The Radian and Angles

KNOW	DO	UNDERSTAND
How to recognize angles in radians. What quadrant an angle is in.	Determine coterminal angles to a given angle. Determine the trig ratios of an angle. How to use the unit circle and special triangles	*Function Characteristics*: Why the (x, y) coordinate on the unit circle is $(\cos\theta, \sin\theta)$

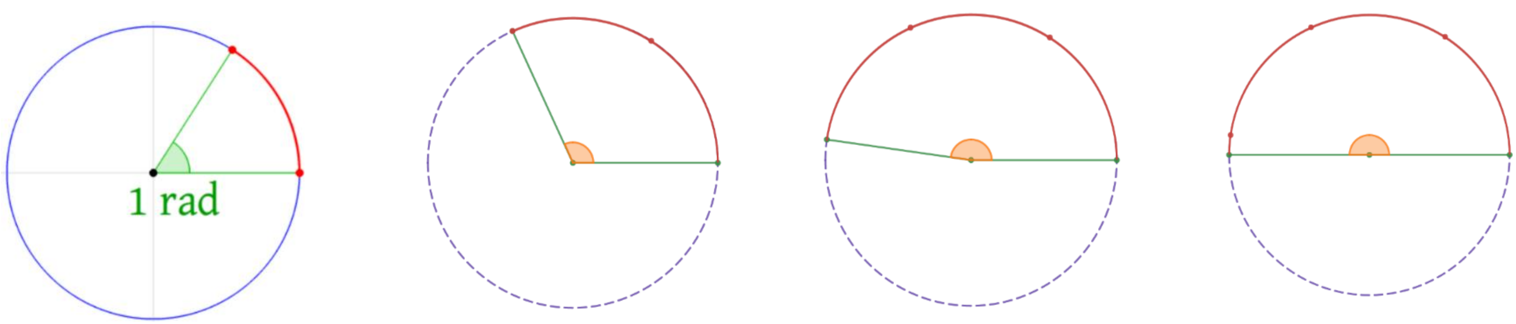
Vocab & Notation

- Radian
- Co-terminal
- Special Triangle
- Unit Circle
- Secant, Cosecant, Cotanget

Why is there $360°$ in a full rotation?

You think trigonometry is about triangles, but really it is about circles.

Definition: One **radian** is equal to the angle made when the arc of a circle is equal to the radius. In general, it is the ratio of the arc to the radius.



$$\text{Angle (radians)} = \frac{\text{Arc Length}}{\text{radius}} \quad \Rightarrow \quad \theta = \frac{a}{r}$$

Where θ is the angle in radians. This may look like a formula, but it is the definition of the radian. This is similar to how π is defined as the ratio between the circumference and the diameter of a circle.

If we go all around the circle, then:

Example:

$\frac{\pi}{6}$ $\frac{\pi}{4}$

$\frac{\pi}{2}$ $\frac{\pi}{3}$

$100°$ $250°$

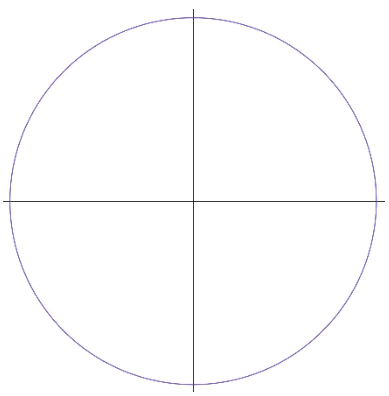
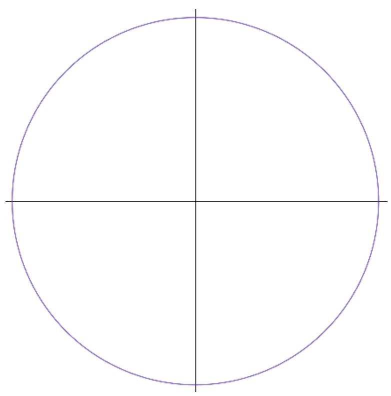
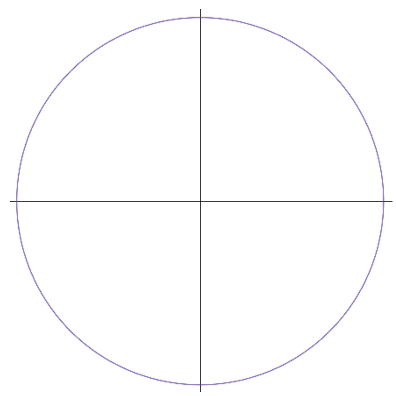
**Recall that positive angles move counter-clockwise around the circle, and negative angles move clockwise.

Definition: Angles are considered **co-terminal** if they have the same terminal arms.

Any multiple of $360°$ or 2π will wrap around back to the same terminal arm so we say the **general form** (which represents all the possible solutions) as

$$\theta + 2\pi n, \qquad n \in \mathbb{Z}$$

Examples:

Angle	General Form	Domain	Coterminal Angles
$\frac{19\pi}{4}$		$-2\pi \leq \theta < 4\pi$	
$\frac{11\pi}{3}$		$-3\pi \leq \theta < \pi$	
$\frac{-17\pi}{6}$		$0 \leq \theta < 8\pi$	

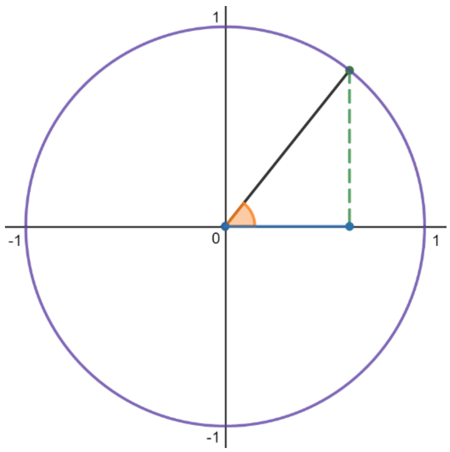
When defining the angle around a circle, it is useful to think of that circle on a grid centered at the origin. Such a circle is defined by the equation:

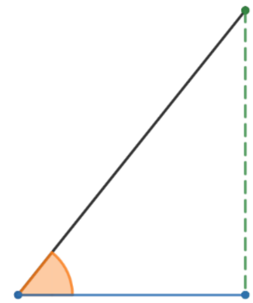
$$x^2 + y^2 = r^2$$

Where r is the radius of the circle.

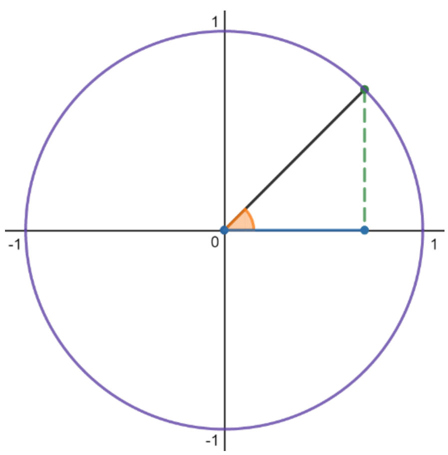
As the angle does not change as the radius changes the **unit circle** is the circle with radius 1, centered about the origin.

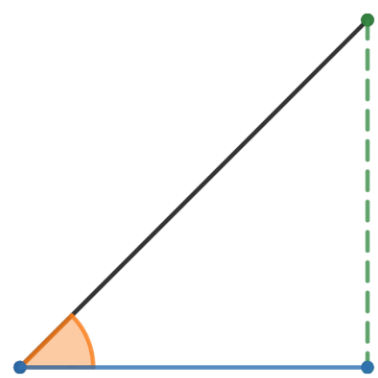
$$x^2 + y^2 = 1$$

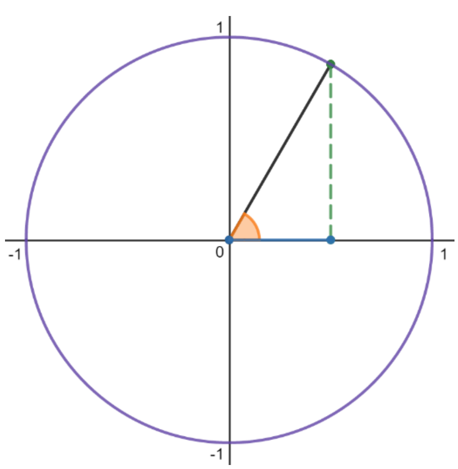


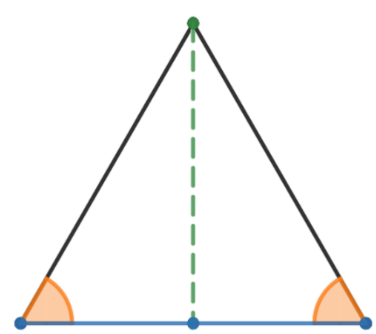


There are a few special angles notice that we build an isosceles right-angle triangle.









Angle, θ	$\sin\theta$	$\csc\theta$	$\cos\theta$	$\sec\theta$	$\tan\theta$	$\cot\theta$
0						
$\frac{\pi}{6}$						
$\frac{\pi}{3}$						
$\frac{\pi}{2}$						
$\frac{3\pi}{4}$						
π						
3.5						
4						
4.5						
5						
6						
9						

Practice Problems: 4.1 page 175 – 176 # 1-13
4.3 page 200 – 203 # 1-6, 9, 12-14, 16, 17