Log Laws

KNOW	DO	UNDERSTAND
The basic log laws and the change of base law	Can use the log laws to simplify expressions and evaluate logs of different bases.	*Function Characteristics*: Can determine the domain of a sum of logarithms *Transformations:* Can relate horizontal transformations to vertical transformations using log laws
Vocab & Notation • Change of base		

In grade 9 and 10 you learned about the exponent laws and know that

$$b^x \cdot b^y = b^{x+y} \qquad (b^x)^y = b^{xy} \qquad b^{-1} = \frac{1}{b}$$

$$\underbrace{b \cdot b \cdots b}_{x} \cdot \underbrace{b \cdot b \cdots b}_{y} = \underbrace{b \cdot b \cdots b}_{x+y} \qquad \underbrace{b^x \cdot b^x \cdots b^x}_{y} = b^{\overbrace{x+\cdots+x}^{y}} \qquad \text{By definition}$$

Using function notation if $g(x) = b^x$ then the above laws for exponents give unique and defining characteristics:

$$g(n) \cdot g(m) = g(n+m) \qquad g(n)^m = g(nm)$$

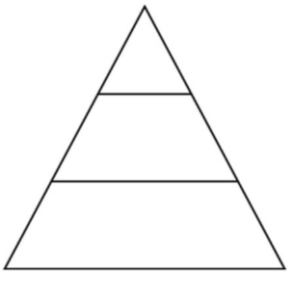
Logarithms, being the inverse of exponentials, have similar laws:

Product Law: $\log_b(m \cdot n) = \log_b m + \log_b n$

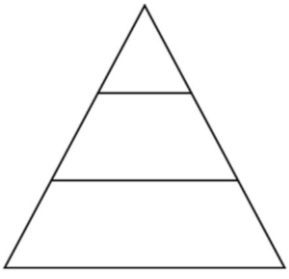
Power Law: $\log_b(x^n) = n \cdot \log_b x$

Quotient Law: $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

Inside → Outside Operations



Outside → Inside Operations



Product Law Proof:

Power Law Proof:

So just like we would simplify exponential functions we can simplify logs.

$e^x \cdot \left(\frac{e^y}{e^z}\right)^2$ $\ln x + 2(\ln y - \ln z)$

Practice: Use log laws to simplify the following into a single log:

$\log 7 - \log 3 + \log 6$ $3\ln 6 - \ln 9 - \ln 8$

$-\frac{1}{2}\ln 81 - 2\ln 3$

$-3\log 2 + (2\log 7 - \log 5)$

$2\log_2(12+3) - (\log_2 5 + \log_2 4)$

$\frac{\ln 10}{\ln 5}$

We need to be careful about the domain when we simplify log functions:

Example: Simplify the following and state the overall domain.

$$f(x) = -\ln(x+2) + 2\ln(1-x) - \ln\big(x(x+1)\big)$$

Practice: Simplify the following and state the overall domain

$$g(x) = \log x + 2\log(x+1) - \log\big((x+1)(x-2)\big)$$

Change of Base Law: $\log_b a = \frac{\log_x a}{\log_x b} =$

Proof:

Practice: Evaluate the following logarithms

$\log_2 20$ $\log_5 1000$

$\log_\pi e$ $\log_{\sqrt{2}} \sqrt{8}$

Practice Problems: 8.3 page 400-403 # 1-3, 5-12, 20, C1, C2