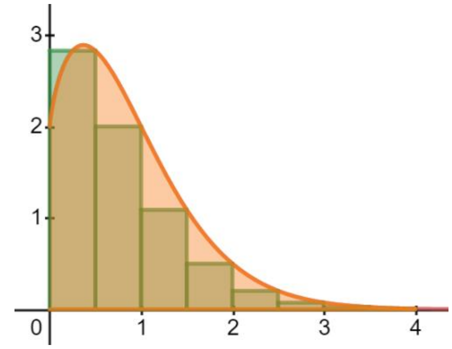
Area Approximation Solutions

a. Using 8 subintervals we have $x_0 = 0, x_1 = 0.5, \dots, x_7 = 3.5, x_8 = 4$

$$\Delta x = \frac{4}{8} = \frac{1}{2}$$

$$\text{Area} = \frac{1}{2}\sum_{k=1}^{8} 2 \cdot x_k^{-x_k} = \sum_{k=1}^{8} x_k^{-x_k}$$
$$= 0.5^{-0.5} + 1^{-1} + \cdots + 3.5^{-3.5} + 4^{-4}$$
$$= 3.363 \dots$$

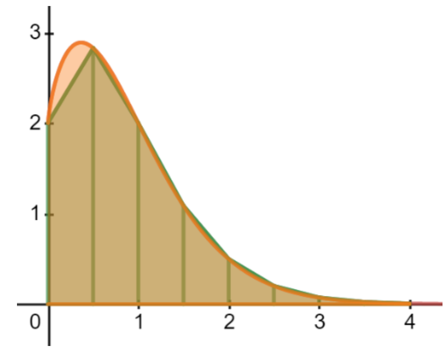
This will be an underestimation since f is decreasing (mostly)



b. Using trapezoids, we have

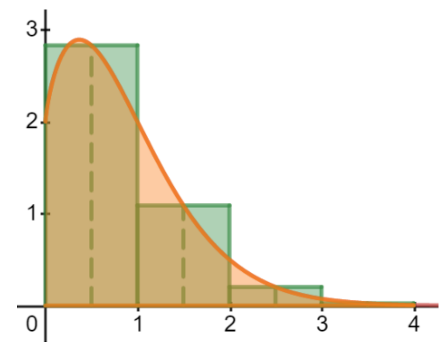
$$\text{Area} = \frac{1}{2}\left[\sum_{k=0}^{7} x_k^{-x_k} + \sum_{k=1}^{8} x_k^{-x_k}\right]$$
$$= \frac{1}{2}\big(f(x_0) + 2f(x_1) + \cdots + 2f(x_7) + f(x_8)\big)$$
$$= 0^0 + 2 \cdot 0.5^{-0.5} + \cdots + 2 \cdot 3.5^{-3.5} + 4^{-4}$$
$$= 3.861 \dots$$

Note that 0^0 for us is 1 as $f(0) = 2$. Overall, this is likely very close to the true value. It will underestimate around $[0,1]$ as the function is concave down and will overestimate the rest of the time. But the concavity is much greater on $[0,1]$. That it underestimates much more than we overestimate.



c. Using midpoint and 4 subintervals we have $\{x_1 = 0.5, x_2 = 1.5, x_3 = 2.5, x_4 = 3.5\}$ and $\Delta x = 1$

$$\text{Area} = \sum_{k=1}^{4} 2 \cdot x_k^{-x_k}$$
$$= 2(0.5^{-0.5} + 1.5^{-1.5} + 2.5^{-2.5} + 3.5^{-3.5})$$
$$= 4.144 \dots$$



d. With $n = 100$ we have

Type	Sigma Notation	Value
Right Endpoint	$\frac{4}{100}\sum_{k=1}^{100} 2 \cdot x_k^{-x_k}, \quad x_k = \frac{4k}{100}$	3.9465 ...
Left Endpoint	$\frac{4}{100}\sum_{k=0}^{99} 2 \cdot x_k^{-x_k}, \quad x_k = \frac{4k}{100}$	4.0261 ...
Midpoint	$\frac{4}{100}\sum_{k=0}^{99} 2 \cdot x_k^{-x_k}, \quad x_k = \frac{4k}{100} + \frac{2}{100}$	3.9883 ...
Trapezoid	$\frac{4}{100}\left(1 + \left(\sum_{k=1}^{99} 2 \cdot x_k^{-x_k}\right) + 4^{-4}\right), \quad x_k = \frac{4k}{100}$	3.9828 ...