Differentiability and Continuity

Goal: • Can determine if a function is differentiable based on its graph and using the limit definition. • Understands why differentiability implies continuity.
Terminology: • Local linearity • Differentiable $\sin x \approx x$ line

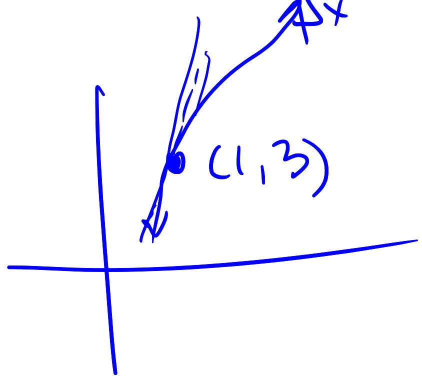
Last class we eneded before consider this problem: Given that f is continuous and $f'(1) = 2$ and that $f(1) = 3$, how could we approximate $f(1.05)$?

slope is $\frac{\Delta y}{\Delta x}$

slope @ $x=1$ is 2

$1.05 \approx 1$

use tangent line



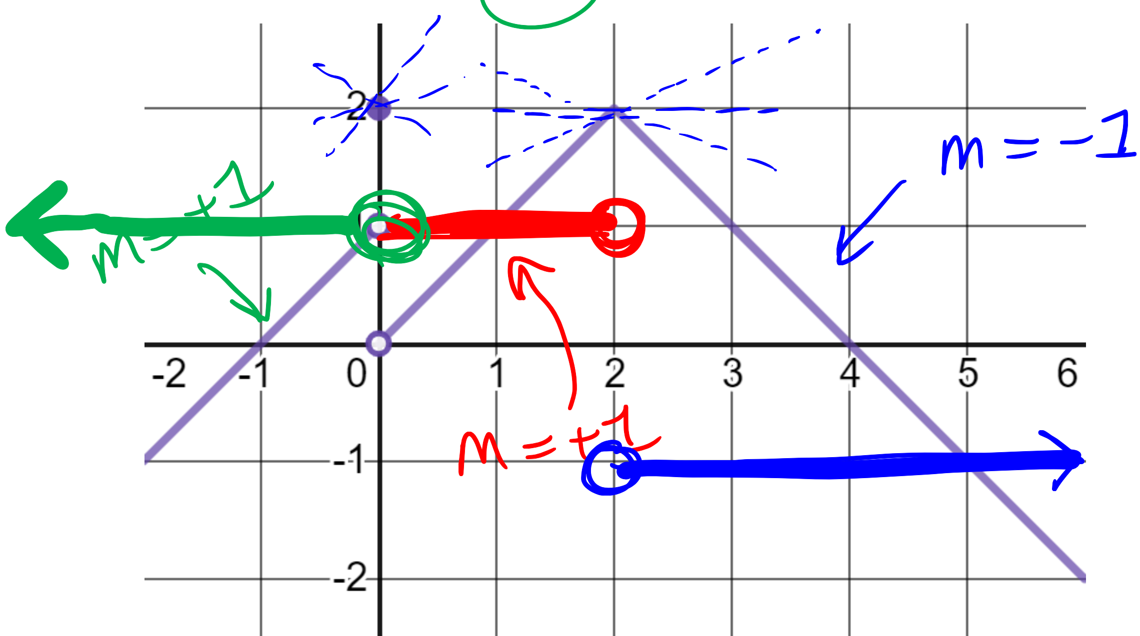
tangent line

$y = 2(x-1)+3 = 3.1$ put in 1.05

The idea is that if we don't move far away, then the tangent line looks like the graph: That is the graph is locally linear if it has a derivative.

Consider the following graph of f, we want to find f' slope

$f'(2) =$ a lot of outputs



$f'(0)$, $f'(2)$ is undefined

★ not be ∞

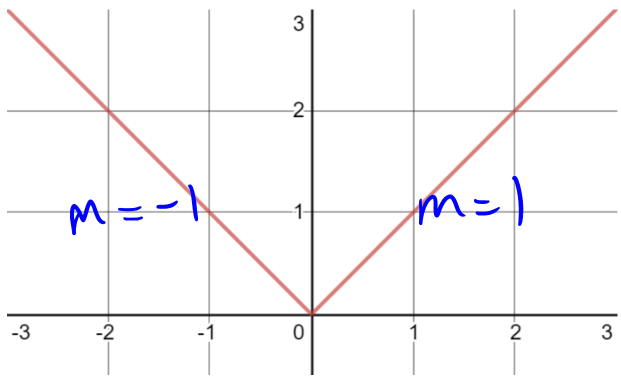
We say a function is **differentiable** at $x = c$ if $f'(c)$ exists. This means the limits below exist

$$\lim_{h\to 0}\frac{f(c+h)-f(c)}{h} = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$

★ both sides

This leads us to a few conditions that would make the function NOT differentiable at a point.

1. Corner, such as $f(x) = |x|$ at $x = 0$



find $f'(0)$ and I want this to be a problem

$$\lim_{x\to 0}\frac{|x|-0}{x-0} = \lim_{x\to 0}\frac{|x|}{x}$$

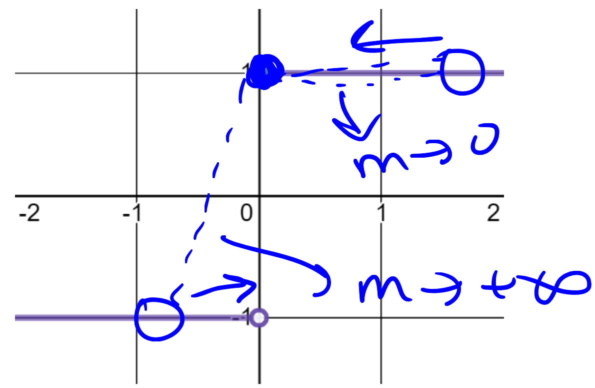
$$\lim_{x\to 0^+}\frac{x}{x} = 1$$

$$\lim_{x\to 0^-}\frac{|x|}{x} = \lim_{x\to 0^-}\frac{-x}{x} = -1$$

$$\lim_{x\to 0^-}\frac{|x|}{x} \neq \lim_{x\to 0^+}\frac{|x|}{x}$$

NOT Differentiable!

2. Jump, such as $g(x) = \begin{cases}\frac{|x|}{x}, & x \neq 0\\ 1, & x = 0\end{cases}$ at $x = 0$

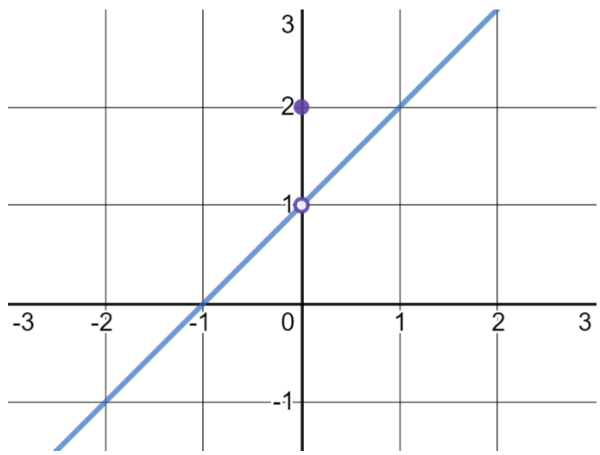


$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$$

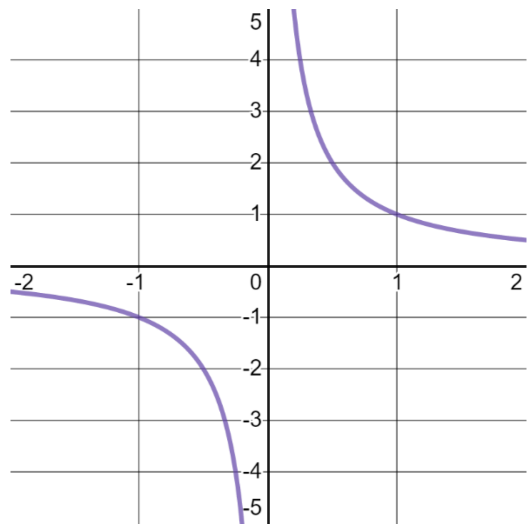
$$\lim_{x\to 0}\frac{f(x)-1}{x}$$

See V2 for complete

3. Hole, such as
$h(x) = \begin{cases} x+1, & x \neq 0 \\ 2, & x = 0 \end{cases}$ at $x = 0$

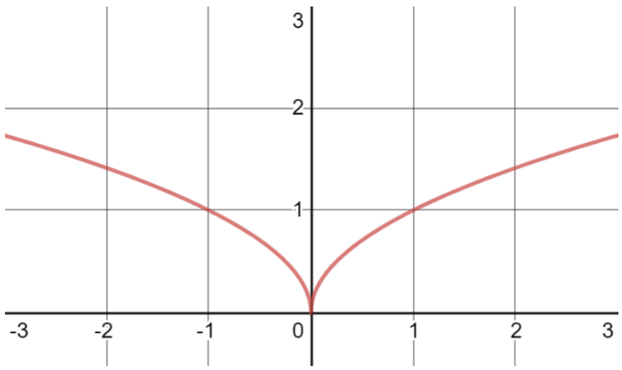


4. Asymptote, such as $k(x) = \frac{1}{x}$ at $x = 0$

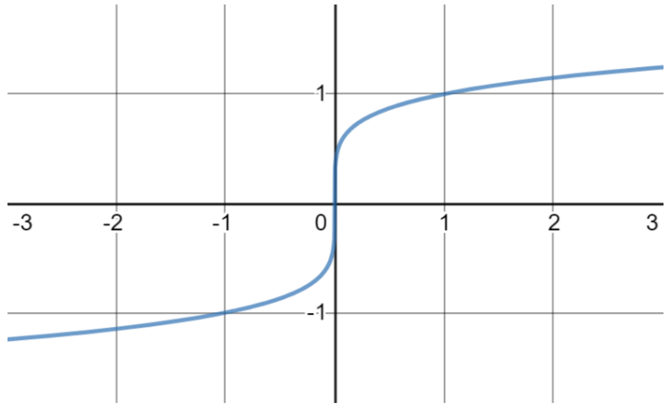


See v2 for complete

5. Cusp, such as $p(x) = \sqrt{|x|}$ at $x = 0$



6. Vertical Tangent, such as $q(x) = \sqrt[5]{x}$ at $x = 0$



see V2 for complete

A criticial consequence of these cases is that: If a function is differentiable at $x = c$ is MUST be continuous at $x = c$.

Proof: If differentiable $\Longrightarrow$ continuous

then/implies

assume f is differentiable @ $x=c$

$$\lim_{x \to c} \frac{f(x)-f(c)}{x-c} \in \mathbb{R}$$

want to show $\boxed{\lim_{x \to c} f(x) = f(c)}$

$\equiv$ show $\lim_{x \to c} \left(f(x) - f(c) \right) = 0$

$$\lim_{x \to c} \left(f(x) - f(c) \right) \frac{(x-c)}{x-c} = \lim_{x \to c} \frac{f(x)-f(c)}{x-c} \cdot (x-c)$$

$$= \lim_{x \to c} \frac{f(x)-f(c)}{x-c} \cdot \lim_{x \to c} (x-c)$$

$$= f'(c) \cdot 0 = 0 \quad \square$$

QED

Differentiable $\equiv$ smooth

Practice Problems: 2.4 #1-6 (select), 7, 8, 19-22 3.1 #2-10, 25	
Textbook Readings: Page	
Workbook Practice: Page 91-99	
Next Day: Online learning derivative rules (chapter 3 of the textbook and workbook)	