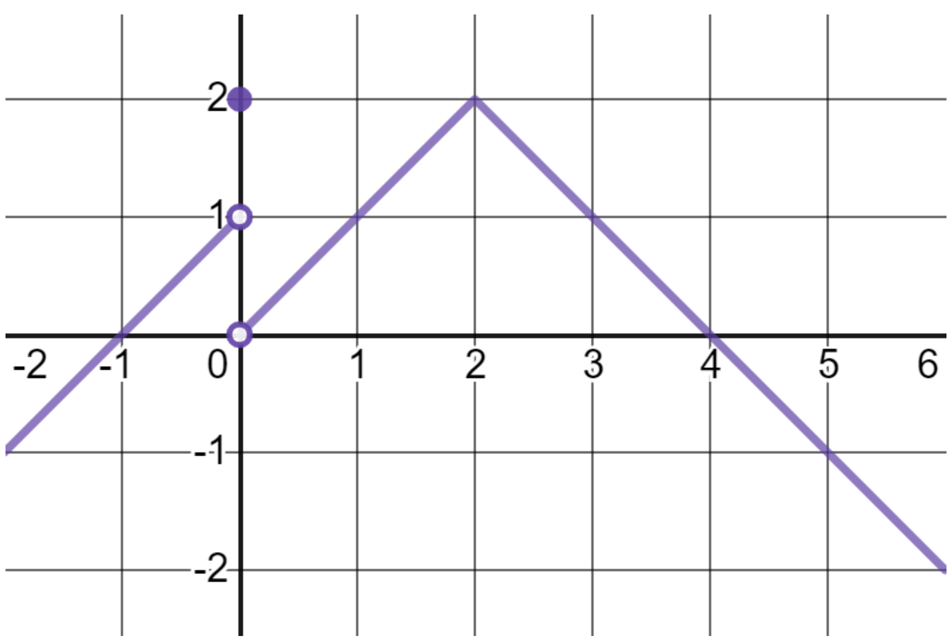
Differentiability and Continuity

Goal:
• Can determine if a function is differentiable based on its graph and using the limit definition. • Understands why differentiability implies continuity.
Terminology:
• Local linearity • Differentiable

Last class we eneded before consider this problem: Given that f is continuous and $f'(1) = 2$ and that $f(1) = 3$, how could we approximate $f(1.05)$?

The idea is that if we don't move far away, then the tangent line looks like the graph: That is the graph is locally linear if it has a derivative.

Consider the following graph of f, we want to find f'

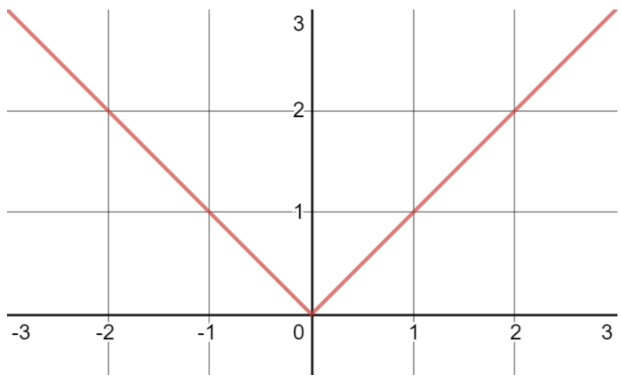


We say a function is **differentiable** at $x = c$ if $f'(c)$ exists. This means the limits below exist

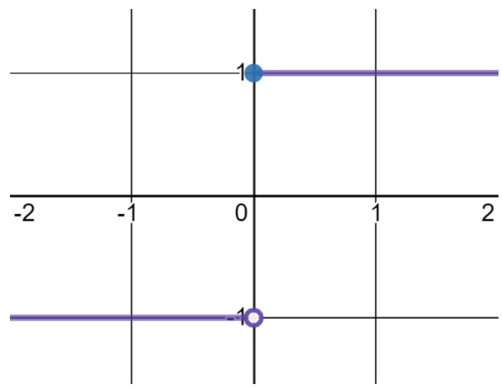
$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

This leads us to a few conditions that would make the function NOT differentiable at a point.

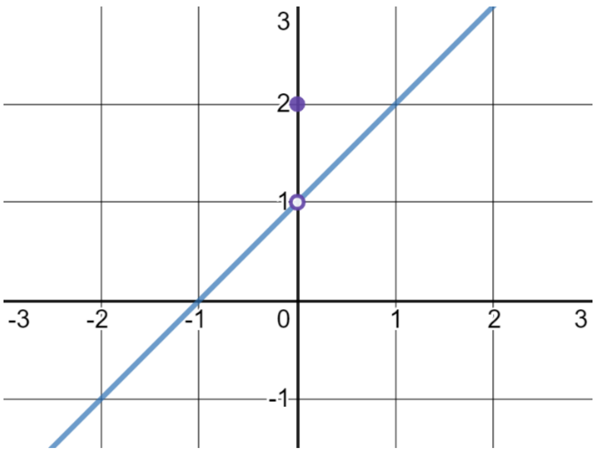
1. Corner, such as $f(x) = |x|$ at $x = 0$



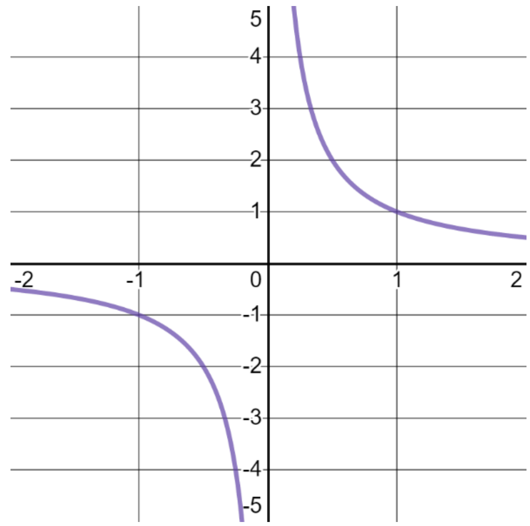
2. Jump, such as $g(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ at $x = 0$



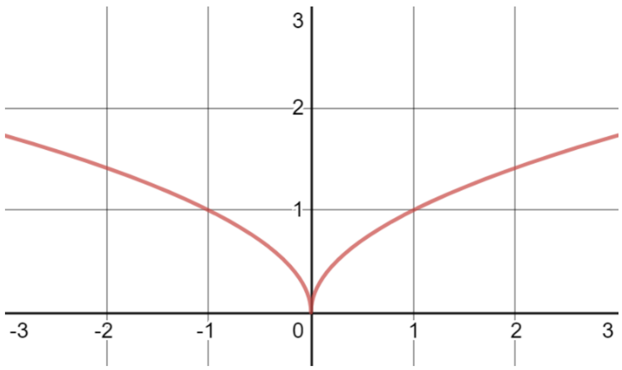
3. Hole, such as
$h(x) = \begin{cases} x+1, & x \neq 0 \\ 2, & x = 0 \end{cases}$ at $x = 0$



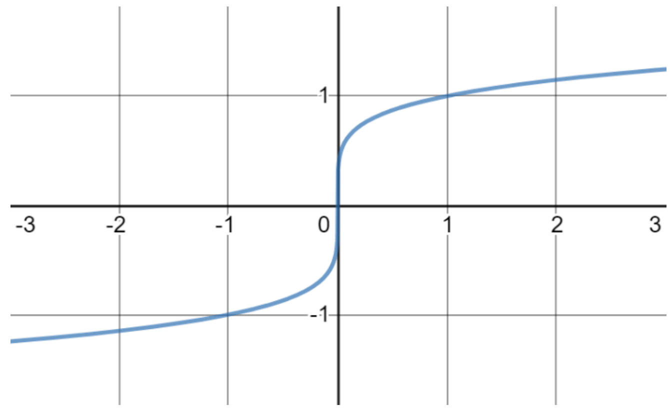
4. Asymptote, such as $k(x) = \frac{1}{x}$ at $x = 0$



5. Cusp, such as $p(x) = \sqrt{|x|}$ at $x = 0$



6. Vertical Tangent, such as $q(x) = \sqrt[5]{x}$ at $x = 0$



A criticial consequence of these cases is that: If a function is differentiable at $x = c$ is MUST be continuous at $x = c$.

Proof:

Practice Problems:	2.4 #1-6 (select), 7, 8, 19-22 3.1 #2-10, 25
Textbook Readings:	Page
Workbook Practice:	Page 91-99
Next Day:	Online learning derivative rules (chapter 3 of the textbook and workbook)