LEVELS OF LEARNING

There are 3 different levels of learning I am going to assess you on throughout the quarter. The first is how well you **KNOW** the content (this is necessary to reach a C), the second is how proficiently you **DO** things with the content (this is necessary to reach a B), and finally, how well you **UNDERSTAND** the content (what you need to reach an A).

1. KNOWING

The first objective is that you know the basic of what the course is about. At the end of the course, you should be able to read the table of contents in the textbook and know what each section is talking about. Questions I would ask would start with a "what", "where", "which" such as:

- "What is [the domain of the following function]"
- "Where are [the zeros of the following function]"
- "What are [the multiplicities of the zeros]"
- "Which one is [a cubic polynomial]"
- "What does [the graph of $\sin\frac{2x}{\pi}$ look like]"

2. DOING

The second objective is that you can use the basic course content in more analytic and meaningful way. During the course I want you to be able to use the content to solve problems and build skills in the following categories

- Reasoning
  - Can you estimate reasonably? (akin to solving without technology)
  - Can you use technology to demonstrate relationships/characteristics of functions?
  - Can you use strategic and flexible choices to solve a problem?
- Solving
  - Can you solve problems accurately with technology?
  - Can you use drawings and diagrams to solve a problem?
- Communicating
  - Can you represent your ideas in pictorial and symbolic forms (graphs and equations are accurate)?
  - Can you use correct math vocabulary and write using proper mathematical language and logic?

3. UNDERSTANDING

The final objective is that you leave the class with a deep understanding of the core big ideas of the course so that you are best prepared for calculus when you analyze functions at a deeper level and look to model more complex behaviour.

- **BIG IDEA 1: CHARACTERISTICS OF FUNCTIONS**: During this course we are going to study functions through polynomials, exponential functions, and trig functions. These functions have unique characteristics such as domain, range, symmetry, one-to-oneness, growth and periodic behaviour. To demonstrate a strong understanding, I will assess you with questions such as:
  - Can you model a problem in context?
  - When you use or build your model (or use a given model) to make a prediction, is your solution presented in a way that is easily understood?
  - Can you explain why certain classes of functions have particular characteristics?

For a simple example, consider a glass of water that is brought out of the fridge and is $4°\text{C}$ at 1:00 pm, it is left in a room at $20°\text{C}$ and an hour later it has warmed to $14°\text{C}$. I would ask the following:

1. Determine an equation for the temperature of the water at time t.
   SOLUTION: Let t be the time in hours (pm) and $T(t)$ the temperature of water at that time. Then [work]
   $$T(t) = -16\left(\frac{3}{8}\right)^{t-1} + 20$$

2. Given the equation you found above, what time will the water be $18°\text{C}$?
   SOLUTION: Set $T = 18$ and solve for t [work] and find that
   $$t = 3.12 \rightarrow 3{:}07 \text{ pm}$$
3. Why is the 16 negative and what is the significance of the $" + 20"$ term?
   SOLUTION: We have exponential behaviour that grows toward an asymptote (the room temperature) hence the plus 20 term. We need the 16 to be negative so there is a vertical reflection, otherwise the graph would decrease to the asymptote instead of increasing toward the asymptote.

4. If $f(x) = 4 - x^2$ and $g(x) = \log_2 x$ then what is the domain of $f \circ g$ and $g \circ f$?
   SOLUTION: $f\colon \mathbb{R} \rightarrow (-\infty, 4]$ and $g\colon (0, \infty) \rightarrow \mathbb{R}$
   Therefore $f \circ g\colon (0, \infty) \rightarrow (-\infty, 4]$ and $g \circ f\colon A \rightarrow (0,4] \rightarrow B$ where $A = [-2, 2]\backslash\{0\}$ and $B = (-\infty, 2)$

- **BIG IDEA 2: INVERSES**: We'll start analyzing the inverse very early and we will continue to use it throughout to course as inverses are the foundation to solving equations and they extend to manipulating functions algebraically. To demonstrate a strong understanding, I will assess you with questions such as:
  - Can you manipulate functions algebraically in a fluent manner?
  - Can you solve functions that are not one-to-one fluently?
  - Can you justify the behaviour of a function based on its inverse?

Something for this might look like, given a one-to-one function g, consider the function
$$f(x) = \ln(g(x) - 1)$$

1. Determine the value of a such that $f(a) = 0$ in terms of g or g^{-1}
   SOLUTION: $0 = \ln(g(a) - 1) \Rightarrow e^0 = g(a) - 1 \Rightarrow 1 + 1 = g(a) \Rightarrow g^{-1}(2) = a$

2. If g was not one-to-one such as $g(x) = 3\cos x$, how does that change the prior solution?
   SOLUTION: As before we have $2 = g(a) = 3\cos x$, then $\cos x = \frac{2}{3} \Rightarrow x = \pm\arccos\left(\frac{2}{3}\right) + 2\pi n, n \in \mathbb{Z}$

3. If the range of f is all real numbers, what is the domain of g^{-1}?
   SOLUTION: The domain of g^{-1} is the same as the range of g, since the range of $\ln x$ is all real numbers when $x > 0$ we must have that $g(x) - 1 > 0$, therefore $g(x) > 1$. So, the range of g is all numbers greater than 1 which is the domain of g^{-1}.

- **BIG IDEA 3: TRANSFORMATIONS:** Along with the functions we are going to analyze, we will study how transformations of space and shapes (shifting and stretching) affect the characteristics of functions and relations. To demonstrate a strong understanding, I will assess you with questions such as:
  - Can you explain how space has been transformed after being given an image, function, or map?
  - Can you predict how the inverse relation will be transformed or the inverse of the transformation?
  - Can you connect transformations to function characteristics?
  - Can you relate certain transformations to each other?

This might be presented as, consider the following transformation:

$$(x, y) \mapsto (2x - 3, -y + 1)$$

1. How has space been transformed?
   SOLUTION: In order, we have expanded space horizontally by a factor of 2 and reflected it over the x-axis. Then we shifted it up 1 unit and left 3 units.

2. If a one-to-one function f was transformed by the above transformation into the function g, how would g^{-1} compare to f^{-1}?

   SOLUTION: It is the same transformations but switch horizontal and vertical. Therefore, we take all the points on f^{-1} and in order, expand space vertically by a factor of 2 and reflect it over the y-axis. Then we shift it right 1 unit and down 3 units.
   This is correct as $g(x) = -f\left(\frac{1}{2}(x+3)\right) + 1$ and $g^{-1}(x) = 2f^{-1}\big(-(x-1)\big) - 3$ and $g\big(g^{-1}(x)\big) = x$

3. If $h(x) = \cos x$ was transformed under the transformation described above, what would the period and amplitude of the new function be?
   SOLUTION: The period is 2π and horizontal space has been expanded by 2 so the period is now 4π. The amplitude was 1 and space has not been expanded or compressed so the amplitude is still 1.

4. If $k(x) = \ln x$ was transformed under the transformation described above, why would it be correct to say the graph did NOT get expanded or compressed horizontally and instead just experienced a reflection and a translation left and up?
   SOLUTION: Under the transformation $\ln x$ would become
   $$-\ln\left(\frac{1}{2}(x+3)\right) + 1 = \ln 2 - \ln(x+3) + 1 = -\ln(x+3) + 1.69\ldots$$
   Using log properties the multiplication inside the log becomes addition outside so no expansion, only a different shift up.