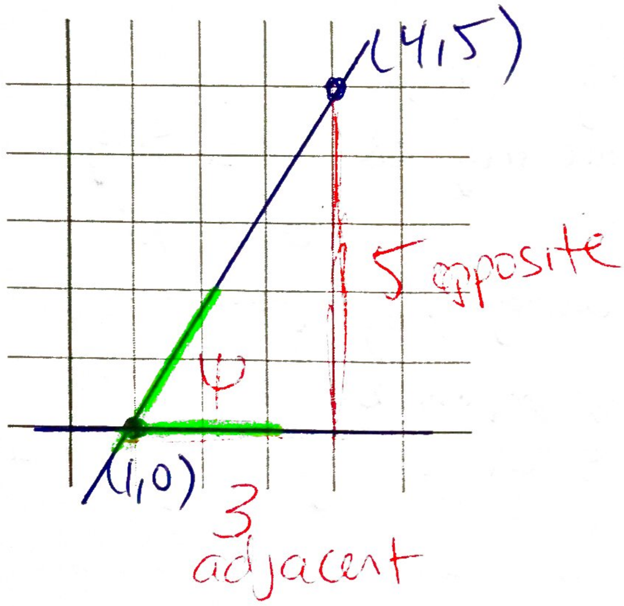
soh cah $\boxed{\text{toa}}$

$\tan\theta = 3/5$

$\theta = \arctan(3/5)$

$= 31°$

Discuss: Determine the angle between the line through (1,0) and (4,5) and the x-axis?



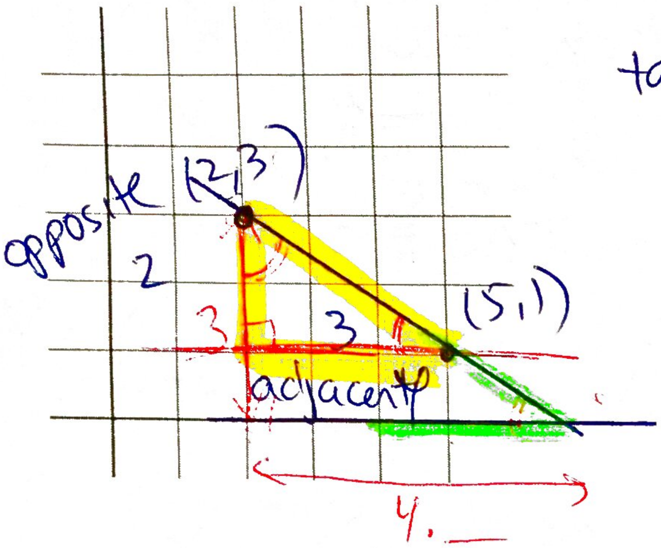
soh cah $\boxed{\text{toa}}$

$\tan\psi = \frac{5}{3}$

$\psi = \tan^{-1}(5/3)$

$= 59°$

Discuss: Determine the angle between the line through (2,3) and (5,1) and the x-axis?



$\tan\varphi = \frac{2}{3}$

$\varphi = \tan^{-1}\left(\frac{2}{3}\right)$

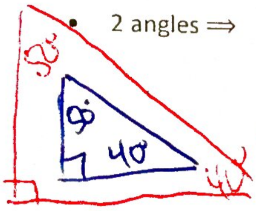
$= 33.7°$

☆ in general the angle between the line thru (0,0) and (x,y) and the x-axis is

$\tan^{-1}\left(\frac{y}{x}\right)$ — y → opposite, x → adjacent

SUMMARY: We know that a right-angle triangle will have 3 distinct sides and 2 acute angles. The minimal amount of information we need is TWO THINGS!

- 2 sides ⇒ Find missing side using Pythagoras
  Find acute angles arc trig OR inverse trig
- 1 side and 1 angle ⇒ Find missing angle by subtracting 90°
  Find missing sides ~~##~~ using trig operators
- 2 angles ⇒ Find missing sides is impossible
  Can find ratio of sides using trig operator



Assigned Problems: 3.1 page 107 – 112: # 4, 5, 6b, 7a, 8, 11, 13a, 19

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3.2 page 120 – 123: # 3, 5, 6ac, 12, 13

15:

Key Ideas on page 107 and 119