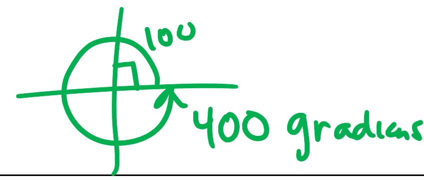
# The Radian and Angles

| KNOW | DO | UNDERSTAND |
|---|---|---|
| How to recognize angles in radians.<br>What quadrant an angle is in. | Determine coterminal angles to a given angle.<br>Determine the trig ratios of an angle.<br>How to use the unit circle and special triangles | *Function Characteristics*:<br>Why the $(x, y)$ coordinate on the unit circle is $(\cos\theta, \sin\theta)$ |

**Vocab & Notation**

- Radian
- Co-terminal
- Special Triangle
- Unit Circle
- Secant, Cosecant, Cotanget

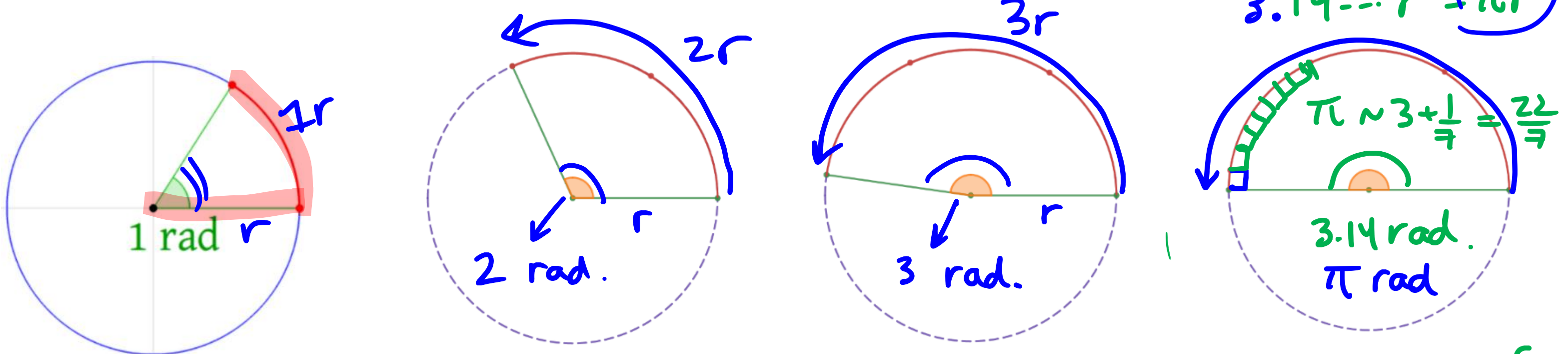



Why is there 360° in a full rotation? 360 ~365.25...

360 has a lot of factors ⇒ Arbitrary

You think trigonometry is about triangles, but really it is about circles.

**Definition:** One **radian** is equal to the angle made when the arc of a circle is equal to the radius. In general it is the ratio of the arc to the radius.




$C = 2r\pi \qquad \pi = \frac{C}{d}$

$$\text{Angle (radians)} = \frac{\text{Arc Length}}{\text{radius}} \Rightarrow \boxed{\theta = \frac{a}{r}}$$

Where $\theta$ is the angle in radians. This may look like a formula, but it is the definition of the radian. This is similar to how π is defined as the ratio between the circumference and the diameter of a circle.

If we go all around the circle, then:

no units

$$\text{arc} = 2\pi r \Rightarrow \theta = \frac{2\pi r}{r} = 2\pi \text{ rad.} = 360^\circ$$

$$\pi = 180^\circ$$

**Example**:

$\frac{\pi}{6} \cancel{rad} \times \frac{360^\circ}{2\pi \cancel{rad}} = 30^\circ$

$\frac{\pi}{4} = \frac{1}{2} \cdot \frac{\pi}{2} = 45^\circ$

$\frac{\pi}{2} = 3\frac{\pi}{6} = 90^\circ$

$\frac{\pi}{3} = 2 \cdot \frac{\pi}{6} = 60^\circ$

$10\cancel{0}^\circ \times \frac{2\pi}{36\cancel{0}^\circ} = \frac{5}{9}\pi = 0.56\pi$

$25\cancel{0}^\circ \times \frac{2\pi}{36\cancel{0}} = \frac{25}{18}\pi = 1.4\pi$

**Recall that positive angles move counter-clockwise around the circle, and negative angles move clockwise.

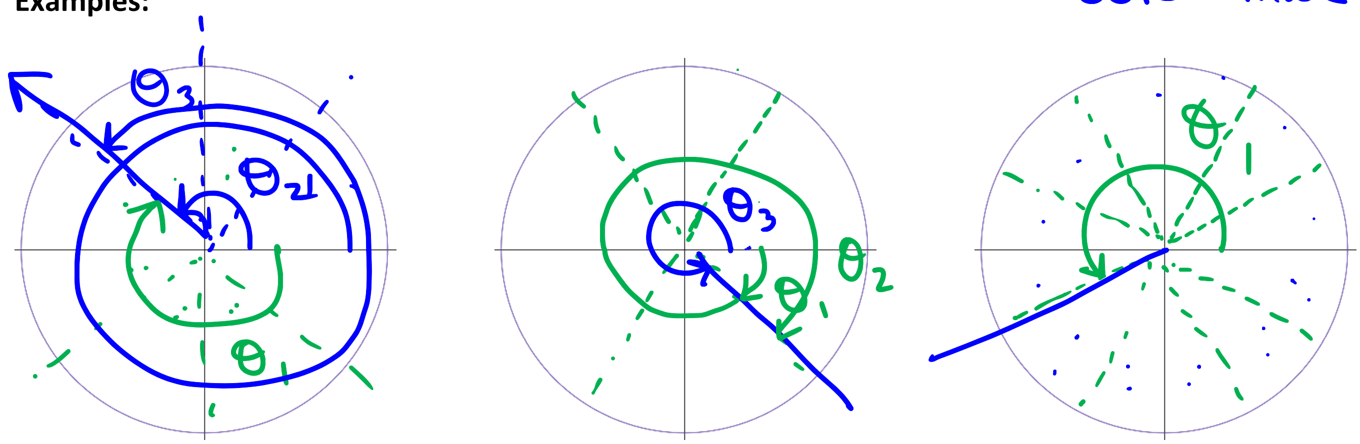
**Definition**: Angles are considered **co-terminal** if they have the same terminal arms.

Any multiple of 360° or 2π will wrap around back to the same terminal arm so we say the **general form** (which represents all the possible solutions) as

$$\theta + 2\pi n, \quad n \in \mathbb{Z}$$

∞ many coterminal angles

**Examples:**




$\frac{19}{4} = 19$ quarters size

| Angle | General Form | Domain | Coterminal Angles |
|---|---|---|---|
| $\frac{19\pi}{4}$ | $\frac{19\pi}{4} + 2\pi n,\ n \in \mathbb{Z}$ | $-2\pi \le \theta < 4\pi$ | $\theta_1 = -\frac{5}{4}\pi$, $\theta_2 = \frac{3}{4}\pi$, $\theta_3 = \frac{11\pi}{4}$ (+ $\frac{8}{4}\pi$, + $\frac{8\pi}{4}$) |
| $\frac{11\pi}{3}$ | $\frac{11\pi}{3} + 2\pi n,\ n \in \mathbb{Z}$ | $-3\pi \le \theta < 3\pi$ | $\theta_1 = -\frac{\pi}{3}$, $\theta_2 = -\frac{7\pi}{3}$, $\theta_3 = \frac{5\pi}{3}$ ($-\frac{6\pi}{3}$) |
| $-\frac{17\pi}{6}$ | $-\frac{17\pi}{6} + 2\pi n,\ n \in \mathbb{Z}$ | $0 \le \theta < \frac{8\pi}{4}$ | $\theta_1 = \frac{7\pi}{6}$, $\theta_2 = \frac{19\pi}{6}$, $\theta_3 = \frac{31\pi}{6}$, $\theta_4 = \frac{43}{6}\pi$ |

When defining the angle around a circle, it is useful to think of that circle on a grid centered at the origin. Such a circle is defined by the equation:

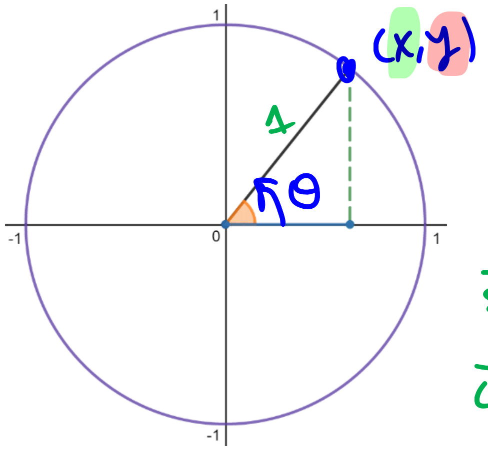
$$x^2 + y^2 = r^2$$

Where $r$ is the radius of the circle.

As the angle does not change as the radius changes the **unit circle** is the circle with radius 1, centered about the origin.

$(\cos\theta, \sin\theta)$

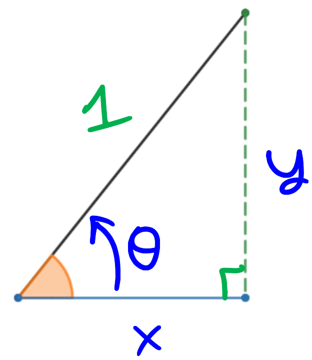
$$x^2 + y^2 = 1$$




$\sin\theta = y$

$\cos\theta = x$

$\tan\theta = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$

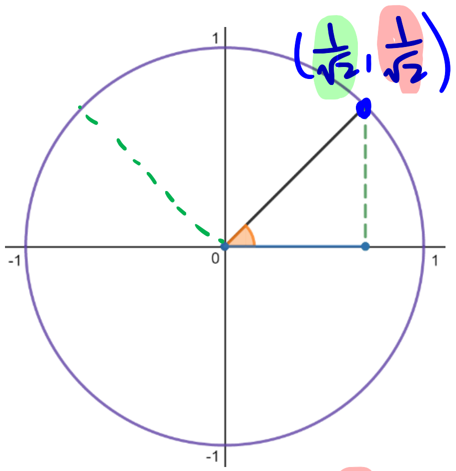



$\frac{1}{\sin\theta} = \csc\theta = \frac{1}{y}$ (cosecant)

$\frac{1}{\cos\theta} = \sec\theta = \frac{1}{x}$ (secant)

$\frac{1}{\tan\theta} = \cot\theta = x/y$ (cotangent)

There are a few special angles notice that we build an isosceles right-angle triangle.

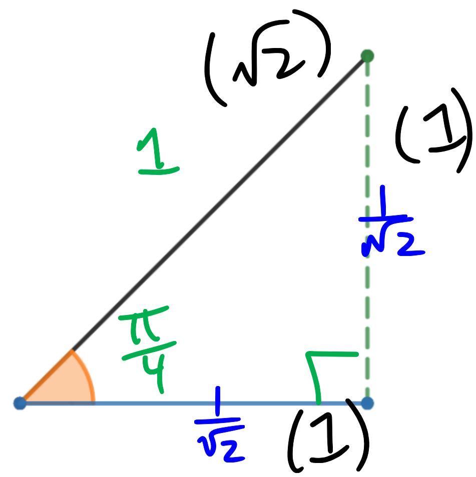


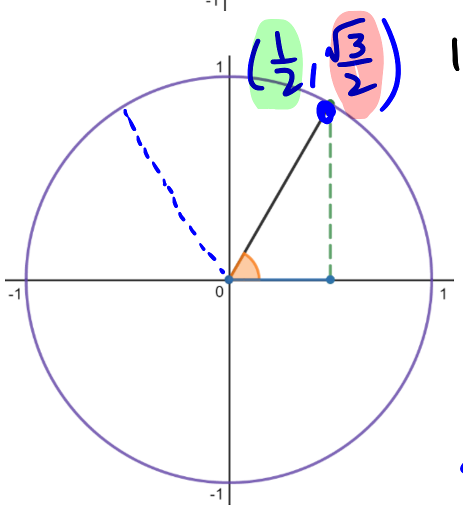

$a^2 + a^2 = 1 \rightarrow a = \frac{1}{\sqrt{2}}$

$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\tan\frac{\pi}{4} = 1$ $\cot\frac{\pi}{4} = 1$

$\csc\frac{\pi}{4} = \sqrt{2}$ $\sec\frac{\pi}{4} = \sqrt{2}$



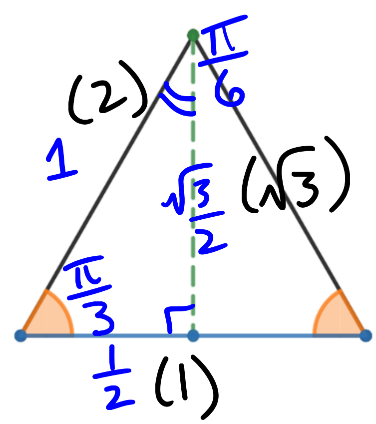

$1/\sin(\pi/4) \longrightarrow \sin(\pi/4)$ enter $x^{-1}$

$1 = b^2 + \frac{1}{4} \rightarrow b = \sqrt{3}/2$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\cos\frac{\pi}{3} = \frac{1}{2}$

$\tan\frac{\pi}{3} = \sqrt{3}$

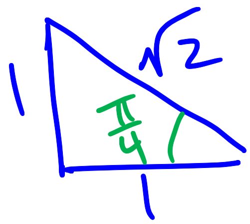
$\sin\frac{\pi}{6} = \frac{1}{2}$ $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

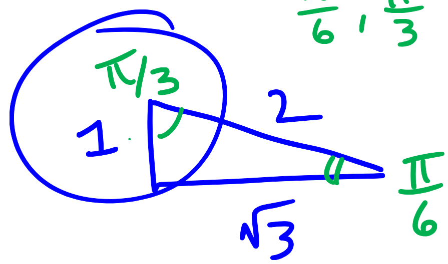


$1 - 1 - \sqrt{2}$

$1 - 2 - \sqrt{3}$

$\frac{\pi}{6}, \frac{\pi}{3}$

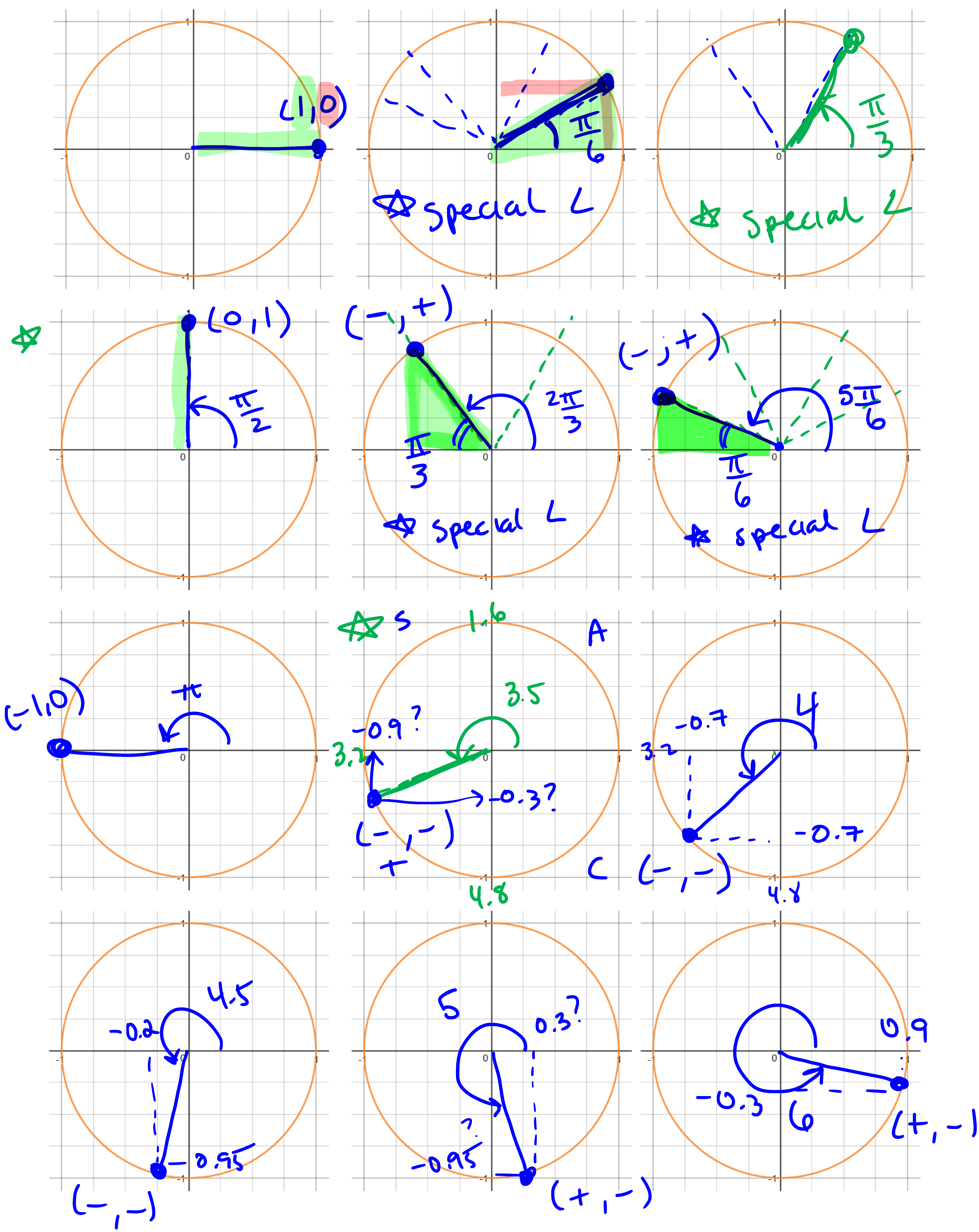



| Angle, $\theta$ | $\sin\theta$ | $\csc\theta$ $\left(\frac{1}{\sin\theta}\right)$ | $\cos\theta$ | $\sec\theta$ $\left(\frac{1}{\cos\theta}\right)$ | $\tan\theta$ $\left(\frac{\sin\theta}{\cos\theta}\right)$ | $\cot\theta$ $\left(\frac{\cos\theta}{\sin\theta}\right)$ |
|---|---|---|---|---|---|---|
| 0 | 0 | undef. asym. | 1 | 1 | 0 | undef. |
| $\frac{\pi}{6}$ | $1/2$ | 2 | $\sqrt{3}/2$ | $2/\sqrt{3}$ | $\frac{1}{\sqrt{3}}$ | $\sqrt{3}$ |
| $\frac{\pi}{3}$ | $\sqrt{3}/2$ | $2/\sqrt{3}$ | $1/2$ | 2 | $\sqrt{3}$ | $\frac{1}{\sqrt{3}}$ |
| $\frac{\pi}{2}$ | 1 | 1 | 0 | undef | undef | 0 |
| $\frac{2\pi}{3}$ | $+\sqrt{3}/2$ | $2/\sqrt{3}$ | $-1/2$ | $-2$ | $-\sqrt{3}$ | $-\frac{1}{\sqrt{3}}$ |
| $\frac{5\pi}{6}$ | $\frac{1}{2}$ | 2 | $-\sqrt{3}/2$ | $-2/\sqrt{3}$ | $-1/\sqrt{3}$ | $-\sqrt{3}$ |
| $\pi$ | 0 | undef. | $-1$ | $-1$ | 0 | undef |
| 3.5 | $-0.35$ | $-2.85$ | $-0.94$ | $-1.07$ | 0.37 | 2.67 |
| 4 | $-0.76$ | $-1.32$ | $-0.65$ | $-1.53$ | 1.16 | 0.86 |
| 4.5 | $-0.98$ | $-1.02$ | $-0.21$ | $-4.74$ | 4.64 | 0.21 |
| 5 | $-0.96$ | $-1.04$ | 0.28 | 3.52 | $-3.38$ | $-0.30$ |
| 6 | $-0.28$ | $-3.58$ | 0.96 | 1.04 | $-0.29$ | $-3.44$ |

★ $\cos\theta = x$ $\sin\theta = y$





**Practice Problems**: 4.1 page 175 – 176 # 1-13
4.3 page 200 – 203 # 1-6, 9, 12-14, 16, 17