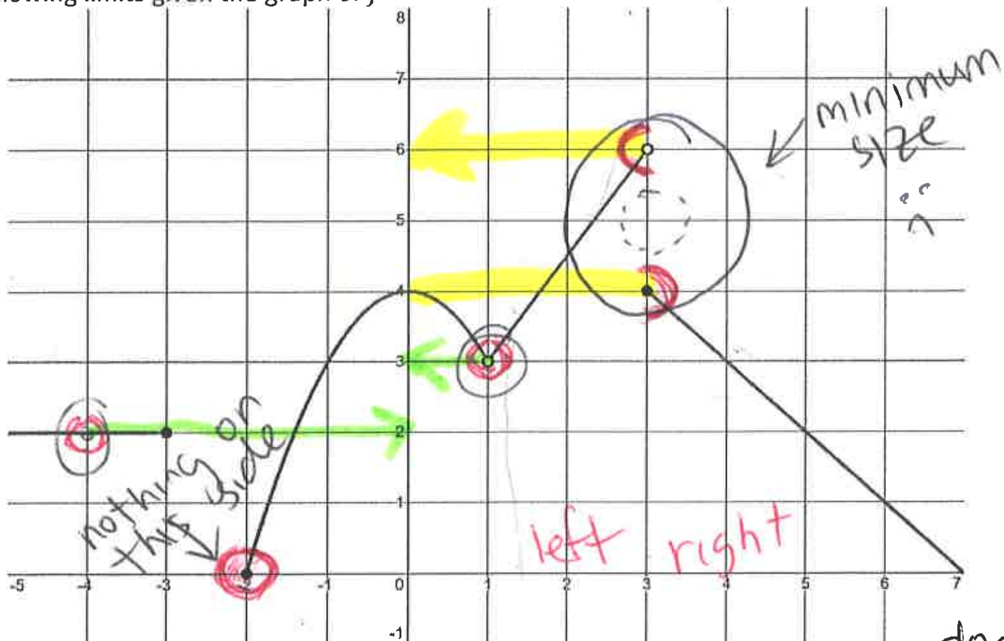
One-Sided Limits

Goal:	
• Can determine the value of the limit using left and right-hand approaches • Can use the definition of continuity alongside piecewise functions	
Terminology:	
• Continuous	

Review

Determine the following limits given the graph of f



1. $\lim_{x \to -4} f(x) =$ 2

2. $\lim_{x \to 1} f(x) =$ 3

3. $\lim_{x \to 3} f(x) =$ DNE does not exist

$\lim_{x \to -4^+} f(x) = 2$

$\lim_{x \to -4^-} f(x) = 2$

$\lim_{x \to 1^+} f(x) = 3$

$\lim_{x \to 1^-} f(x) = 3$

$\lim_{x \to 3^+} f(x) = 4$ right

$\lim_{x \to 3^-} f(x) = 6$ left

Group: What about $\lim_{x \to -2} f(x)$? Note that $f(x)$ is undefined for $x \in (-3, -2)$.

$\sqrt{x}, x \geq 0$

$\lim_{x \to -2^+} f(x) = 0$ $\quad$ $\lim_{x \to -2^-} f(x)$ is undefined

with endpoints $\quad$ $\lim_{x \to -2} f(x) = 0$ because we can only approach -2 from the right

This gives us another definition of the limit as x approaches c.

$$\lim_{x \to c} f(x) = L \overset{\text{equivalent}}{\Longleftrightarrow} \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$$

We are going to use this definition in conjunction with the definition of continuity.

Continuity Definition: A function is continuous at the point c if and only if the following is true.

value $\rightarrow$ $f(c) = \lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x)$ $\leftarrow$ behaviour around value

**Note that this implies two things aside from the obvious that the limit is the value of the function

1. $f(c)$ must be defined.
2. The limit must exist.

Example: Determine when the following function is **discontinuous** (not continuous)

$$f(x) = \begin{cases} 1 + x, & x < 0 \\ \sqrt{1 + x}, & 0 < x < 3 \\ 2, & x \geq 3 \end{cases}$$

what about $x = 0$?

$f(0) =$ undefined

$\Rightarrow$ discontinuous @ $x = 0$

what about $x = 3$? $\Rightarrow$ continuous!

$f(3) = 2$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} 2 = 2$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \sqrt{1+x} = 2$

Practice: Determine when the following function is **discontinuous** (not continuous) and add statements to make it continuous.

$$g(x) = \begin{cases} (x + 2)^2, & x \leq -1 \\ 2x + 3, & -1 < x < 4 \\ x + 8, & x > 4 \end{cases}$$

$g(-1) = (-1+2)^2 = 1$

$\lim_{x \to -1^+} g(x) = \lim_{x \to -1^+} 2x+3 = 1$

$\lim_{x \to -1^-} g(x) = \lim_{x \to -1^-} (x+2)^2 = 1$

$\Rightarrow$ continuous

$g(4) =$ undefined

$\Rightarrow$ discontinuous

$\lim_{x \to 4^+} g(x) = \lim_{x \to 4^+} x+8 = 12$

$\lim_{x \to 4^-} g(x) = \lim_{x \to 4^-} 2x+3 = 11$

Nothing can fix this!

Practice Problems: 1.3: # 1-4*, 5-10 (every other), 11, 12, 14

13, Problems Plus

* are warm up questions – do what you need