Fundamental Theorem Solutions

1.
   a. We see that $g'(x) = f(x)$ so to find the extrema we need to look for when $f(x)$ changes sign and the endpoints.
   We have that f changes sign at $x = 0.5$ (local minimum) and at $x = 4$ (local max)

$$g(-1) = \int_{-1}^{-1} f(t)dt = 0$$

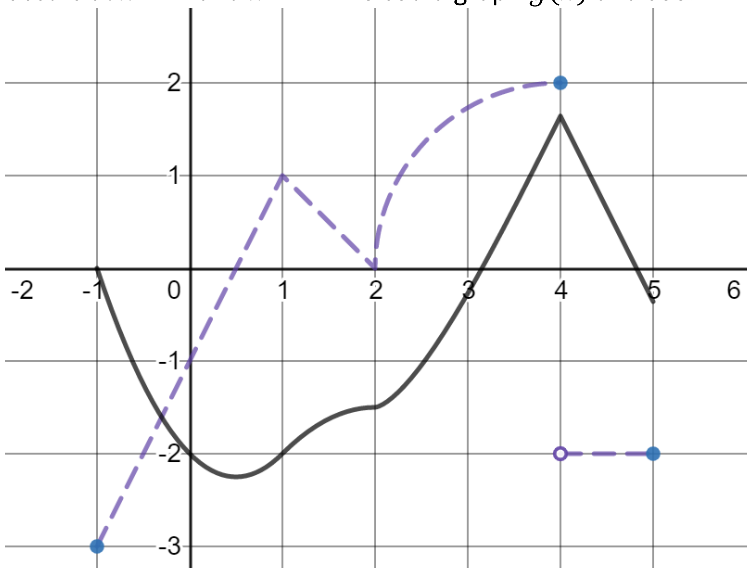
$$g(0.5) = \int_{-1}^{0.5} f(t)dt = -2.25, \qquad \text{(triangle)}$$

$$g(4) = \int_{-1}^{4} f(t)dt = -2 + 0.5 + \pi \approx 1.64, \qquad \text{(trapezoid + tri. +circle)}$$

$$g(5) = \int_{-1}^{5} f(t)dt = \pi - 1.5 - 2 \approx -0.36, \qquad \text{(trap. +tri. +circ. +tri.)}$$

   Thus, we see that $g(4) = \pi - 1.5$ is the absolute max and $g(0.5) = -2.25$ is the absolute min.

   b. To find the inflection points we have that $g''(x) = f'(x)$ so we look for when $f'(x)$ changes sign. This occurs at $x = 1$ and $x = 2$. We could graph $g(x)$ and see:



2.
   a. We need to find an antiderivative to $f(x) = e^x + \frac{3}{x-2} - \frac{1}{(x+1)^3}$ which is

$$F(x) = e^x + 3\ln|x-2| + \frac{1}{2(x+1)^2}$$

$$\Rightarrow \int_0^1 e^x + \frac{3}{x-2} - \frac{1}{(x+1)^3}\, dx = e^x + 3\ln|x-2| + \frac{1}{2(x+1)^2}\Big|_0^1$$

$$= e + 3\ln 1 + \frac{1}{8} - \left(e^0 + 3\ln 2 + \frac{1}{2}\right)$$

$$= e - \frac{11}{8} - 3\ln 2$$

   b. We need to use long division before finding an antiderivative.

$$f(x) = \frac{2x^2 - x + 4}{1 - 2x} = -x + \frac{4}{1-2x}$$

$$\Rightarrow \int -x + \frac{4}{1-2x}\, dx = -\frac{x^2}{2} - 2\ln|1-2x| + C$$