Log Laws

KNOW	**DO**	**UNDERSTAND**
The basic log laws and the change of base law	Can use the log laws to simplify expressions and evaluate logs of different bases.	*Function Characteristics*: Can determine the domain of a sum of logarithms *Transformations:* Can relate horizontal transformations to vertical transformations using log laws

Vocab & Notation

- Change of base

In grade 9 and 10 you learned about the exponent laws and know that

$$b^x \cdot b^y = b^{x+y} \qquad (b^x)^y = b^{xy} \qquad b^{-1} = \frac{1}{b}$$

$$\underbrace{b \cdot b \cdots b}_{x} \cdot \underbrace{b \cdot b \cdots b}_{y} = \underbrace{b \cdot b \cdots b}_{x+y} \qquad \underbrace{b^x \cdot b^x \cdots b^x}_{y} = b^{\overbrace{x+\cdots+x}^{y}} \qquad \text{By definition}$$

Using function notation if $g(x) = b^x$ then the above laws for exponents give unique and defining characteristics:

$$\underbrace{g(n) \cdot g(m)}_{\text{outside product}} = g(\underbrace{n+m}_{\text{inside sum}}) \qquad \underbrace{g(n)^m}_{\text{outside power}} = g(\underbrace{nm}_{\text{inside product}})$$

Logarithms, being the inverse of exponentials, have similar laws:

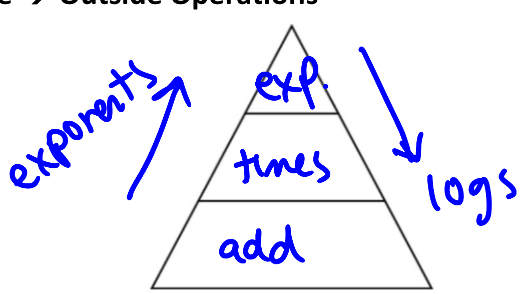
Product Law: $\log_b(\underbrace{m \cdot n}_{\text{inside product}}) = \underbrace{\log_b m + \log_b n}_{\text{outside sum}}$

Power Law: $\log_b(x^n) = \underbrace{n \cdot \log_b x}_{\text{outside product}}$

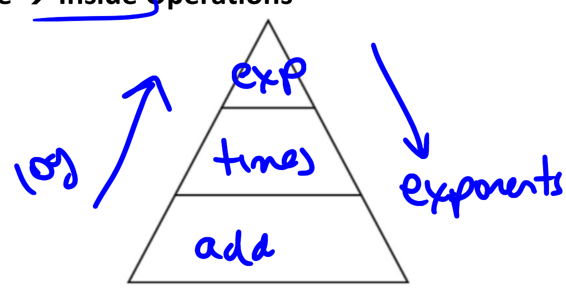
inside power

Quotient Law: $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

Inside → Outside Operations



Outside → Inside Operations



Product Law Proof: want to show $\log X\cdot Y = \log X + \log Y$

let $\log XY = A$ $\log X = B$ $\log Y = C$

$\Rightarrow$ $10^A = XY$ $10^B = X$ $10^C = Y$

$\Rightarrow$ $10^A = 10^B \cdot 10^C = 10^{B+C} \Rightarrow A = B + C$ $\square$

Power Law Proof:

$$\ln(x^n) = \ln(\underbrace{x\cdot x \cdot \ldots x}_{n \text{ times}}) = \underbrace{\ln x + \ln x + \ldots + \ln x}_{n \text{ times}}$$

$$= n \ln x \quad \square$$

So just like we would simplify exponential functions we can simplify logs.

$$e^x \cdot \left(\frac{e^y}{e^z}\right)^2$$

$$e^x \cdot \left(e^{y-z}\right)^2$$

$$e^{x+2y-2z}$$

:)

$$\ln x + 2(\ln y - \ln z)$$

$$\ln x + 2\ln\left(\frac{y}{z}\right)$$

$$\ln x + \ln\left(\frac{y^2}{z^2}\right)$$

$$\ln\left(\frac{x\cdot y^2}{z^2}\right)$$

Practice: Use log laws to simplify the following into a single log:

$$\log 7 - \log 3 + \log 6$$

$$\log\left(\frac{7\cdot 6}{3}\right)$$

$$= \log(14)$$

$$3\ln 6 - \ln 9 - \ln 8$$

$$\ln 6^3 - \ln 9 - \ln 8$$

$$\ln\left(\frac{6^3}{9}\right) - \ln 8$$

$$\ln\left(\frac{6^3}{9\cdot 8}\right) = \ln 3$$

$$-\frac{1}{2}\ln 81 - 2\ln 3$$

$\ln 81^{-1/2} - \ln 3^2$

$\ln \frac{1}{9} - \ln 9 = \ln\left(\frac{1}{81}\right)$

$$-3\log 2 + (2\log 7 - \log 5)$$

$-\log 2^3 + \log 7^2 - \log 5$

$= \log\left(\frac{49}{40}\right)$

$$2\log_2(12+3) - (\log_2 5 + \log_2 4)$$

$\log_2 15^2 - \log_2 20$

$\log_2\left(\frac{45}{4}\right)$

$\neq \ln 10 - \ln 5$

$$\ln\frac{10}{5} \neq \frac{\ln 10}{\ln 5} \neq \ln(10-5)$$

$\frac{1}{\ln 5}\cdot \ln 10$

$= \ln\left(10^{\frac{1}{\ln 5}}\right)$

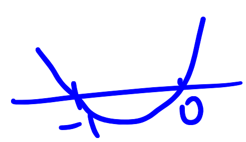
We need to be careful about the domain when we simplify log functions:

Example: Simplify the following and state the overall domain.

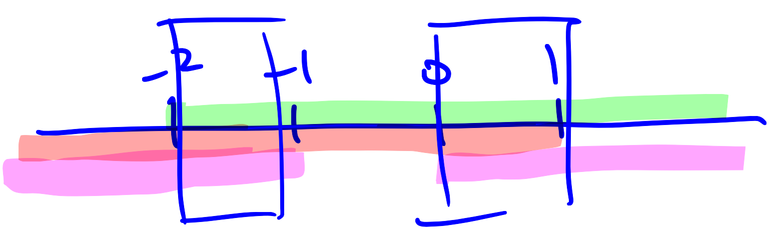
$$f(x) = -\ln(x+2) + 2\ln(1-x) - \ln\big(x(x+1)\big)$$

$f(x) = \ln\left(\frac{(1-x)^2}{(x+2)\,x\,(x+1)}\right)$

$x+2>0$ $\quad 1-x>0$ $\quad x(x+1)>0$



$x>-2$ $\quad x<1$ $\quad x>0$ or $x<-1$



Domain

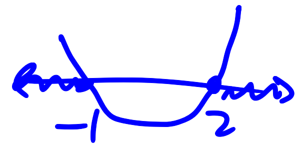
$x \in (-2,-1)\cup(0,1)$

Practice: Simplify the following and state the overall domain

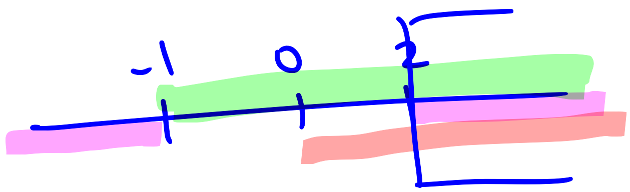
$$g(x) = \log x + 2\log(x+1) - \log((x+1)(x-2))$$

$$g(x) = \log\left(\frac{x \cdot (x+1)^2}{\cancel{(x+1)}(x-2)}\right) = \log\left(\frac{x(x+1)}{x-2}\right)$$

$x > 0$ $x+1 > 0$ $(x-2)(x+1) > 0$



$x > -1$ $x > 2$ or $x < -1$



Domain $x > 2$

Change of Base Law: $\log_b a = \frac{\log_x a}{\log_x b} = \frac{\ln a}{\ln b} = \frac{\log a}{\log b}$

Proof: $\log_b a = Y \Rightarrow a = b^Y \Rightarrow \ln a = \ln b^Y$

$\ln a = Y \ln b$

$\frac{\ln a}{\ln b} = Y = \log_b a$

Practice: Evaluate the following logarithms

$\log_2 20$

$\frac{\ln 20}{\ln 2} = 4.32$

$\log_5 1000$

$\frac{\log 1000}{\log 5} = \frac{3}{\log 5} = 4.29$

$\log_\pi e$

$\frac{\ln e}{\ln \pi} = \frac{1}{\ln \pi} = 0.87$

$\log_{\sqrt{2}} \sqrt{8}$

$\frac{\log 8^{1/2}}{\log 2^{1/2}} = \frac{\log 8}{\log 2} = 3$

Practice Problems: 8.3 page 400-403 # 1-3, 5-12, 20, C1, C2