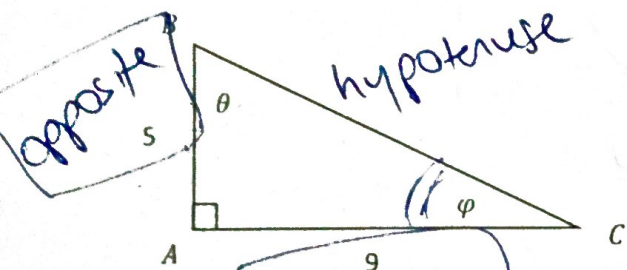# Lesson 3 – Solving Triangles

Goal:
- Given a right-angle triangle with 2 known measurements, you can determine the missing lengths and angles. (Can be applied to contextualized problems)
- Understands that connected triangles can be solved in unison.
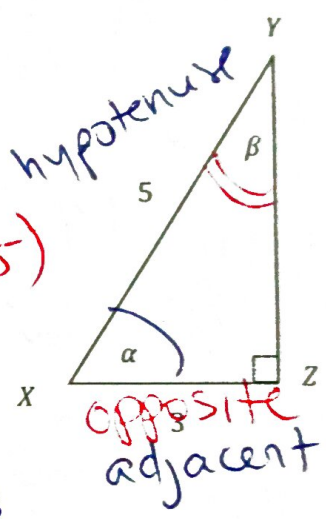
New Terminology:
- "Solve the triangle"

Review: Determine the missing angles of the triangles. Use inverse sine, cosine, and tangent all at least once.

$$\sin \beta = \frac{3}{5}$$
$$\beta = \arcsin(3/5)$$
$$= 36.9°$$

$$\tan \varphi = 5/9$$
$$\varphi = \tan^{-1}(5/9)$$
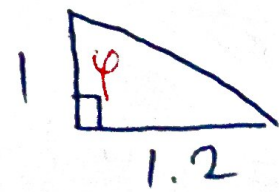$$= 29°$$
$$\star \quad \theta = 90° - 29° = 61°$$

$$\cos \alpha = \frac{3}{5}$$
$$\alpha = \arccos(3/5)$$
$$= 53.1°$$

Review: What does $\arcsin(0.35)$ and $\tan^{-1}(1.2)$ represent?

$$\arcsin(0.35) = \theta \quad \leftarrow \text{angle}$$
$$\text{ratio} = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\tan^{-1}(1.2) = \varphi \quad \text{angle}$$
$$\text{ratio} = \frac{\text{opposite}}{\text{adjacent}}$$

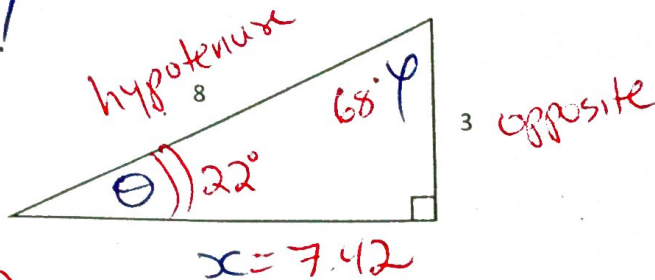Recall, we can find all the information for a triangle if we know 2 sides OR 1 side and 1 angle.

Example: Solve the following triangle:

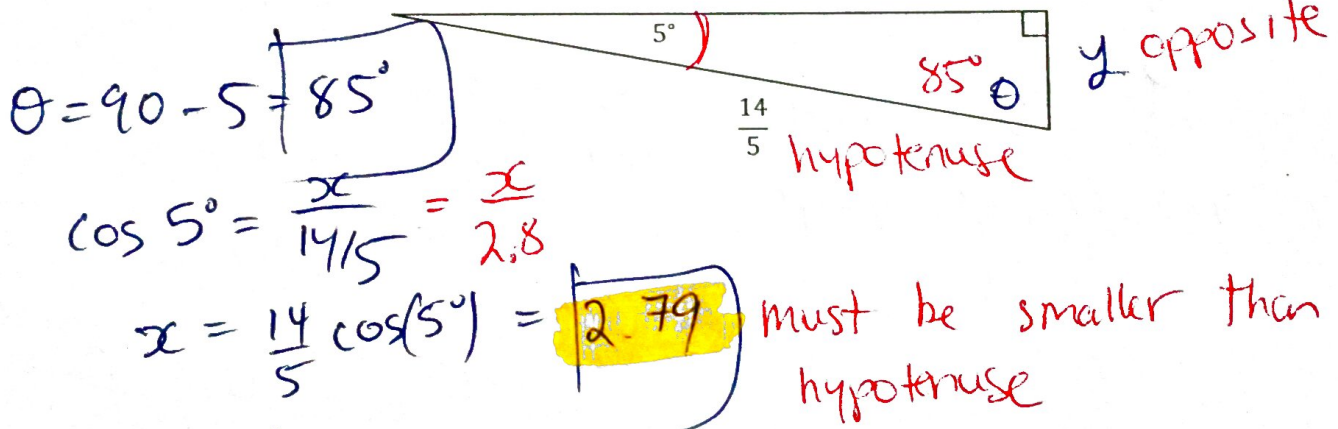Find everything!

$x^2 + 3^2 = 8^2$

$x^2 = 64 - 9$

$x = \sqrt{55} = \boxed{7.42}$

hypotenuse 8   68° φ 3 opposite

θ)) 22°

$x = 7.42$

$\sin \theta = \dfrac{3}{8} \Rightarrow \theta = \sin^{-1}(3/8) = \boxed{22.0°}$

$\varphi = 90 - 22 = \boxed{68.0°}$

Practice: Solve the following triangle:

adjacent x

5°

$\dfrac{14}{5}$ hypotenuse   85° θ y opposite

$\theta = 90 - 5 = \boxed{85°}$

$\cos 5° = \dfrac{x}{14/5} = \dfrac{x}{2.8}$

$x = \dfrac{14}{5} \cos(5°) = \boxed{2.79}$ must be smaller than hypotenuse

$\sin 5° = \dfrac{y}{2.8}$

$y = 2.8 \sin 5° = \boxed{0.24}$

Example: Find the perimeter of the following triangle:

$$180 - 100 - 45 = \theta = 35°$$

$$\cos 35° = \frac{p_2}{4} \implies p_2 = 4\cos 35° = 3.28$$

$$\sin 35° = \frac{h}{4} \implies h = 4\sin 35° = 2.29$$

$$\sin 45° = \frac{2.29}{y} \implies y\sin 45° = 2.29$$

$$\implies y = \frac{2.29}{\sin 45°} = 3.24$$

$$\frac{\tan 45°}{1} = \frac{2.29}{p_1} \implies \frac{1}{\tan 45°} = \frac{p_1}{2.29}$$

$$\implies \frac{2.29}{\tan 45°} = p_1 = 2.29$$

$$\text{Perimeter} = 2.29 + 3.24 + 3.28 + 4$$

$$= 12.81$$

Practice: Determine the perimeter of the following shape:

adjacent
x

opposite

~~hypotenuse~~

25°

adjacent

3

25° h 65°

y
hypotenuse

65° z opposite

$90 - 25 = 65°$

~~$\tan$~~ ~~$\cos 25 = \dfrac{x}{3}$~~

~~$x = 3\cos 25 = 2$~~

$\tan(25°) = \dfrac{3}{x} \Rightarrow x = \dfrac{3}{\tan 25} = 6.43$

$\sin(25°) = \dfrac{3}{h} \Rightarrow h = \dfrac{3}{\sin 25} = 7.10$

⭐ to solve we need 1 piece of information

1 angle   or   1 side

to solve we will we assume they are
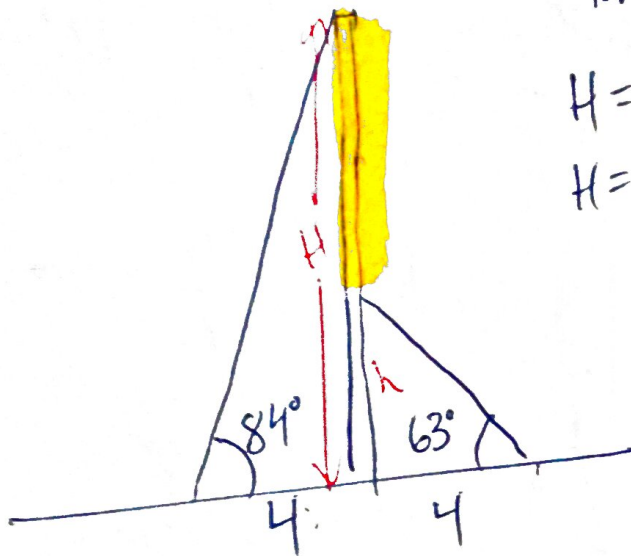similiar

$\cos 25° = \dfrac{7.10}{y} \Rightarrow y = \dfrac{7.10}{\cos 25} = 7.83$

$\tan 25° = \dfrac{z}{7.10} \Rightarrow z = 7.10 \tan 25° = 3.31$

$P = 3.31 + 7.83 + 3 + 6.43 = 20.57$

⭐ with assumption $\angle = 30° \Rightarrow P = 21.73$

$\angle = 20° \Rightarrow P = 19.57$

Discuss: A telephone pole is supported by two guide wires anchored at the ground 4 m away from the base of the pole. One of the wires connects from the top and makes an angle of elevation of 84°, the other connects somewhere in the middle of the pole and makes an angle of elevation of 63°. How far apart are the two wires on the telephone pole?

$$\tan 84° = \frac{H}{4} \qquad \tan 63° = \frac{h}{4}$$
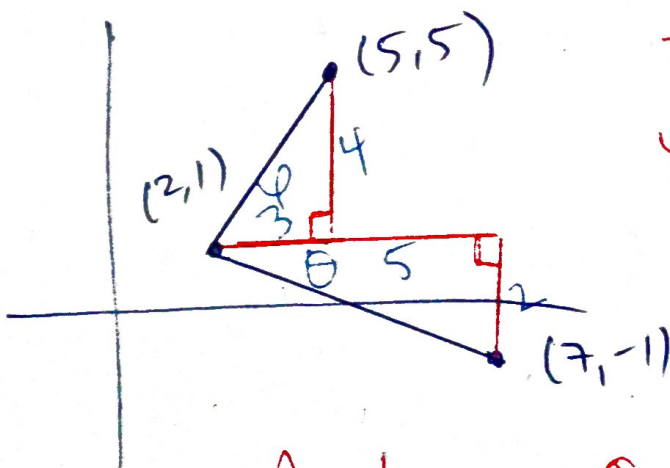
$$H = 4\tan 84° \qquad h = 4\tan 63°$$

$$H = 38.1\,m \qquad h = 7.9\,m$$

$$d = H - h$$
$$= 38.1 - 7.9$$
$$= 30.2\,m$$

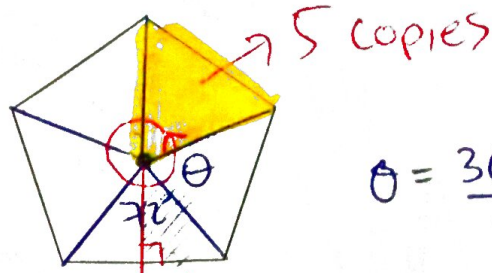**Discuss:** Determine the angle formed when you connect the points (5,5), (2,1), and (7,−1) together in order.

$$\tan\varphi = \frac{4}{3} \qquad \tan\theta = \frac{2}{5}$$

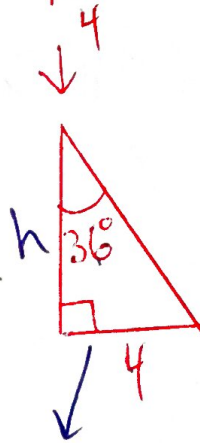$$\varphi = \tan^{-1}\left(\frac{4}{3}\right) \qquad \theta = \tan^{-1}\left(\frac{2}{5}\right)$$

$$= 53.1° \qquad\qquad = 21.8°$$

Angle is $\theta + \varphi = 74.9°$

Discuss: Determine the area of a regular pentagon with side length 8 cm.

$\rightarrow$ 5 copies

$$\theta = \frac{360°}{5} = 72°$$

$$\tan 36° = \frac{4}{h}$$

$$h = \frac{4}{\tan 36°}$$

$$= 5.5 \text{ cm}$$

$$Area = \frac{4 \times 5.5}{2} = 11 \text{ cm}^2$$

$$Area_{pent.} = 10 \times 11 \text{ cm}^2 = 110 \text{ cm}^2$$

Assigned Problems: 3.3 page 131 – 135 # 1-3, 5, 7, 9, 11-14

15, 16

Multi-Step Trig Problems Worksheet

Multi-Step Angle Worksheet

Key Ideas on page 131