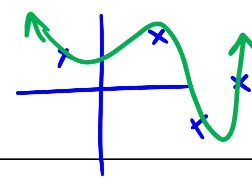
Polynomial Characteristics

KNOW	DO	UNDERSTAND
Identify the end behaviour and intercepts of a polynomial. Identify the zeros from a factored form.	Sketch a graph of a polynomial by hand. Use desmos/geogebra to graph polynomials and reason how their characteristics combine. Determine when a polynomial is positive or negative.	*Function Characteristics*: Can explain why the multiplicity of the zero causes the observed behaviour. Can justify why polynomial behaviour is mostly determined by its degree and only by its components near zero.

Vocab & Notation

- End behaviour
- Multiplicity
- Intervals of positivity



Definition: Recall that a polynomial is anything of the form:

leading term

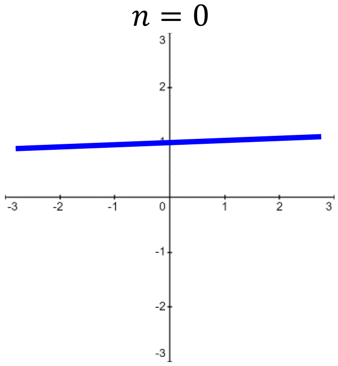
$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x^1 + a_0 x^0 \quad , a_n \neq 0$$

degree coefficient

Where $a_0, \ldots, a_n \in \mathbb{R}$, and n is a whole number.

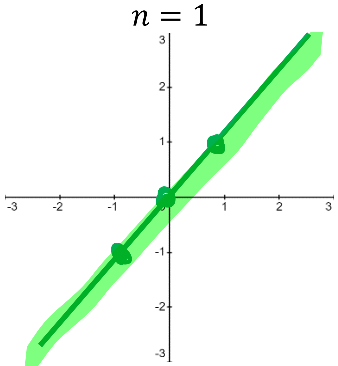
We are going to mostly focus on when $n \leq 5$ but we should be able to predict behaviour for any degree.

$n = 0$



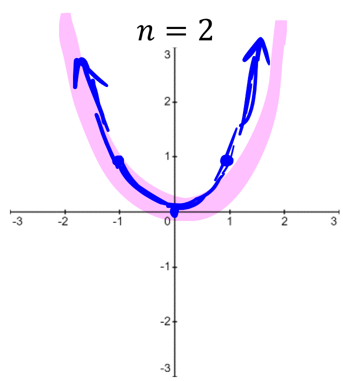
$P_0(x) = 1$

$n = 1$



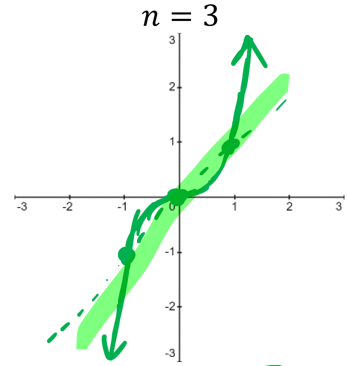
$P_1(x) = x$

$n = 2$



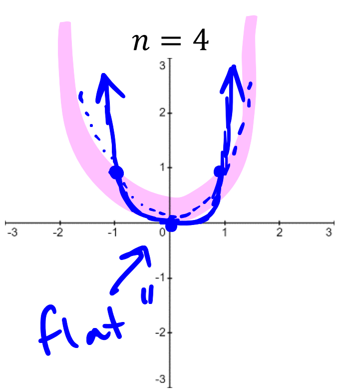
$P_2(x) = x^2$

$n = 3$



$P_3(x) = x^3$

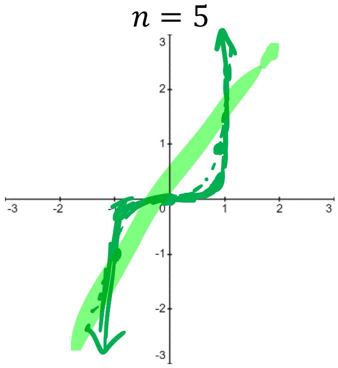
$n = 4$



"flat"

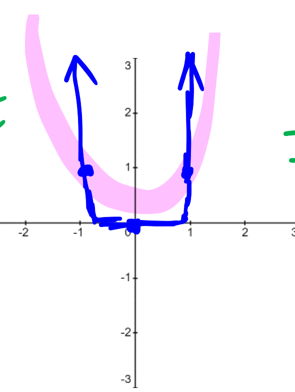
$P_4(x) = x^4$

$n = 5$



$P_5(x) = x^5$

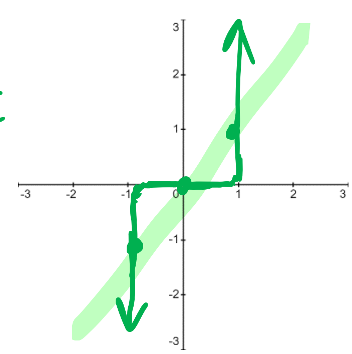
II



I

n is even

x^{100}



n is odd

x^{101}

Note that the y-intercept is always at zero, if we want to change it we need to apply a vertical shift

$p(x) + d$

If we want the polynomial to move in a different direction we need to reflect over x-axis

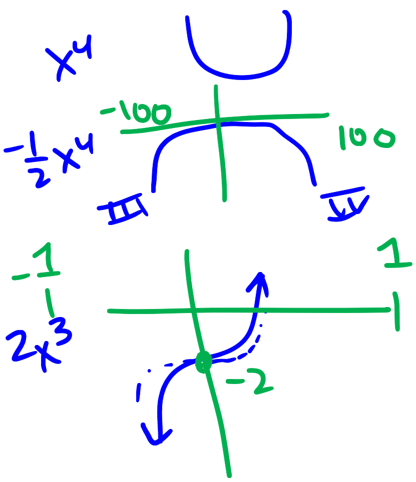
$-p(x)$

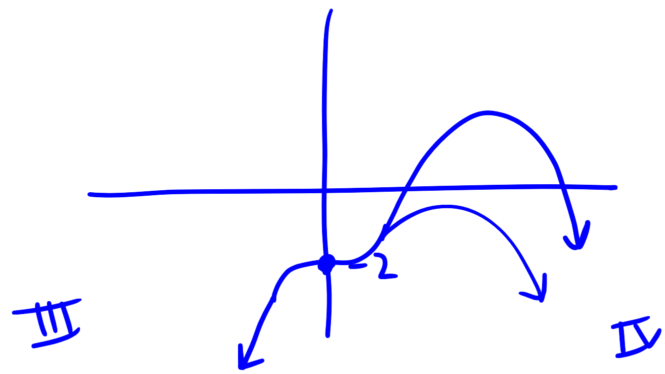
Since a general polynomial is just the sum of these basic components, I like to think of the terms as ingredients and the strength/amount of the ingredient depends on the degree and coeffient.

Example: Graph the polynomial:

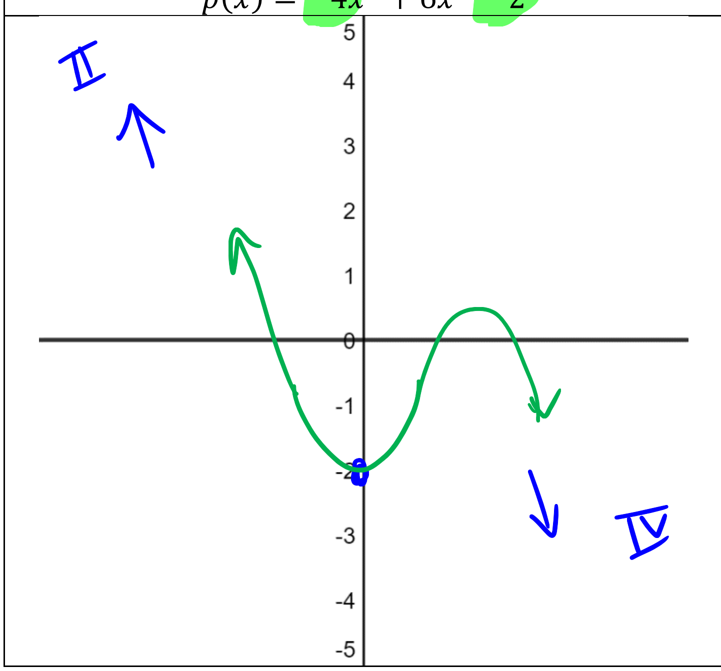
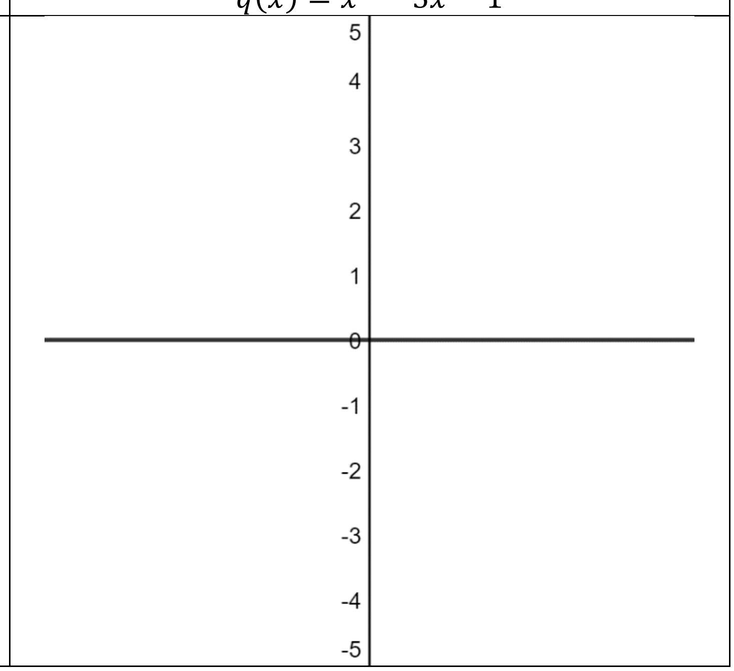
$$p(x) = -\frac{1}{2}x^4 + 2x^3 - 2$$

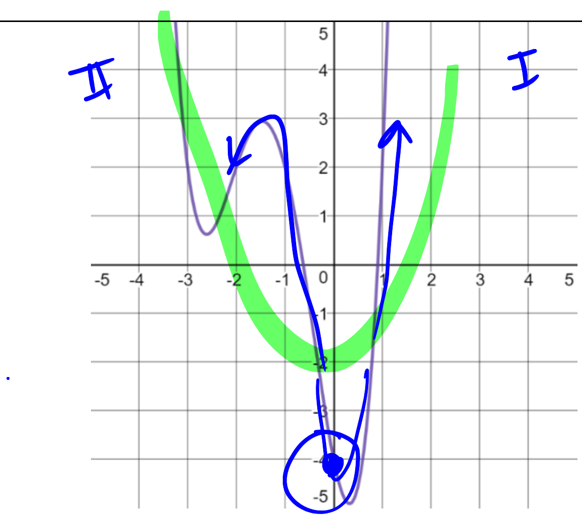
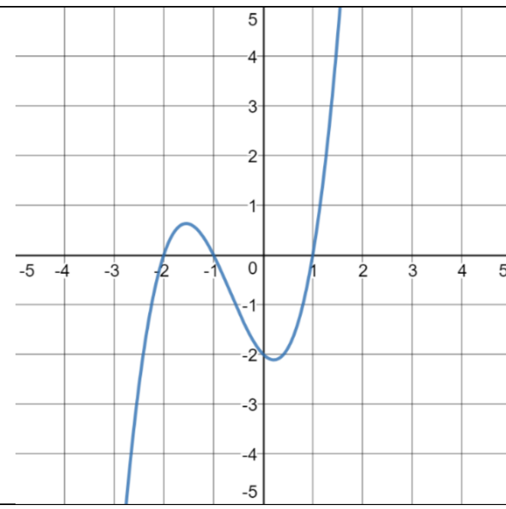
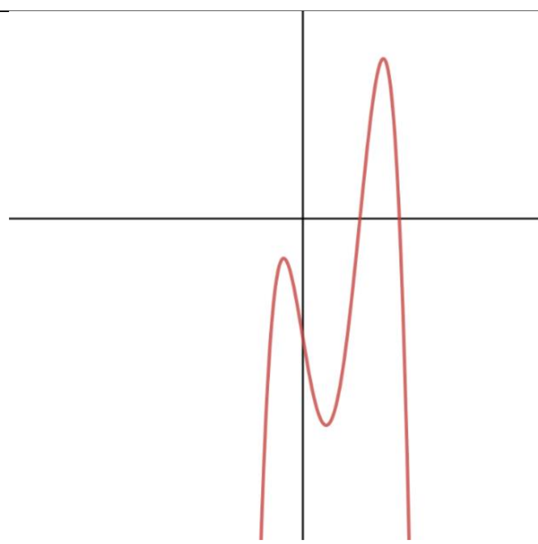
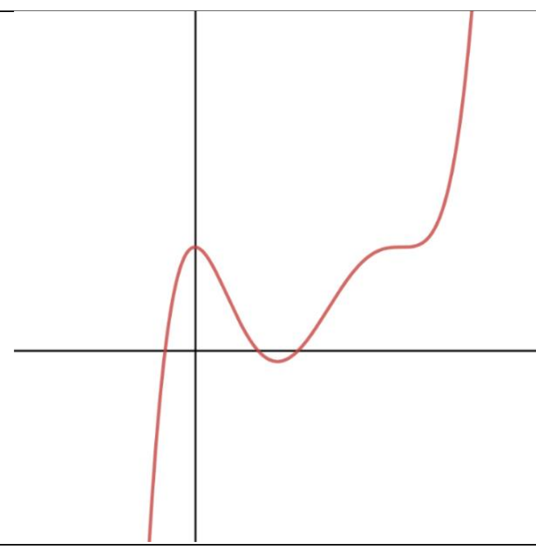
leading term y-int





Practice: Sketch the polynomials and build possible equations for the graphs

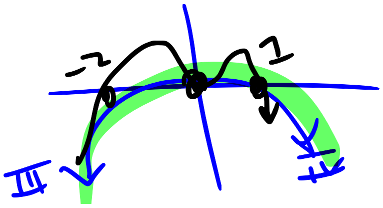
$p(x) = -4x^3 + 6x^2 - 2$	$q(x) = x^4 - 3x - 1$
	

$P(x) = x^3 - 2x^2 - 3x + 1$	$Q(x) = -x^4 + 3x^3 + x$
	
$0.5x^4 + 3x^3 + 2x^2 - 6x - 4$	
	
	

We are going to learn tomorrow how to factor polynomials, but we can analyze the zeros right now. We know that we can factor quadratics and similarly we can factor higher degree polynomials.

Example: $p(x) = -x^2(x-1)(x+2)^3$ $= -x \cdot x \cdot (x-1) \cdot (x+2)(x+2)(x+2)$

$= 0$ $\quad = -x^6 + \dots + -8x^2 + 0$



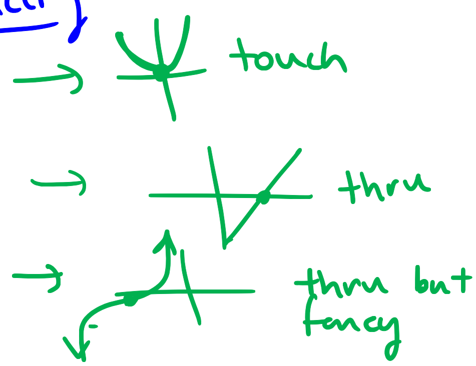
$\Rightarrow -x^2 = 0$ OR $(x-1) = 0$ OR $(x+2)^3 = 0$

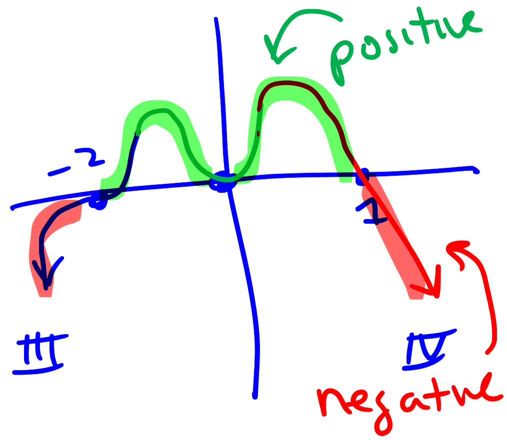
$x = 0$ multiplicity 2 $\quad x = 1$ mult 1 $\quad x = -2$ mult 3

There are two ideas to help us graph this accurately:

Idea 1: Use the end behaviour and the parity *(multiplicity)* **of the zeros**

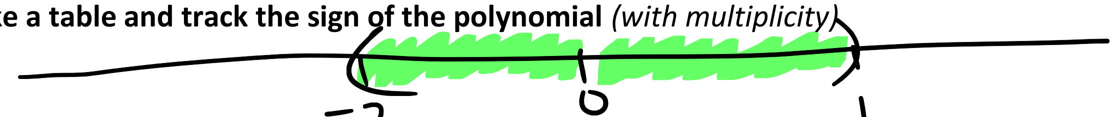
zeros	multiplicity
$x = 0$	2
$x = 1$	1
$x = -2$	3





$p(x) > 0$ when $x \in (-2, 1), x \neq 0$ // $-2 < x < 0$ or $0 < x < 1$

Idea 2: Make a table and track the sign of the polynomial *(with multiplicity)*

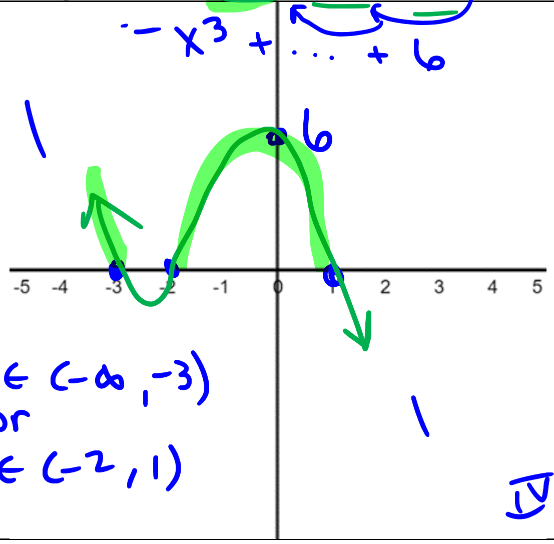


		-2		0		1	
$-x^2$	$-$		$-$	0	$-$		$-$
$x-1$	$-$		$-$		$-$	0	$+$
$(x+2)^3$	$-$	0	$+$		$+$		$+$
	$-$	0	$+$	0	$+$	0	$-$

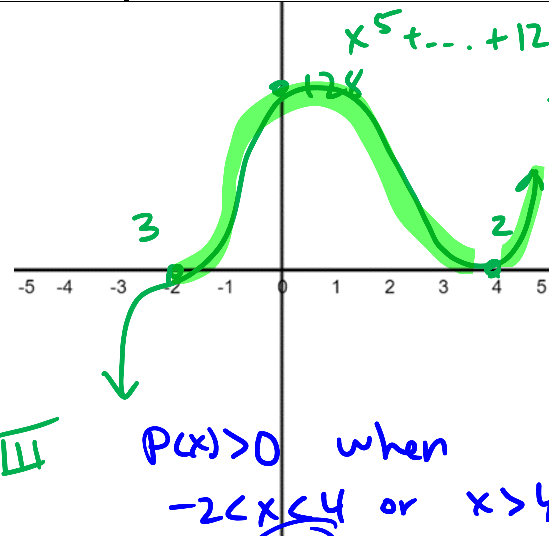
$p(x) > 0$ when $-2 < x < 0$ or $0 < x < 1$

Sketch the polynomials and state the intervals they are positive.

$p(x) = -(x-1)(x+2)(x+3)$	$q(x) = (x+2)^3(x-4)^2$
$-x^3 + \dots + 6$	$x^5 + \dots + 128$



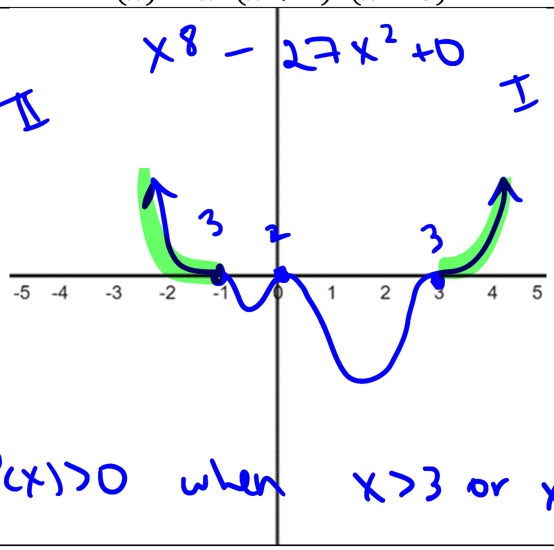
$x \in (-\infty, -3)$ or $x \in (-2, 1)$



$P(x) > 0$ when $-2 < x < 4$ or $x > 4$

$P(x) = x^2(x+1)^3(x-3)^3$

$x^8 - 27x^2 + 0$

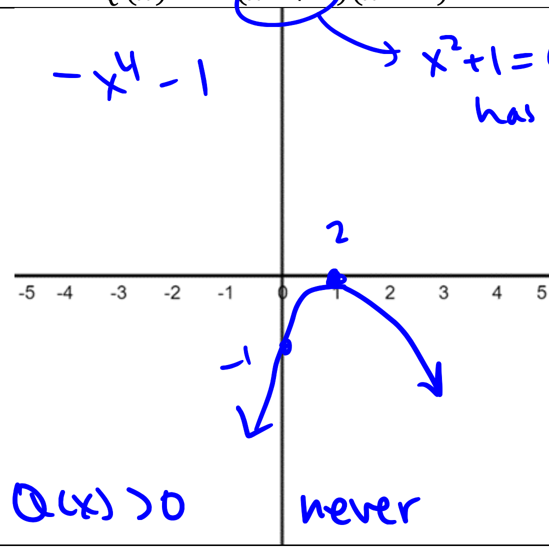


$P(x) > 0$ when $x > 3$ or $x < -1$

$Q(x) = -(x^2+1)(x-1)^2$

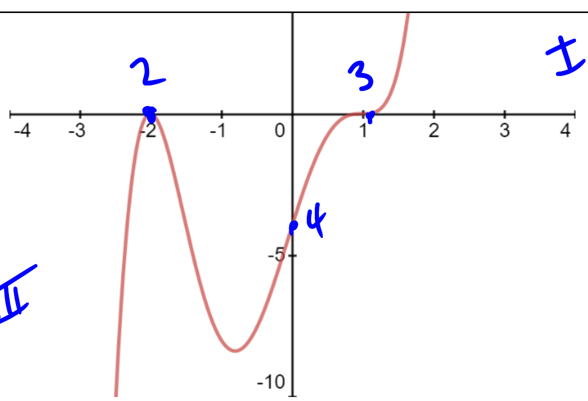
$-x^4 - 1$

$x^2 + 1 = 0$ has no soln.

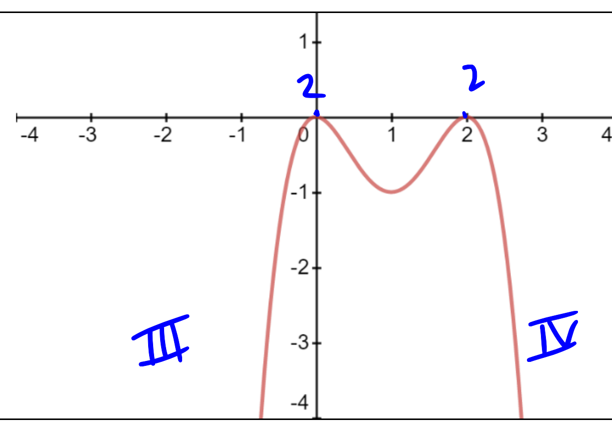


$Q(x) > 0$ never

$(x+2)^2(x-1)^3$



$-x^2(x-2)^2$



Practice Problems: 3.1 page 114 – 116 # 1-5, 12, 13, C1, C2, C3
3.4 page 147 – 152 # 1-6, 10, 11, 14, 20, 22, C1, C2, C3 (#7-9 are good, but leave for the weekend)