Polynomial Characteristics

KNOW	DO	UNDERSTAND
Identify the end behaviour and intercepts of a polynomial. Identify the zeros from a factored form.	Sketch a graph of a polynomial by hand. Use demos/geogebra to graph polynomials and reason how their characteristics combine. Determine the interval when a polynomial is positive or negative.	*Function Characteristics*: Can explain why the multiplicity of the zero causes the observed behaviour. Can justify why polynomial behaviour is mostly determined by its degree and only by its components near zero.

Vocab & Notation

- End behaviour
- Multiplicity
- Intervals of positivity

Definition: Recall that a polynomial is anything of the form:

$p: \mathbb{R} \to$

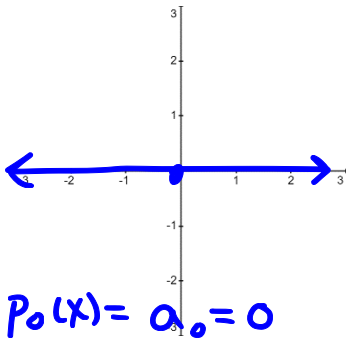
coefficients are $\mathbb{R}$

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \quad , x \in \mathbb{R}$$

Where $a_0, \ldots, a_n \in \mathbb{R}$, and $n \in \mathbb{Z}, n \geq 0$. degree

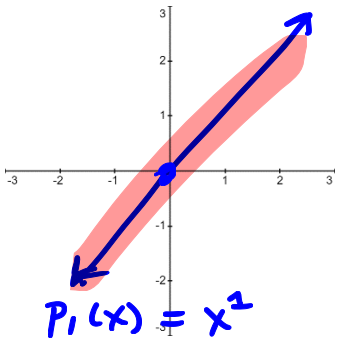
We are going to mostly focus on when $n \leq 5$ but we should be able to predict behaviour for any degree.

$n = 0$



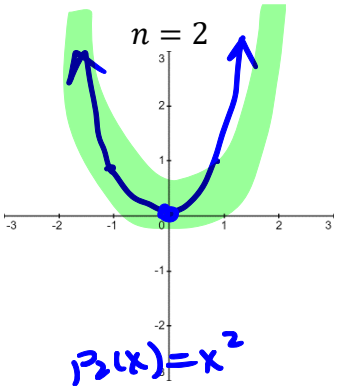
$P_0(x) = a_0 = 0$

$n = 1$



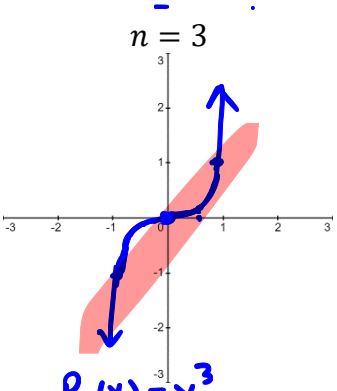
$P_1(x) = x^1$

$n = 2$



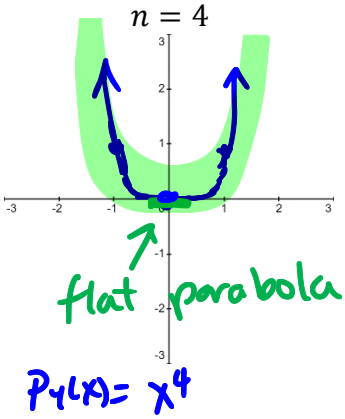
$P_2(x) = x^2$

$n = 3$



$P_3(x) = x^3$

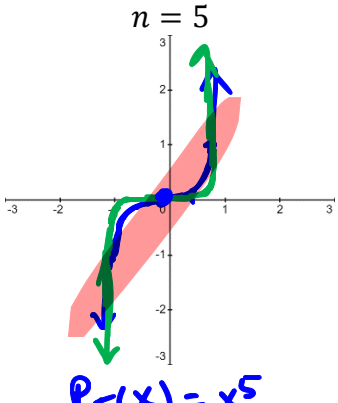
$n = 4$



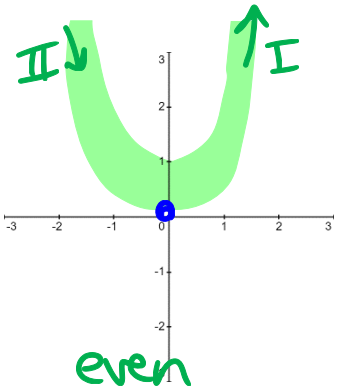
flat parabola

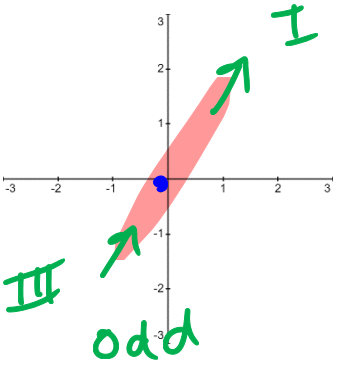
$P_4(x) = x^4$

$n = 5$



$P_5(x) = x^5$





Note that the y-intercept is always at zero, if we want to change it we need to translate up/down

$q(x) = p(x) + d$ ← constant term

If we want the polynomial to move in a different direction we need to reflect over x-axis

$q(x) = -p(x)$ ⇒ changes quadrants moved thru

Since a general polynomial is just the sum of these basic components, I like to think of the terms as ingredients and the strength/amount of the ingredient depends on the degree (and the coefficient to a lesser amount).

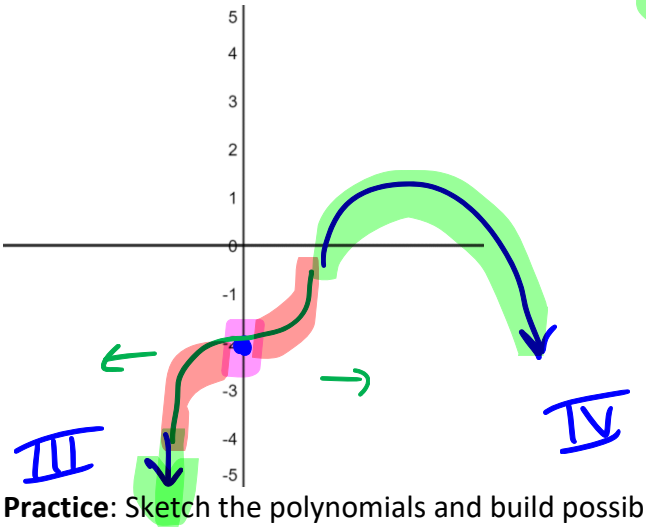
Example: Graph the polynomial:

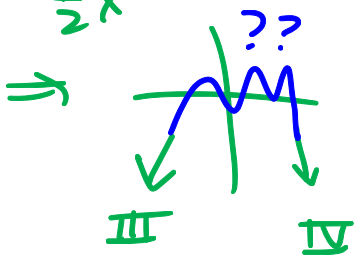
$$p(x) = -\frac{1}{2}x^4 + 2x^3 - 2$$

← leading term is dominant

↑ y-int.

$-\frac{1}{2}x^4$ ⇒ ??





Practice: Sketch the polynomials and build possible equations for the graphs

$p(x) = -4x^3 + 6x^2 - 2$	$q(x) = x^4 - 3x - 1$
$P(x) = x^3 - 2x^2 - 3x + 1$	$Q(x) = -x^4 + 3x^3 + x$

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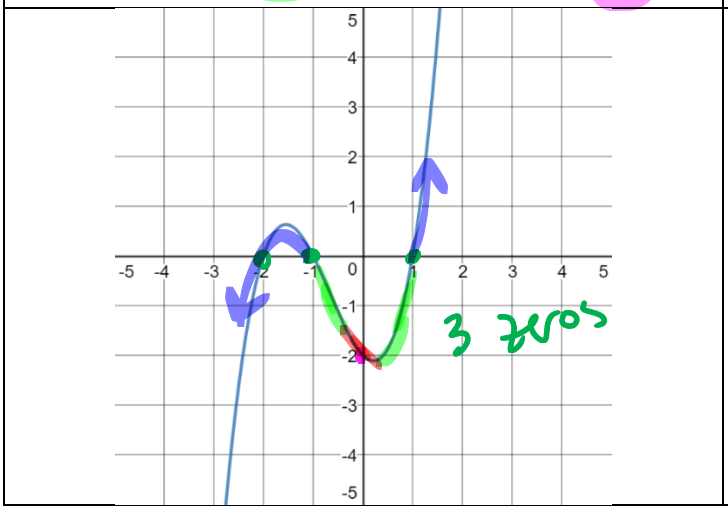
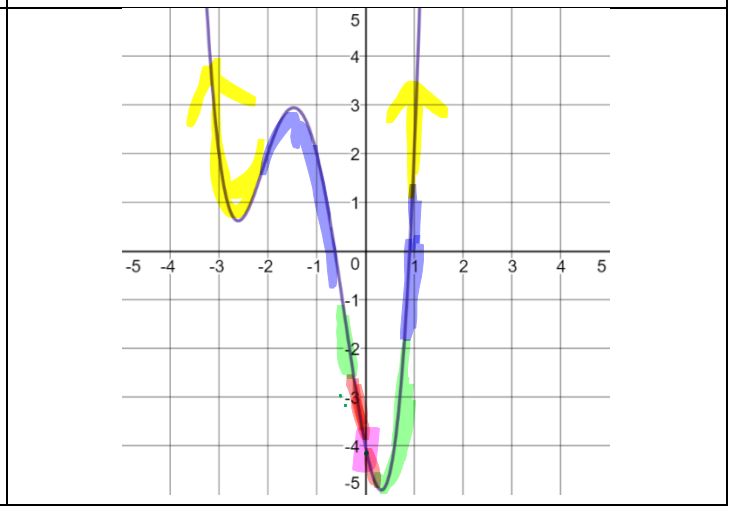
added after to make x^2 more prominent

added after to make parabola longer + see the x^3

$x^3 + 2x^2 - x - 2$



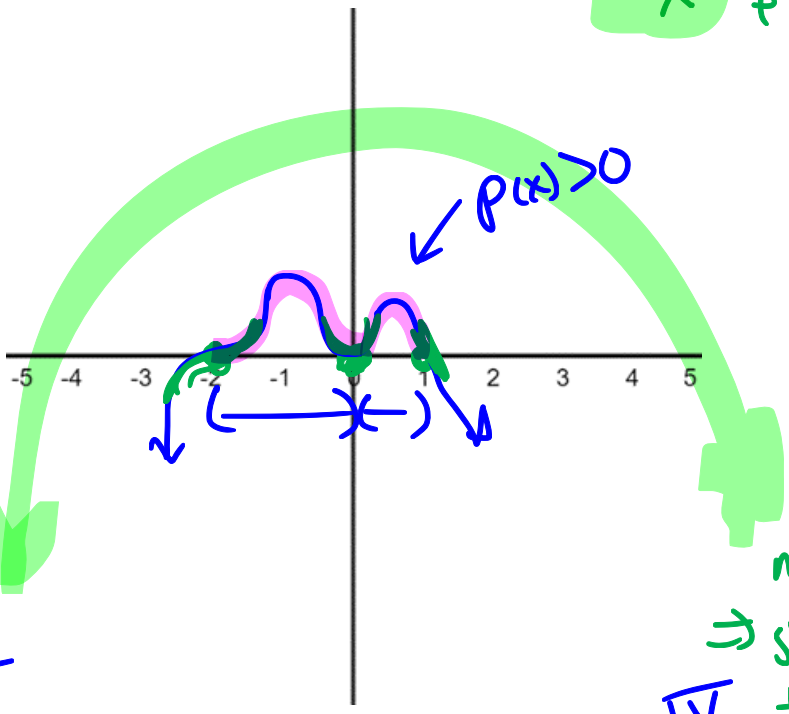
$x^4 + 5.2x^3 + 6x^2 - 4x - 4$



We are going to learn tomorrow how to factor polynomials, but we can analyze the zeros right now. We know that we can factor quadratics and similarly we can factor higher degree polynomials.

Example: $p(x) = -x^2(x-1)(x+2)^3$ $(x+2)(x+2)(x+2)$ $= -x^6 + \dots + 8x^2$

zeros	mult.
$x = 0$	2
$x = 1$	1
$x = -2$	3



$p(x) = 0$ when $x = 0$, $x = 1$, $x = -2$

$x^2 = 0$ $\quad x - 1 = 0$ $\quad (x+2)^3 = 0$

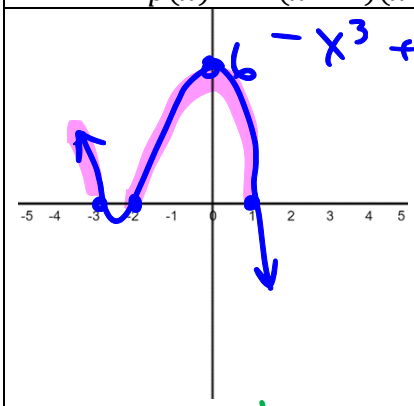
multiplicity $\Rightarrow$ shape near that zero

III IV

$p(x) > 0$ when $x \in (-2, 0) \cup (0, 1)$

Sketch the polynomials or determine their equation and state the intervals they are positive.

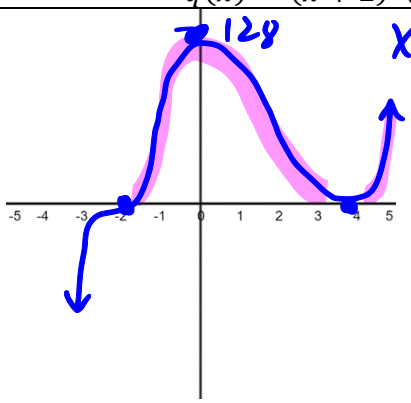
$p(x) = -(x-1)(x+2)(x+3)$



$-x^3 + \ldots + 6$

$p(x) > 0$ when $x \in (-\infty, -3) \cup (-2, 1)$

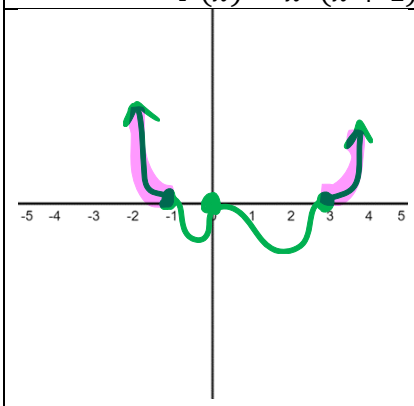
$q(x) = (x+2)^3(x-4)^2$



$x^5 + \ldots + 128$

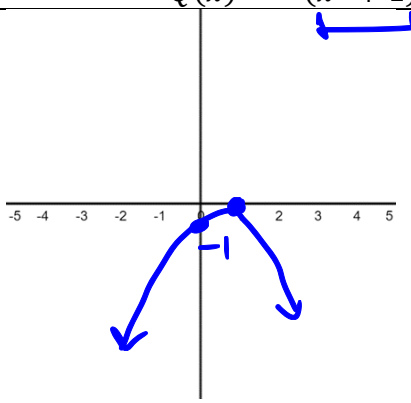
$q(x) > 0$ when $x \in (-2, \infty), x \neq 4$

$P(x) = x^2(x+1)^3(x-3)^3$



$P(x) > 0$ when $x > 3$ or $x < -1$

$Q(x) = -(x^2+1)(x-1)^2$



$x^2 + 1 = 0$

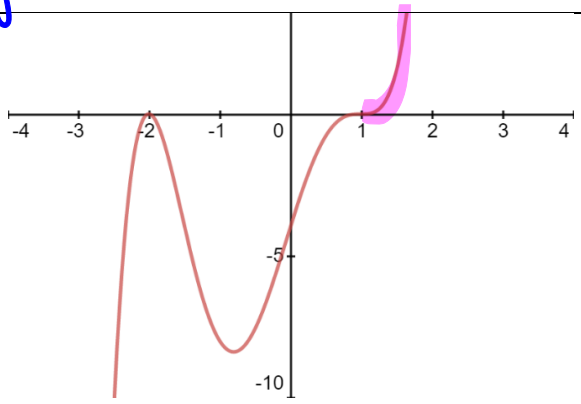
$x^2 = -1$

$x = \pm\sqrt{-1} \notin \mathbb{R}$

$Q(x) > 0$ Never

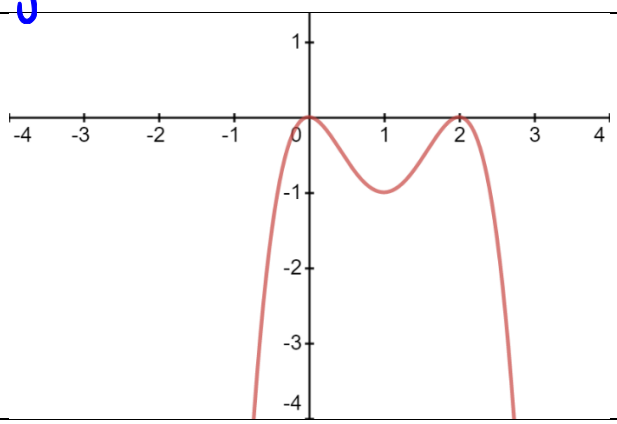
$y = (x+2)^2(x-1)^3$

$y > 0$ when $x > 1$



$y = -x^2(x-2)^2$

$y > 0$ never



Practice Problems: 3.1 page 114 – 116 # 1-5, 12, 13, C1, C2, C3

3.4 page 147 – 152 # 1-6, 10, 11, 14, 20, 22, C1, C2, C3 (#7-9 are good, but leave for after tomorrow)